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**PRE-SERVICE PRIMARY TEACHERS' UNDERSTANDINGS OF  
MATHEMATICAL PROBLEM POSING AND PROBLEM SOLVING:  
EXPLORING THE IMPACT OF A STUDY INTERVENTION**

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**Submitted to Mary Immaculate College,**

**University of Limerick,**

**June 10<sup>th</sup> 2016**

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Maura Walsh

Abstract.

The results of national assessments – National Assessment in Mathematics and English Reading (Shiel, Kavanagh and Millar, 2014) and international assessments- Programme for International Student Assessment (2011) and Trends in Mathematical and Science Studies (2012) reveal the need for improvement in the Mathematical skills of “Applying and problem solving” and “Reasoning” as laid out in the Revised Primary Mathematics Curriculum (NCCA 1999).

Many studies have emphasised the valuable role problem solving plays in the classroom. Central to this is the quality of the problems posed. The primary aim of this study was to explore the effect of a study intervention on the problem posing skills of pre-service Primary teachers.

The study intervention took a pre-test/post-test format. A questionnaire exploring participants’ knowledge of and attitudes towards problem posing and problem solving was administered to First Year Bachelor of Education students in the Second Semester of their Four Year teacher education course. The study intervention, comprised of a series of lectures and tutorial sessions on problem solving/problem posing, then followed. The original questionnaire was again administered.

The data were analysed and pre-test/ post-test changes evaluated. This evaluation revealed that the students’ conception of what constituted a mathematical problem had greatly improved as shown by the variety and quality of the post-intervention posed problems.

The study recommendations outline the need for the inclusion of problem solving/problem posing modules in initial teacher education courses.

*“The formulation of a problem is often more essential  
than its solution which may be merely a matter of  
mathematical or experimental skills.”*

*Albert Einstein*

## Declaration

I hereby declare that this project is entirely my own work other than the counsel of my Supervisor and that it has not been submitted for any academic award, or part thereof, at this or any other educational establishment.

Signed: \_\_\_\_\_

Date: \_\_\_\_\_

## Acknowledgement

I wish to acknowledge Dr. Aisling Leavy for her unfailing help, guidance, support, positivity and encouragement throughout the writing of this thesis. She steered me through the research and writing up of the dissertation and constantly encouraged me to persevere. Special thanks for your understanding when I came up against the “wall”.

A special mention to Dr. Mairéad Hourigan for all her words of encouragement. “A friend in need is a friend indeed” Thank you sincerely for all your help-above and beyond the call of duty.

Thank you to all the Mary Immaculate student teachers who participated so willingly in this study. Your enthusiasm was inspiring.

To Karen in the IT Department, Mary Immaculate College, thank you for all your help with the technology side of things. Mysterious stuff!

To the staff of the Graduate School, Mary Immaculate College, thank you all for your help.

To the staff of Mary Immaculate Library, thank you for all your help in locating various books and journals.

Finally, to my friend Antoinette, thank you for your somewhat bemused support for me as I worked through the thesis.

## **Dedication**

I wish to dedicate this dissertation to my husband Ignatius

and our children Rosaleen, Liam and Elaine.

Thank you for always believing in me.

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## **List of Abbreviations**

CBMS	The Conference Board of the Mathematical Sciences.
DES	Department of Education and Skills.
ITE	Initial Teacher Education.
NAMER	National Assessment in Mathematics and English Reading.
NCCA	National Council for Curriculum and Assessment.
NCTM	National Council of Teachers of Mathematics.
OECD	Organisation for Economic Co-operation and Development.
PISA	Programme for International Student Assessment.
PSMC	Primary School Mathematics Curriculum
QUASAR	Quantitative Understanding Amplifying Student Achievement and Reasoning.
TIMSS	Trends in International Mathematics and Science Studies.

## **Chapter One**

### **Introduction**

#### **1.1 What is Mathematics?**

The Irish Primary School Mathematics Curriculum (NCCA, 1999) defines mathematics as:

“The science of magnitude, number, shape, space and their relationship and also is a universal language based on symbols and diagrams. It involves the handling (arrangement, analysis, manipulation and communication) of information, the making of predictions and the solving of problems through the use of language that is both concise and accurate.”

National Council for Curriculum and Assessment (NCCA, 1999, p.2)

This very broad definition specifically mentions problem solving, and by association, problem posing which is the focus of this study. This points to the importance of this area of research in mathematics. Recent international comparative studies in mathematics education also highlight problem solving as deserving further attention in the Irish mathematics education field.

#### **1.2 National and International Assessments in Mathematics**

The Trends in International Mathematics and Science Study (TIMSS) is an international comparative study in mathematics and science instituted in 1995. This study takes place every four years and is offered to students in fourth and eighth grade. TIMSS is designed to align broadly with the mathematics and science curricula of the participating countries. The results therefore suggest the degree to which students have learned the mathematics and science concepts and skills likely to have been taught in their schools.

Ireland participated in TIMSS in 1995 at both grade levels and in TIMSS in 2011 at Fourth Class level. Most recently in 2015, Ireland participated in TIMSS at both class levels, the results will be released in December 2016. The results of the 2011 survey show that Irish Fourth Class students were ranked average coming in at 17<sup>th</sup> out of 47 participating countries.

However, only 9% of Irish students performed at the advanced benchmark compared with, for instance, 24% of students in Northern Ireland. At the other end of the scale 23% of students performed at or below the low benchmark (Close, 2013). This is cause for concern.

The Programme for International Student Assessment (PISA) is a triennial international survey, instigated in 2000, which aims to evaluate education systems worldwide by testing skills and knowledge of 15 year old students. PISA is unique because it develops tests which are not directly linked to the school curriculum. The tests are designed to assess to what extent students can apply their knowledge to real-life situations and be fully equipped to participate in society. The tests are a mixture of open-ended and multiple choice questions that are organised in groups based on a passage setting out a real-life situation. Countries who participate in successive surveys can compare their students' performance over time and assess the impact of education policy decisions. Schools in the participating countries are randomly selected.

Ireland has taken part in the six cycles of PISA assessments- 2000, 2003, 2006, 2009 2012 and 2015. As mentioned earlier, PISA has a 'real-life' approach to mathematics. Central to the PISA mathematics framework is the idea of "mathematisation" which involves starting with a problem, in a real world context, identifying the mathematics relevant to solving the problem, re-organising the problem according to the mathematics concept identified, gradually trimming away the reality so that the problem can be solved and making sense of the mathematics solution in terms of the real solution.

PISA 2003 saw Ireland achieve an overall mean score in mathematics that was not significantly different from the OECD average-achieving an overall ranking of 17<sup>th</sup> among 29 OECD countries and a mean score that was well below the OECD average in Shape and Space. PISA 2012 was the first PISA cycle in which Irish students performed above the OECD average in mathematics. In this cycle of PISA (2012), Irish 15 year olds achieved a mean score on print mathematics significantly higher than the average for OECD countries ranking 13<sup>th</sup> out of 34

OECD countries and 20<sup>th</sup> out of 65 participating countries/ economies. 11% performed at Level 5- the highest level- compared with 13% on average across OECD countries. This represents an improvement on 2009 results in which Ireland demonstrated a remarkable plunge down the rankings but was not significantly different from 2003 and 2006 results. Again results in Shape and Space were below average. This content area includes aspects of geometry, problem solving and spatial reasoning. (Perkins, Shiel, Merriman, Cosgrove, Moran, 2013)

The poor results of PISA 2009 caused considerable concern and resulted in the Department of Education and Skills (DES) drawing up the National Strategy to improve Literacy and Numeracy in Children and Young People. This strategy was rolled out to schools in 2011 and set targets to be achieved by 2020. Reforms outlined in this document include revised curricula in English and mathematics for primary schools, new approaches to assessment, lengthening the duration of initial teacher education degree courses with increased focus on Literacy and Numeracy. These recommendations have all been introduced and are in process in primary schools since 2011 and with the introduction of the four year B.Ed. programme in Colleges of Education in 2012. (DES, 2011)

The National Assessment of Mathematics and English Reading (NAMER) has been in place for several decades and assesses the achievements of primary school pupils on behalf of the DES. When the NAMER report of 2009 results was published much concern was expressed when it was revealed that there hadn't been an appreciable rise in standards since the 1980's in spite of the introduction of the Revised Primary School Curriculum in 1999. Earlier national assessments covered a range of grade/age levels, the DES decided that from 2014 they would be implemented in Second and Sixth Class only and would include both English Reading and Mathematics. In spring 2014, 8000 pupils in a representative sample of 150 schools completed secure tests of English reading and Mathematics and responded to a questionnaire while their principals, teachers and parents also completed a questionnaire. The poor results of PISA 2009 and the findings of NAMER 2009 led to the National Strategy to Improve Literacy and

Numeracy in Children and Young People. Therefore, there was much interest in the NAMER 2014 data. Did the National Strategy bear any benefits?

The NAMER 2014 data showed a great improvement in reading standards. However, the same could not be said for standards in mathematics. In the National Assessment 2014 pupils in both Second Class and Sixth Class showed some improvement from the 2009 results across all areas of mathematics. However, both classes showed a drop in performance in the mathematical skill of 'Apply and Problem Solve' more marked than in any of the other four mathematical processes. (Shiel, Kavanagh and Millar, 2014) In the Performance Report it is stated that:

“Scope for further substantial progress is probably greater in mathematics than in reading. For example there seems to be scope for further growth in problem solving in mathematics”

(Shiel, Kavanagh and Millar, 2014, p. 68)

The findings of these three reports (TIMSS 2011, PISA 2012, NAMER 2014) provide support and impetus for this study. It appears that one route to bringing about improvement in Irish pupils' performance in problem solving is through improving the skills and understandings of their teachers. The primary pre-service teachers enrolled in this College of Education (ITE provider for primary teachers) provide a perfect opportunity to investigate the attitudes and aptitudes of future teachers in relation to problem solving. This study aims to shed light on the ways in which the design of experiences in mathematics education may bridge the gap between knowledge and practice in problem solving and problem posing. This study sets out to examine the ways in which an intervention in the form of lectures and tutorials changes the way pre-service primary teachers think about and understand problem solving and problem posing. Can intervention improve these student teachers' understanding of what constitutes a problem? Can it improve the quality of the problems the student teachers themselves pose? These are the questions and considerations at the heart of this study.

### **1.3 Aims and objectives of this study**

In undertaking this research project my aim is to examine the gap between theory and practice as it presents for pre-service primary teachers in Mary Immaculate College in the area of mathematical problem posing. My specific objectives are:

- To examine entry level pre-service primary teachers' understandings of what constitutes a mathematical problem and their ability to pose a mathematical problem.
- To identify the effects of an intervention focusing on problem posing on pre-service primary teachers understanding of what constitutes a mathematics problem and their ability to pose a mathematics problem.

### **1.4 Outline of Study**

Much work has been carried out in recent years on problem solving in the primary classroom. However, the focus is now changing to the study of problem posing. After all, as they say: "a good question is half the answer." All pre-service teachers' experience of mathematical problems has come from their own primary or secondary school days. The problems they have come across have mostly been from textbooks. These problems were usually word problems, require one step to arrive at a solution and have only one correct answer. (Lave, 1992; Ny, 2002) The problems were set by their teachers to be 'answered'. These classroom practices are long established and are, therefore, hard to change. However, problem posing; both as an act of mathematical enquiry and of mathematics teaching; is a component part of mathematics education that seeks to promote mathematics as a worthy intellectual activity (Crespo and Sinclair 2008). Therefore, if we hope to develop pre-service primary teachers' understandings of what constitutes a mathematical problem, we must provide them with the opportunity to experience new and different sorts of problems themselves as mathematics students.

Research has shown that because of their formative experience of problems, student teachers when asked to write mathematical problems will construct one-step word problems using the usual format and phrases with which they are familiar and comfortable. These word problems require quick, accurate, one-correct-solutions and tend to be narrow and restrictive in their focus (Gonzales, 1994; Silver and Cai, 1996; English, 1998; Crespo, 2003). Furthermore, research also reveals that students are able to write better problems when they have had experience of solving more open-ended problems, when they have been invited to pose problems outside of the classroom and when they have been prompted by informal contexts such as pictures. Thus, the outcomes of research suggest that if we hope to change pre-service primary teachers' approaches to and understanding of problem posing we must expose them to many varied types of problems. We must provide them with the opportunity to construct problems and in doing so hope that they will take this knowledge into their classrooms. Indeed, Schoenfeld noted, in 1985, that problems provide students with opportunities to do and learn mathematics and they convey messages about the nature of the discipline, what it entails and what is worth knowing and doing. (Schoenfeld, 1985) This is supported by recent work from Crespo which indicates that the way teachers go about choosing and discarding which problems to bring to their classrooms carries a lot of weight in opening or closing learning opportunities for their students. (Crespo, 2003) Therefore, it follows that teachers need some benchmark in deciding what makes some problems better than others.

Some guidance has been offered to pre-service teachers regarding the various types of problems that they present to their students. The Revised Primary Mathematics Curriculum (1999) outlines seven different types of problems: Word problems, practical tasks, open-ended investigations, puzzles, games, projects and mathematical trails (NCCA, 1999).

Examining this list in itself may be a very useful exercise in opening pre-service primary teachers' eyes to what exactly a mathematical problem is. They are not just the one step word

problems with which they are familiar. It may encourage the pre-service primary teachers to approach problem posing in a more open and exciting manner.

I believe it is vital to mathematics education that the teachers of the future and indeed the present are provided with opportunities to gain insights and generate rich understandings of what constitutes a good mathematical problem. This ensures better outcomes for our pupils and therefore a better outcome for our society. The magic and beauty of mathematics is unlocked by the exploration of good mathematical problems so, to do our pupils justice, our teacher education courses must address this issue.

## **1.5 Methodology**

The participants in this study were a group of first year pre-service primary teachers in an initial teacher education programme in a College of Education (ITE provider) in Ireland. These students were in the second semester of the first year of their four year degree programme and have started their second school placement since the start of semester two.

The study was a pre-test/post-test design. Participants were asked to complete a questionnaire probing their understanding of problem solving and their ability to pose mathematical problems (See Appendix A). Participants' replies to this questionnaire were analysed and categorised according to the various responses. Participants were then engaged in the intervention involving input from lecturers and tutors on the topic of problem solving and problem posing. Following on from the intervention, participants were presented with the post-test. Their responses were analysed and coded using the original coding system. The differences in the participants' responses will form the basis of this study.

## **1.6 Structure of the Thesis**

This study will be presented using the following headings: literature review, methodology, results and conclusion/recommendations.

In Chapter 2, the literature review, I will provide the reader with insights into previous research undertaken in the fields of problem solving and problem posing. This chapter outlines the findings of many researchers on what problem solving is and what strategies have been found to improve this area in teaching and learning. The question of what constitutes a good problem is also addressed in an effort to ascertain the best way forward in achieving confident problem posing in the classroom as found in the research.

Chapter 3 outlines the methodology undertaken in conducting this research. It provides an overview of the questionnaire administered to the pre-service primary teachers and the intervention used. It presents the background of the participants, describes the tools and processes used in analysis of the results and describes the steps taken to improve the reliability and validity of the study.

The results obtained from the various data sources are described in Chapter 4. The majority of the data collected for this study was qualitative. The data was hand-coded with common themes being identified across the data.

Chapter 5 presents the main findings of the study. The conclusions to be drawn from the study will be outlined and recommendations for further study will be made as well as how we can further improve problem posing in Irish classrooms.

## **1.7 Conclusion**

I undertook this study to examine what pre-service primary teachers understanding of what a good mathematical problem is and to ascertain if intervention could improve their understanding of this aspect of mathematics teaching. I hope to show from the existing research and from the results of this study that this is an intervention that bears fruitful results.

## Chapter Two

### Problem Posing- A Literature Review

#### 2. 1. Problem Solving

Although the main focus of this study is problem posing, it is necessary to research some work on problem solving as these topics are clearly closely related.

##### 2. 1.1. Why use a Problem Solving Approach in Mathematics Education?

Problem solving is considered central to school mathematics. It is deemed to be a key factor of change in mathematics education. This opinion is supported by many national curricula, for example, within the context of the United States, the National Council of Teachers of Mathematics (NCTM) states:

“Instructional programs should enable all students to build new mathematics knowledge through problem solving, solve problems that arise in mathematics and other contexts; apply and adapt a variety of appropriate strategies to solve problems and monitor and reflect on the progress of mathematical problem solving.”

(NCTM, 2000, p. 52)

Similarly, in the Mathematics Primary School Curriculum, the Irish Government, Department of Education and Skills, states:

“Developing the ability to solve problems is an important factor in the study of mathematics. Problem-solving also provides a context in which concepts and skills can be learned and in which discussion and co-operative working may be practised. Moreover, problem solving is a major means of developing higher-order thinking skills.”

(NCCA, 1999, p. 52)

Goldin also reported, in 1997, that mathematics education has evolved to stress conceptual understanding, higher level problem-solving processes and children's internal constructions of mathematical meanings in place of, or in addition to, procedural and algorithmic learning. (Goldin, 1997)

Studies in almost every area of mathematics have demonstrated that problem solving provides an important context in which students can learn about number and other mathematical topics. (Kilpatrick, Swafford and Findell, 2001) Problem solving can also provide the site for learning new concepts and for practicing learned skills.

### **2.1.2. What is a Mathematical Problem and what is Mathematical Problem-solving?**

*What is a mathematical problem?* The Oxford English School Dictionary defines a problem as “something that is difficult to deal with or understand.” (Oxford English School Dictionary, 2013) In the world of mathematics, while some people construe problems as routine exercises for the consolidation of newly learned mathematical techniques, others view them as tasks whose complexity makes them problematic or non-routine. (Schoenfeld, 1985) More recently, Van de Walle states that problems have no clear solution method. (Van de Walle, 2003)

The eminent mathematician George Polya asserts that solving a problem is finding a way out of a difficulty, a way around an obstacle or attaining an aim which was not immediately attainable. Polya outlines two types of problems:

1. Problems to find; in which we are asked to construct, to obtain, to identify, what is the unknown? e.g. What did he say?
2. Problems to prove; in which we are asked is this true or false, what is the conclusion. e.g. Did he say that? (Polya, 1973)

These two types of problems require different approaches from the problem solver. Another way to conceptualise problems is those that are purely mathematical or those that are applied. Blum and Niss (1991), Chapman (2008) and Xenofontos (2014) all identify this delineation as being present in contemporary international assessments of mathematics. The TIMSS (2011) test items concentrate on purely mathematical problems whereas PISA (2012) test items concentrate on applied mathematical problems. The latter test has a real-world focus in which relevant data is imbedded in the text and commonly take the form of word or story problems.

*Mathematical problems in the classroom context.* In the classroom, problem solving can be seen in a number of ways. Firstly, problem solving can be seen as a process in which the use of Polya-style heuristics (See Section 2.1.4.) encourage the use of higher-order thinking skills that guide the search for a solution and enable the problem-solver to select from a set of alternatives and order their solution process in a sequence of steps. Some studies support this heuristic-guided approach, notably Verschaffle, De Corte and Borghart (1996), and Hensberry and Jacobee (2012). However, this approach is disputed by Sweller, Clark and Kirschner, (2010) who argue that no systematic body of evidence has emerged that provides support for the effect of any general problem-solving strategies.

An alternative approach, which could be described as teaching about problem solving focuses on repeated worked examples, a process which encourages the recognition of similar problem types and therefore, analogous reasoning is supported by the research of Andrescu (2008). Marshall (1995) states that if adopting this approach, students are taught well-defined approaches to a particular problem and given regular opportunities to practice them.

Problem solving can also be seen as a curricular goal in the classroom. This approach is common when a teacher wishes to satisfy the curricular guidelines laid down by the jurisdiction in which he/she works.

Problem solving may also take the form of an instructional approach in situations when a teacher uses a problem-based approach to teach mathematics in their classroom. Problems are posed to structure pupil learning of mathematical content and this new content is related to prior mathematical knowledge. In these situations, problem solving is to the forefront of the teacher's mind.

Of course, what teachers understand or believe mathematical problems and mathematical problem-solving to be is crucial in the context of what is taught, how it gets taught and what gets learnt in our classrooms. Ernest (1989), Thompson (1984) Chapman (2002) and Aguirre and Speer (1999) all assert that the conceptions, personal ideologies, world views and values that shape practice and orient knowledge of a teacher impact hugely on their classroom approach. It therefore follows that teachers need to be very sure of what constitutes a mathematical problem and what mathematical problem solving entails. As already stated most mathematics curricula give problem solving centre stage as a very important goal in mathematics education. (See Section 2.1.1.) However, it is not merely enough to state a goal without making that goal explicit. This may lead to misunderstanding and perpetuate teacher confusion of what exactly this area of mathematics education involves. Xenofontos (2014) in research with teachers in Cyprus and the United Kingdom, found that little information seems to have been provided to teachers with respect to what constitutes a mathematical problem and how problem solving is systematically perceived.

### **2.1.3. The Education of Pre-service Teachers in Problem Solving Skills**

If a problem solving approach is being promoted, then the next question is how can we ensure that problem solving is given its proper place in mathematics education? There is consensus that the key to better problem solving in our schools is better teacher preparation in this very vital area. Schoenfeld (1985) contends that the quality of the pupils' exposure to

problem solving all depends on the teacher's own approach to this very important topic. He identifies four aspects of pupils' problem solving that can be used to give guidance to teachers: resources, heuristics, control and beliefs. Teachers must play a central role in helping their pupils choose resources, implement heuristics or pathways to solutions, control their problem solving actions and develop useful beliefs about mathematics.

The preparation of our teachers to take on this vital role is at the centre of the issue. Pre-service primary teachers must be given the chance to develop an understanding of problem solving from a pedagogical perspective. Their understanding of problem solving and their ability as problem solvers will obviously affect their implementation of problem solving in their classrooms.

A study of in-service teachers by Chapman (2000) which addressed problem solving as mathematical thinking and as a method of instruction, found that this approach was effective in expanding and deepening the in-service teachers' understanding of problems, problem solving and problem solving pedagogy as well as enquiry-based teaching. The participants' thinking shifted from predominantly traditional exercises or word problems to an understanding of what constitutes worthwhile mathematics problems.

In a later study of pre-service secondary school mathematics teachers, Chapman (2005) found that most of the participants had a limited view of problem solving, associating problems with traditional routine problems they themselves had experienced prior to entering the teacher education programme. This study indicates that teachers need help in the development of their understanding of problem solving from the perspective of the learner and the teacher.

The goal of Roddick, Becker and Pence's (2000) study was to influence pre-service teachers' problem solving, problem posing, modelling and beliefs about the role of problem solving in teaching mathematics. The participants were furnished with rich and varied problem solving experiences. They spent time on topics such as: what is a problem, problem solving in

traditional and innovative curricula, equity issues in problem solving and assessment and the use of technology. The pre-service teachers reflected on their problem solving and concentrated on specialising, generalising and justifying their work. They also participated in substantial in-class time working in groups on problems and giving presentations and justifications to the class. The effect of the course varied from not much discernible implementation of the new approaches studied to substantial integration of problem solving in their teaching. This case study demonstrates the changes that can occur in beliefs and instruction as a result of an intensive year long course that immerses prospective teachers in being reflective problem solvers themselves.

The importance of supporting the problem solving practices of pre-service teachers is supported in the Recommendations for Elementary Teacher Preparation where they state:

“The first priority of pre-service maths programs must be to help prospective teachers to engage in problem solving with classroom experiences in which THEIR ideas for problem solving are elicited and taken seriously. Their sound reasoning affirmed and their missteps challenged in ways that help them make sense of their errors.”

Conference Board of the Mathematical Sciences (CBMS, 2011, p. 17)

The role of choosing appropriate tasks to further develop reasoning and problem solving skills is an important consideration identified by a number of researchers. Hiebert, Carpenter, Fennema, Fuson, Werne and Murray (1997), Knapp and Pearson (1995), Stigler and Hiebert (1999) in their research found that teachers at all levels, including, mathematics instructors of prospective teachers, need to understand the important role of choosing problematic tasks. These instructors must help pre-service teachers to consciously make reasoning and understanding salient features of learning for their pupils. These findings provide important insights and guidance for those charged with the remit of designing courses for pre-service mathematics teachers. These pre-service teachers must be given full and extensive

recourse to relevant modules on the very important areas of problem posing and solving in mathematics education.

#### **2.1.4 Learning to Teach Mathematical Problem Solving**

George Polya one of the best known writers on the topic of mathematical problem solving states:

“Solving a problem means finding a way out of a difficulty, a way around an obstacle, attaining an aim which was not immediately attainable.”

(Polya, 1973)

Polya considers solving problems to be a skill which can be learned by imitation and practice. He asserts that if you wish to become a problem solver you have to solve problems. He encourages teachers and pre-service teachers to become problem solvers themselves in order to better understand the processes that their students go through when engaging in this topic. The teacher should put themselves in the student’s place and try to understand what is going on in their student’s mind and what difficulties they may have.

Polya’s belief that problem solving is a practical art that can be learned through practice and imitation belief led him to develop four main stages to guide students in their quest to solve problems. These are: Understand the problem, Devise a plan, Carry out the plan, and Look back.

The first stage, ‘Understanding the problem’, is the exploratory stage where the problem is discussed. Questions are asked such as ‘What is the unknown?’ ‘What data are we given?’ ‘How is the unknown connected to the given data?’

The second stage is ‘Devising a plan’. It is necessary to plan when it is known which calculations, computations or construction must be performed in order to obtain the unknown.

The road from understanding the problem to devising a plan may be long and hard. The main achievement in the solution of a problem is to conceive the idea of a plan. This may be a gradual process or a flash of inspiration. Good ideas are based on past experience and formerly acquired knowledge. At this stage, teachers often ask the question ‘Do you know a related problem?’ These questions often start the right train of thought and depend entirely on the teacher understanding the problem and knowing their pupils’ capabilities.

The third stage is ‘Carry out the plan’. This is much easier than devising a plan. The main problem here is helping the students to stick to the plan. The teacher must insist that the student checks each step. Prompt questions may include: Can the pupil SEE the solution? Can the pupil PROVE the solution?

Polya’s problem solving process ends with the fourth and final stage – ‘Looking back’. This is very important and instructive phase of the work but sometimes the most neglected. By looking back at the path to the solution the pupil consolidates their knowledge and develops their ability to solve problems. Constructive questions posed by the teacher at this stage could be: ‘Can you check the result?’ ‘Can you check the argument?’ ‘Can you derive the result differently?’ ‘Can you see it at a glance?’

Polya believed that students who were taught to follow these four steps would become efficient problem solvers in the mathematical sense of the term. Polya also states that a teacher introducing problems to his class should have two aims. The first aim is to help the student to solve the problem at hand, and the second aim is to give the students skills to solve future problems independently. Therefore, common sense and generality are paramount. The students who are given these opportunities will internalise these skills and use them again and again.

Polya asserts that the teacher who wishes to develop his/her students’ ability to solve problems must give them plenty of opportunity for imitation and practice, using appropriate questions. Also, when the teacher models problem solving in class he/she should put the same questions to him/herself so that the students see the process in practice. (Polya, 1975)

These methods and rules of discovery and invention are called heuristics. Polya firmly believed in his four point plan or heuristic for solving problems. However, Schoenfeld (1985) argues that research has shown again and again that heuristic strategies in and of themselves are not sufficient to ensure competent problem-solving. He argues that heuristics are complex, subtle and highly abstract and no substitute for subject matter knowledge. Smith (1973) carried out a study which found that the transfer of heuristic learning was far less than was hoped for, therefore concurring with Schoenfeld's (1985) contention.

There is some agreement in the literature that active discovery-led learning where the pupils are engaged and motivated is the best approach. This is coherently stated by G.C. Lichtenberg in his book *Aphorismen* and quoted by Polya:

“What you have been obliged to discover by yourself leaves a path in your mind which you can use again when the need arises.”

(Polya, 1973)

Leavy and O'Shea (2011) emphasise the importance of children understanding the work presented to them. They state that

“The relationship between understanding and problem solving is symbiotic. The tasks must be accessible to the learners in that they build on knowledge that the learners already have while at the same time engaging and drawing on contexts and situations that are new to them.”

(Leavy and O'Shea, 2011, p.9)

Understanding usually comes with maturation therefore the pupils' stage of development will be the guiding light for teachers.

There can be no doubt but that teaching is a very complex activity influenced by many factors such as the social context in which the teaching takes place, the curriculum, the pupils, the parents and the other teachers in a school. All these factors lean heavily on the mathematics classroom also. However, we must strive to ensure that our teachers are as fully prepared as

possible to impart the best knowledge to their pupils ensuring that the next generation gets the very best that research and study in this field has to offer.

## **2.2. Problem Posing**

### **2.2.1. What is Problem Posing?**

Mathematics in general is a tool for understanding our world, for understanding how society operates and for understanding and discussing science as defined by Abu-Elwan (2009). There can be little doubt but that many of the situations we encounter in everyday life involve problems and how to solve them. These may not always be mathematical in nature but the development of a problem solving attitude is very important as it a very necessary talent in our ever-changing world. Problem solving is emphasised in most national curricula but surely problem posing should be the starting point. The quality of the problems posed, without doubt, will impact greatly on the quality of the problem solving process. How do we improve and/or influence pre-service teachers' problem posing ability? This is the focus of this study.

### **2.2.2. Types of Problem Posing.**

Problem posing according to Duncker (1945) is the generation of a new problem or the reformulation of a given problem. Silver (1994) also defines problem posing as the generating of new problems and questions aimed at exploring a given situation as well as the re-formulation of a problem during the process of solving it. Stoyanova (1996) defined mathematical problem posing as the process by which, on the basis of concrete situations, meaningful mathematical problems are formulated. There are benefits associated with re-formulating existing problems. This strategy is based on the idea that modifying the attributes or demand of a given problem could generate new and intriguing problems. Southwell (1998)

found that posing problems based on given problems could be a valuable strategy for developing problem solving abilities of pre-service teachers. Problem posing skills could be developed by giving students an ill-formulated or a partially formulated problem and asking them to re-state it. Silver, Kilpatrick and Schlesinger (1990) and English (1998) consider generating new problems from given mathematics situations to be the main activity of posing problems. Polya's 'looking back' phase could be used as a vehicle to accommodate this re-formulation idea as this is a time when other aspects of a problem may come to light which may in fact result in another problem (See Section 2.1.4.)

The 'What-if-not' strategy of Brown and Walter (1983) suggested a new approach to problem posing and problem solving in mathematics teaching. This strategy is based on the idea that modifying the attributes of a problem could yield new and intriguing problems which may eventually result in some interesting investigations. In using this strategy three steps are recommended namely, re-examine the problem and make a list of the problem's attributes, address the 'What-if-not' question and then suggest alternatives to the given attributes which results in the pupils posing a new problems inspired by the alternatives. For example, the pupil asks him/herself: "What if the attributes of this question were not so? What would the problem be then?" This approach throws up new problems i.e. problem posing through problem re-formulation.

### **2.2.3. Why is problem posing important-its place in National Curricula.**

The selection and construction of worthwhile mathematical tasks is highlighted in the NCTM Professional Standards (1991) as one of the most important pedagogical decisions a teacher needs to make. These decisions open or close the students' opportunity for meaningful mathematics learning. They convey implicit messages about the nature of mathematics, what it is, what it entails and what is worth doing in mathematics. These tasks, must be chosen

carefully in order to engage pupils' intellect, facilitate multiple entry points as well promoting conceptual understanding and connections. (NCTM,1991)

Problem posing is seen as a critical aspect of the work of teachers both in posing problems for their pupils and also in helping pupils to become better problem posers themselves, (Crespo, 2003; Olson and Knott, 2013) Problems that a teacher poses can shape the mathematical learning in their classroom and “further their maths goals for the class”. (NCTM, 2000, p.53)

Problem posing is not just the domain of the teacher- it is also something that the pupils can and should engage in. NCTM (2000) states that the school curriculum should give the students the opportunity to formulate interesting problems based on a wide variety of situations both within and outside mathematics. Pupils are also recommended to make and investigate mathematical conjectures and learn how to generalise and extend problems by posing follow-up questions. This reflects Brown and Walter's (1983) “What-if-not” technique and Polya's ‘looking back’ heuristic. (See Section 2.2.2.)

The Australian National Curriculum, the National Statement on Mathematics for Australian Schools, states:

“Students should engage in extended mathematical activity which encourages problem posing, divergent thinking, reflection and persistence. They should be expected to pose, and attempt to answer their own mathematical questions”.

(Australian Education Council, 1991, p.39)

The NCTM also states:

“Students in Grades 9-12 should have some experience recognising and formulating their own problems an activity which is at the heart of doing mathematics”

(NCTM, 1989, p.138).

This approach ties in perfectly with the constructivist theory of teaching and learning where pupils, using prior mathematical knowledge, construct their own learning based on what they already know.

Francisco and Maher (2005) explain that pupils must be provided with the opportunity to work on complex tasks as opposed to simple tasks as such tasks are crucial to the development of mathematical reasoning. Bonotto (2013) states that pupils should be provided with opportunities to explore, make conjectures and pose meaningful problems.

In the Irish Primary School Mathematics Curriculum (NCCA, 1999) problem posing does not receive any mention. The emphasis was all on problem solving, but, as will be addressed later, the relationship between these two facets of mathematics education are very inter-related. (See Section 2.2.7) Similarly in the National Strategy introduced in 2011, problem posing does not receive any specific mention.

#### **2.2.4. Are Pre-service Teachers and Teachers Capable of Posing Valuable Mathematical Problems?**

Given the importance that problem posing is assigned in National Curricula, we have to ask are pre-service teachers and teachers capable of posing important mathematical problems? In her 2003 study, Sandra Crespo asserted that learning to pose problems is one of the challenges of learning to teach mathematics. In this study, she found that when pre-service teachers and teachers were asked to extend a mathematics problem they did so in a predictable, undemanding, ill-formulated and unsolvable way. Even when they had access to potentially rich, worthwhile problems they lowered their cognitive demand. Therefore, they did not challenge or extend their pupils' knowledge or skills. (Crespo, 2003) Again, in a later study, Crespo and Sinclair (2008) found that pre-service teachers often posed trivial, non-

mathematical or poor problems due to a lack of opportunity to engage in and explore a problem situation before and during the posing process. Worryingly, Ball (1990) concluded that pre-service teachers' own mathematical understanding was inadequate for teaching elementary and secondary school mathematics. Due to the reform measures of recent years, with the introduction of the National Strategy in Literacy and Numeracy (2011) at primary level and Project Maths (2010) at secondary level, we can only hope that the situation is not as bleak as this presently.

Research carried out by Hourigan and O'Donoghue (2007) and Leavy and Sloane (2010) found that post-primary school graduates perform best at lower order mathematical skills such as memorization of procedures and formulae as opposed to thinking creatively, providing reasons for solutions or engaging in mathematical problem solving. These are the very people who are entering our teacher education programmes. Again the intended curriculum of Project Maths has addressed this issue and we wait to see its results in student teacher intakes in the near future.

Corcoran (2005) carried out a study of 71 pre-service primary teachers to whom she administered 11 items from the PISA 2000 test. Their performance was compared against Irish 15 year olds. Less than 10% got items at Level 6 fully correct. More than 20% got items totally incorrect. This was similar to the outcomes for the 15 year olds. Overall she identified a ceiling proficiency of Level 4 for 80% of the pre-service primary teachers. More than 50% of the participants demonstrated concerning low levels of the process skills identified in the Primary School Mathematics Curriculum (1999) of applying and solving problems. They also scored poorly in communicating and expressing skills with formal reasoning. Chapman (2005) found that pre-service teachers had a limited view of what constituted a problem based on their own experience of traditional, routine problems. Leavy and O'Shea, in their 2011 study, state that many primary teachers do not have the necessary skills for creating active classrooms, posing

and solving problems, promoting reflection and metacognition and facilitating broad ranging discussions. Research carried out by Edwards and Mercer (1987) and Vacc (1992) point out that most teachers' questions are closed and factual. These questions mostly focus on memorisation and procedures and have low cognitive demand. Henningsen and Stein (1997) also report that teachers tend to 'dumb down' tasks of high cognitive demand even though these types of tasks are extremely rare.

### **2.2.5. Can Pre-service Teachers' Ability to Pose Mathematically Sound Problems be Improved?**

Findings in Section 2.2.4 all point to the need to improve our pre-service primary teachers' problem posing skills in order for them to be adept and comfortable with passing on this skill to their pupils. Wilson and Berne (1999) assert that if teachers and pre-service teachers are to provide new and different sorts of learning experiences for their pupils it is important that they have such experiences themselves as learners of mathematics. Leung and Silver (1997) found that teachers rarely engage in problem posing activities because they find it difficult to implement and because they themselves do not possess the required skills. Two studies, those of Brown and Walter (1983) and English (1996) found that teachers who were comfortable with their own problem posing introduced their pupils to this skill. These studies both showed that providing pupils with the opportunity to pose their own problems will foster more diverse and flexible thinking, enhance pupils' problem solving skills, broaden their perception of mathematics and enrich and consolidate basic concepts.

Researchers believe that without the immediate need for application pre-service teachers selectively attend to and ignore what may or may not seem relevant for teaching and learning to happen. "Active engagement in authentic activity is considered to be essential for learning to think and act in a community of practice." (Crespo, 2003, p.246).

Lampert's (1985) and Schoenfeld's (1983) work on how practitioners learn also highlight the situated nature of practitioner's knowledge and the importance of learning to reflect and enquire on one's practice in order to become a thoughtful practitioner. In both studies it was found that when practitioners face dilemmas in their practice that tensions between their beliefs and actions bring about and support change in that practice. Change results from tension followed by reflection.

These findings inspired Crespo (2003) to re-design the teacher education course so that it incorporated authentic mathematics teaching experiences involving sustained interactions with pupils and opportunities to reflect on that experience. This study took the form of letter exchange between the pre-service primary teachers and a fourth grade class. This study paralleled and simulated three important aspect of mathematics teaching: posing tasks, analysing pupils' work and responding to pupils' ideas. The design of the study allowed the pre-service teachers to focus their reflections on the work at hand to the exclusion of other aspects of the real classroom. It also slowed down the pace of the work giving the pre-service teachers more time to reflect and practice their skills. The study explored the following ideas: How do pre-service teachers pose mathematical problems to pupils? How do these practices change? What factors contribute to this change? This study took place over eleven weeks in which the pre-service teachers had the opportunity to work with children in classrooms and engage in seminars with peers and tutors. At the outset the pre-service teachers' questions were short, single-step and one-answer problems. They tended to simplify questions and ask leading questions guiding their pupils towards the right answer. These tactics obviously restricted the pupils' work and narrowed the mathematical scope of the problems. However, as the weeks progressed the pre-service teachers' problems tended to be more adventurous, more puzzle like and open ended, encouraged exploration, extended beyond arithmetic and required more than computational facility. The problems became less typical in their structure. Typical problems were extended by showing a picture to prove your answer is right or solve in at least two

different ways. Adaptations to problems were less leading. The findings of this study point to the usefulness of an authentic audience, the value of introducing pre-service teachers to non-traditional mathematical problems and to the power of collaboration in problem posing.

Crespo's (2003) findings follow on from the work of Wilson and Berne (1999) who point out the value of engaging pre-service teachers and teachers in the type of work they are expected to teach. Putnam and Borko's (2000) study emphasised the power of collaboration and the shared experience as important conditions for supporting pre-service teachers' learning. Malaspina, Mallart and Font (2012) carried out a study with pre-service teachers on problem solving and problem posing. One of the main findings of this study was the value of the 'socialization' phase when the participants discussed the rationale behind the problem they had formulated. The discussion about the problem resulted in the enhancement of the skills of the participants and a consequent improvement in the quality of subsequent problems.

How can we best put the results of these studies into operation?

#### **2.2.6. How Can Pre-service Teachers' Problem Posing Skills be Improved?**

In answering this question one comes up with the obvious question-what is a good mathematical problem? In general research has found that pre-service teachers are capable of posing mathematical problems. However, these are often non-mathematical, trivial or poor. Crespo and Sinclair (2008) suggested that this due to a lack of opportunity to engage in and explore problems and their structures.

The Irish Primary School Mathematics Curriculum (NCCA, 1999) states that a problem can be a: word problem, practical task, an open-ended problem, puzzle, game, project or a mathematical trail. (NCCA, 1999) However, when you mention mathematical problems, the first thing many people, including teachers, think about are traditional word problems. This perception brings its own difficulties as the research shows. Many studies

show that this is an issue that needs to be addressed if changes are to take place. Chapman (2005) found that pre-service teachers had a limited view of what constituted a problem based on their own school experience of traditional, routine problems. For many teachers the usual source of mathematical problems is the textbook. Lave (1992) and Ng (2002) found that textbooks provide a high portion of routine, closed problems and problems with exactly sufficient information. Heuristics suggested by curricula are not addressed by textbooks. These textbooks should include more open-ended problems, non-routine problems, authentic problems and problems with insufficient or extra information as well as the traditional problems (Fan and Zhu, 2007). In 2006 Surgenor, Shiel, Close and Millar found that almost one-third of pupils were taught by teachers who used the pupil edition of the class text as their main source in planning lessons, and a further 20% by teachers who drew on the accompanying teacher manual. Other sources, such as the 1999 Primary School Mathematics Curriculum (PSMC) and the School Plan for Mathematics, were used less frequently. This findings underlines the importance of the textbook in the Irish Primary classroom and whereas, there can be little doubt but that textbook content has improved in recent years, we must be aware of the value of the real-life , relevant mathematics problem. Therefore we have to challenge and extend the pre-service teachers' perception of what a good mathematical problem is, thereby giving them skills in how to pose such problems.

The reliance on textbooks and the belief that problems are posed by the teacher for the pupils to solve are enduring and difficult to change. Crespo (2003) asserts that teachers often pose familiar single-step story problems that invite quick accurate responses and re-formulate problems in ways that narrow rather than open the mathematics involved or required by the problem. A lack of concern about sensible connections to real-world situations has been reported in studies involving pre-service teachers.

In 2004 Chapman carried out a study with pre-service teachers in which they closely examined word problems. They came to see word problems in terms of their mathematical and semantic properties. When given the opportunity to closely examine this type of problem their interpretation of word problems improved as did their ability to represent these differently: verbally, symbolically, pictorially and concretely. Arbaugh and Brown (2004) carried out a study focused on helping pre-service teachers develop their understanding of the relationship between a task and the kind of thinking that task requires of pupils with a view to helping them select better problems. However, research carried out by Crespo (2003) also shows that pre-service teachers are able to generate better problems when: they gain personal experience solving such problems, when they have an authentic audience and/or when they are prompted by informal contexts such as pictures rather than symbolic contexts.

It is important for teachers to have mathematical experiences similar to those they intend to provide for their pupils. It is also important to make the practice of problem posing an object of conversation in teacher education because of the central role of problem solving in classrooms. One approach in supporting teachers in gaining these types of experiences is outlined in a study by Crespo and Sinclair (2008). In this study the two researchers designated problems as “tasty” or “nutritious”. This analogy is quite effective as, while food that is nutritious is good and promotes health, we also like food which is tasty. Nutritious in the context of mathematical problems could be seen as factual problems. For instance in the case of tangrams (See Appendix C), a nutritious problem could be ‘What shapes can be identified?’ Tasty problems are categorized as having surprise, novelty and fruitfulness. Fruitfulness; will the problem lead to more questions, will it answer other questions, will it provide insight? A tasty problem in the tangram situation could be ‘Can you make one shape using all the pieces provided?’ Problems can be both tasty and nutritious. One is not exclusive of the other but, as in food, a mix is important. This analogy is very useful as it situates mathematical problems in an everyday context and would be of use to teachers in choosing problems for their pupils. It

provides a very useful framework within which to work in posing questions so that a good mix of questions is achieved.

Vacc (1993) presents an alternative means of categorising problems as factual, reasoning or open. These categories draw attention to the pedagogical qualities of the problems. Given that factual problems provide very little information on whether the pupils actually understand the concept, Vacc recommends that non-fact seeking problems need to be a major part of classroom discourse. Vacc calls for more “reasoning” and “open” types of questions. These require figuring out and explaining why. Open questions elicit information already known but provide a wide range of acceptable answers. Vacc states that open questions provide students with the opportunity to describe phenomena for which they have not learnt a name.

Penrose (1974) argued that visual appeal is an important issue to consider when posing problems. Surprise can arise when a pattern is discovered which was unexpected. ‘A picture tells a thousand words’ so the presentation of visually attractive problems is one that should not be neglected. Wilburne (2006) has explained that the best mathematical problems one can employ in the classroom are non-routine, problems that encourage rich and meaningful mathematical discussions, those that don’t exhibit any obvious solutions and those that require the pupil to use various strategies to solve them.

Abu-Elwan (2009) identified conditions that may encourage pre-service teachers to pose ‘good’ problems. He examined this from both the pedagogical and the mathematical perspective. Focusing on these two aspects of a problem also add to the complexity of what makes a problem ‘good’. Following this study Abu-Elwan suggests the following considerations:

Pedagogical questions may include considerations such as:

- Will my students be able to solve it?
- What kinds of mathematical ideas does the problem involve?
- Will the problem help me to learn about my students' mathematical thinking?

Mathematical questions may include consideration such as:

- Is the problem interesting?
- Are there good techniques for solving it?
- Does it relate to other areas of mathematics?

Silver, Kilpatrick and Schlesinger (1990) also found that students' problem posing skills could be developed by giving them ill-formulated problems and asking them to re-state them (See Section 2.2.2).

Pittalis, Christou, Mousoulides and Pitta-Pantazi (2004) proposed a model of cognitive processes involved with problem posing. These four processes include: filtering quantitative information, translating quantitative information from one form to another, comprehending and organising quantitative information by giving it meaning or creating relations and editing quantitative information from the provided stimuli. Based on their research Pittalis et al. (2004) asserted that these processes correspond to different types of problem posing tasks and that the filtering and editing processes were the most important in posing problems.

As can be seen from the research presented, there are many processes involved in problem posing: the use of heuristics as suggested by Polya (1973), reformulation of existing problems, generating new problems from given situations, keeping in mind the qualities of a 'good' mathematical problem and the cognitive ability of the pupils.

As yet there is no neat, ready-made Polya-like heuristic for problem posing as there is problem solving. Thus problem posing is a highly complex area which the pre-service teacher must be as adequately prepared for as possible, given that it is such a vital part of mathematics education.

### **2.2.7. The Relationship between Problem Posing and Problem Solving.**

Problem posing and problem solving are inextricably linked being the two sides of the one coin. Problem posing can occur before, during or after the solution of a problem. Ellerton (2013) states that research on problem posing and its relation to problem solving has led to new research on the benefits of incorporating problem posing in teacher education programmes. Brown and Walter (1983) state that while problem solving is easily identified as an important aspect of learning mathematics; problem posing has long been considered a neglected aspect of mathematical enquiry.

Problem posing as problem formulation or re-formulation occurs within the process of problem solving. The solver recreates a given problem in some way in order to make it more accessible for solution. The solver transforms a given statement into a version that becomes the focus of solving. Problem formulation relates to planning since it involves posing problems that represent sub-goals for the larger problem. Duncker (1945) said problem solving consists of successive re-formulation of an initial problem. The psychological processes that are involved in problem solving suggests a series of successively more refined problem representations which incorporate relations between the given problem and the desired goal and into which new information is added as sub-goals to be satisfied.

In extended mathematical investigations “problem formulation and problem solution go hand in hand each eliciting the other as the investigation progresses.” (Davis, 1985, p.23)

Cai and Hwang (2002) found a parallel between pupils' thinking when posing and solving problems. They observed that the sequence of pattern-based problems posed by pupils appeared to reflect a common sequence of thought when solving problems-gathering data, analysing data for trends and making predictions. These researchers conjectured that these pupils may have had a solution process in mind when posing problems.

Cai and Cifarelli (2005) described the link between problem solving and problem posing as shown in Figure 2.1 below.

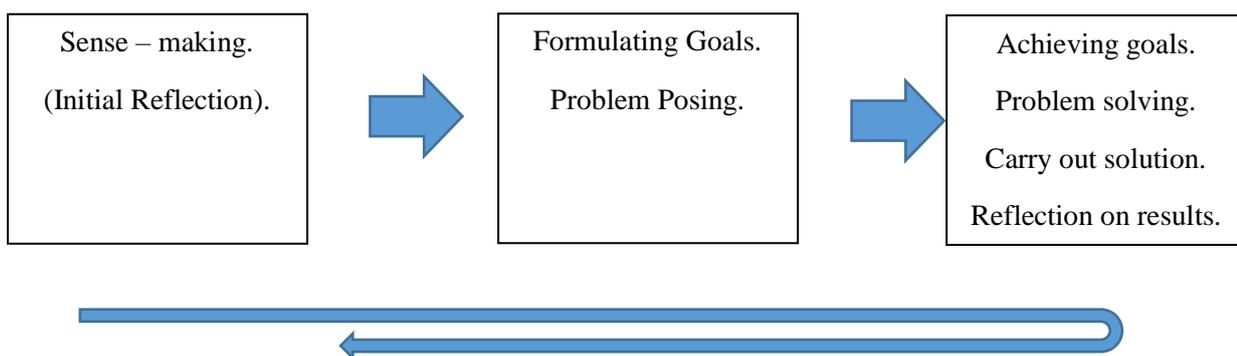


Figure 2.1. Link between problem solving and problem posing. (Adapted from Cai and Cifarelli, 2005)

Cai and Cifarelli (2005) term this the recursive process of problem posing and problem solving. The problem solvers' self-generated questions reframed the problems they were working on and changed the strategy they were using. Cai and Cifarelli (2005) considered both processes to be mathematical exploration.

Building on Polya's 'looking back' stage in problem solving, Brown and Walter (1990) proposed the 'What-if-not' strategy. Abu-Elwan (2009) and Cai and Brook (2006) suggested posing problems through a process of extending or generalising an already solved problem. Gonzales (1998) even suggested this as a fifth step to Polya's four step method. Lavy and Bershadsky (2003) saw the use of the 'What-if-not' strategy as very useful for problem posing. They proposed dividing the activity into two stages. In the first stage, all the attributes included in the original problem are listed. In the second stage, each of these is negated or changed and

alternatives are proposed. Each of the different alternatives creates a new problem situation i.e. problem posing!

Abu-Elwan (2009) suggests that pupils' problem posing activities should correspond directly with their problem solving activities. Ellerton (1986) compared the problems posed by high-ability pupils in contrast to those posed by low-ability pupils asking each group to pose a problem that would be difficult for their friends to solve. The problems posed by the more able pupils were more complex. This finding was also confirmed by Silver and Cai (1996) from research which they carried out with five hundred middle school pupils. Kilpatrick (1987) found that the quality of problems posed by pupils served as an indication of their problem solving ability.

The most frequently cited reason for interest in problem posing is its perceived potential value in assisting students to become better problem solvers. This attitude permeated the NCTM, Professional Teaching Standards (1991). This idea has been prominent for decades as shown by Connor and Hawkins (1936) who argued that pupils, given the chance to generate their own problems, improved their ability to apply mathematical concepts and skills in problem solving. Koenker (1958) included problem posing as one of a list of twenty ways in which to improve students problem solving.

Problem posing has been incorporated into Japanese experimental teaching in which it encourages students to analyse problems more completely thereby enhancing their problem solving competence. Shimada (1977), Hashimoto and Sawada (1984) and Notida (1986) have all described various versions of a style of teaching known as 'open-approach' or teaching with open end or open ended problems. Their descriptions suggest various ways in which problem posing is embedded in instruction. Hashimoto (1987) described an approach in which students pose problems based on one solved the previous day. This approach clarified the connection between the two aspects of a problem for the pupils.

Ellerton, (2013), Singer and Voica (2013), Malaspina (2013) and Bonotto (2013) all stress the relationship between problem posing and problem solving. Bonotto acknowledges:

“There is a common degree of agreement in recommending problem posing and problem solving activities to promote creative thinking in the students and assess it.”

(Bonotto, 2013, p.40)

Research on problem posing and its relation to problem solving has led to new research on the benefits of incorporating problem posing in teacher education programmes. However, in the Irish context, the NCCA, reviewing mathematics in Secondary Schools with reference to problem solving or investigations, states:

“The exploratory, open-ended style associated with investigations does not seem to fit Irish teachers’ and students’ views of mathematics. Possible reasons for this may lie in the culture of mathematics teaching in this country, in the demands that this approach would make on teacher knowledge, skills and attitudes, and in the fact that such work is not currently subject to assessment in the examination.”

(NCCA, 2005, p.5)

With the introduction of Project Maths in 2010, and if the intended syllabus is implemented, we can only hope that the above statement will not reflect the situation in the future.

A few experimental studies have been conducted in the US in which students received a form of mathematics instruction in which problem posing had been embedded contrasted with students who did not receive this. As far back as 1965 Keil found Sixth Grade pupils who had experience of writing and solving their own problems in response to a situation did better in tests than pupils who simply solved textbook problems.

Section 2.1.1 has set out the importance given to problem solving in International Curricula. Section 2.2.3 focuses on the place of problem posing in International Curricula.

Having established the close connection between these two aspects of mathematics education, it is clear that both need equal status in our education system.

### **2.2.8 What is known about the Benefits of Engaging Pupils in Problem Posing Activities?**

What does research tell us about the benefits of problem posing in the classroom? Research has shown that providing pupils with the opportunity to pose their own problems can foster more diverse and flexible thinking, enhance pupils' problem solving skills, broaden their perception of mathematics and enrich and consolidate basic concepts. Brown and Walter (1983), English (1996). Over time, many studies have found that there are numerous benefits to engaging in problem posing in the classroom. Problem posing has freed students and teachers from the tyranny of the textbook. Van den Brink (1987) reported on an experiment in which children wrote a textbook for pupils in another class. In this study, not only were the problems of a good standard but pupils made very few errors. This indicated a strong sense of ownership and understanding of the real reason behind the work. Similar studies were undertaken by Streefland (1987, 1991) as part of Realistic Mathematics Education in the Netherlands and Healy (1993) in the USA where as part of a "Build a Book" project students created their own book based on their geometric investigations. Skinner (1991) reported an Australian study which engaged pupils in an extensive amount of problem posing which they shared with each other and which formed the basis for much problem solving in class.

The problem posing approach in the classroom may also help reduce the dependency on textbooks and teachers and give pupils the feeling of being more engaged in their own education. Cunningham (2004) reported that providing pupils with the opportunity to pose problems enhanced their reasoning and reflection.

Schoenfeld (1992) listed five cognitive aspects of problem solving: 'The knowledge base', 'Problem solving strategies or heuristics', 'Monitoring and control', 'Belief and affects'

and 'Practices'. 'Belief and affects' refer to the pupils' beliefs, the teacher's beliefs and general societal beliefs about the nature of mathematics and doing mathematics. There can be no doubt but the affective response that pupils make to mathematics can colour their whole engagement with the subject. Pupils must believe in the subject's relevance to their lives, in their ability to do it and to the beauty and elegance of the subject. Borwein (2006) refers to this as an aesthetic buzz: This 'light-bulb' moment is one that is wonderful to experience or to witness.

Winograd (1991) also provided another example of the positive results of pupils engaging in problem posing. Over the course of a year, fifth grade pupils wrote, shared and solved original story problems. He did not have a control group in this study but he reported the generally positive impact of this approach on student achievement and disposition towards mathematics. Perez (1985) found similar results that problem posing had a positive effect on pupils' attitude to mathematics.

Problem posing offers a means of connecting mathematics to pupils' interests, a belief emphasised by the NCTM which recommends that students should have opportunities to formulate problems and questions that stem from their own interests. NCTM (1991). However, personal interest is not the sole motivator for posing problems. Within a classroom pupils should be encouraged to pose problems for others in an interesting or novel way. Winograd (1991) reported on this tactic. He found that pupils' sharing their problems with others was a huge motivation.

Pupils who have difficulty with mathematics -a syndrome of fear and avoidance known as mathematics anxiety, also showed a marked change in their disposition towards mathematics when encouraged to pose their own problems. Perez (1985) working with college-age students in a remedial mathematics class found that this approach improved students attitudes towards mathematics as well as their achievement in mathematics. There are no reports of students responding negatively to this approach. However, it should be expected that students who were

comfortable and successful with the more traditional approach might find this approach challenging and filled with uncertainty.

Healy (1993) reports on a study where an emphasis on student generated problem posing was found to humanize and personalize mathematics learning and teaching in profound ways. For many pupils, mathematics has become a neutral body of knowledge filled with abstract ideas and symbolism that others had created and could only be accessed by memorization and imitation. In the course of his work, Healy (1993) found that many pupils became passionately concerned about mathematical ideas when they were investigating problems of personal interest. Therefore, one could expect the passionate personal engagement of students with mathematical ideas to produce learning situations in which affective and cognitive issues would both have great importance.

#### **2.2.9. Problem Posing as a Means to Achieve Other Curricular or Instructional Ends**

Problem posing has a far greater reach in terms of its benefits as indicated by a number of research studies exploring its influence.

*Exceptional mathematical ability and Problem Posing.* The relationship between problem posing ability and exceptional mathematical ability has also been explored. Krutetskii (1976) and Ellerton (1986) compared the problem posing of pupils with different levels of mathematical ability. Krutetskii (1976) argued that the pupils with high ability had the ability to see the problem and pose it quickly. Pupils with less ability either required hints or were unable to pose the problem. Ellerton found that the more able pupils posed problems of greater complexity than did their less able peers.

Because of the association between problem posing and persons with exceptional creativity or talent one might infer that instruction related to problem posing would only be appropriate for 'gifted' students. However, Leung's (1993) findings suggest that problem

posing is an activity not to be reserved only for talented students. In fact, problem posing is an important feature of a broad-based, inquiry oriented approach to education.

***Equality and Problem Posing.*** Problem posing plays a critical role in promoting equality in mathematics teaching and learning. A number of reports highlight that some form of enquiry-oriented instruction has been offered to economic elite groups but has generally been denied to those from less privileged backgrounds. Freire (1970), Gerdes (1985), Ernest (1991) have all shown that an enquiry oriented pedagogy with an emphasis on problem solving and problem posing can be used to challenge rigid hierarchies associated with conventional conceptions of mathematics, mathematics curricula and mathematical ability. Ernest (1991) argues that mathematics can be empowering for all learners. In the USA, the Quantitative Understanding Amplifying Student Achievement and Reasoning (QUASAR) project provides mathematics programmes aimed at high-level thinking, reasoning and inquiry to students in grades 6-8 from economically disadvantaged communities. Silver, Smith and Nelson (1995) all authors writing from a feminist viewpoint, have also shown that enquiry based instruction can be used in ways that respect alternative ways of knowing and solving.

***Creativity and Problem Posing.*** Problem posing has long been viewed as a creative activity or one requiring exceptional mathematical ability. The apparent link between problem posing and creativity is clear from the fact that posing tasks have been included in tests designed to identify creative individuals.

Getzels and Jackson (1962) designed tests to measure creativity. One task asked students to pose mathematical problems. They used the results of this study to measure creativity. Balka (1974) also asked participants to pose mathematical problems from real-world situations. The results were analysed under fluency, flexibility and originality. Fluency referred to the number of questions posed, flexibility to the number of categories of problems generated and originality to how rare a response was.

Leung (1993) studied the relationship between problems posed by a group of pre-service teachers and their performance on tests of creativity and mathematical knowledge. After rating the posed problems she found no relationship with their scores on creativity tests. However, she did find a strong relationship between mathematical knowledge and the quality of the questions they posed.

### **2.3. Conclusion.**

It is clear from all of the research carried out that mathematical problem posing is a very valuable and indeed necessary component of any mathematics curriculum. Its usefulness in many areas of the mathematics classroom has stood up to the rigours of research. The cultivation of a problem posing/problem solving attitude in our pupils could prepare them to be intelligent users of mathematics in order to solve problems of importance or interest to them. This can only benefit the pupils themselves and, in the long run, society in general. Polya (1973) states:

“The first rule of teaching is to know what you are supposed to teach. The second rule of teaching is to know a little more than what you are supposed to teach. Yet it should never be forgotten that a teacher of mathematics should know some mathematics and that a teacher wishing to impart the right attitude of mind towards problems to his students should have acquired that attitude himself”

(Polya, 1973, p. 173)

This quote sums up the reason for this study. ITE should equip pre-service teachers with the necessary skills, confidence and knowledge to impart the same skills, confidence and knowledge to their pupils. The following study is an attempt to research how best this can be achieved.

## **Chapter Three**

### **Methodology**

#### **3.1 Introduction**

In the previous chapter the literature pertaining to research in the areas of problem solving and problem posing in the primary school classroom was reviewed. In this chapter the methods used in this particular study, the rationale for using these methods and the research limitations will be outlined and discussed. The details of the data collection methods used, how the data from these sources were analysed and the demographics of the participants in the study will be explored. Any ethical considerations which are pertinent to the study will also be considered.

#### **3.2 Purpose of the Research**

The purpose of this study is to examine how an intervention could improve the problem posing skills of pre-service teachers. Chapman (2008) found that pre-service teachers made sense of problems in terms of traditional, routine problems that they had experienced in school themselves. However, we know that mathematical problems are much more diverse than this. The Primary School Curriculum (NCCA, 1999) outlines seven different types of problems most of which Chapman's pre-service teachers seem to be unaware of. (NCCA, 1999). In 2005, Corcoran administered eleven items from the PISA 2000 assessment to 71 pre-service teachers. She found that less than 10% of these pre-service teachers got both items at level six completely correct. More than 20% got both items completely incorrect. Overall she found that for up to 80% of the participants, level four was their optimum proficiency. Other Irish studies, such as those of Hourigan and O'Donoghue (2007, 2013) and Leavy and Sloane (2005) also point to the fact that Irish post-primary graduate's performance demonstrates lower order mathematical

skills such as memorisation and procedures as opposed to thinking creatively, reasoning or engaging in mathematical problem solving. Taking all these studies and findings into consideration, there is need for a programme of study which will improve pre-service primary teachers' knowledge of what constitutes a problem, the features of a good problem and in so doing provide pre-service teachers with the necessary skills to pose mathematically sound problems for their pupils. What form should such an intervention take, what points need to be addressed in it and what skills should the intervention endeavour to develop, are questions that constitute the focus of this research.

The researcher herself is most interested in the aspect of problem posing in mathematics education. Over her many years as a teacher the 'problem' of posing good, interesting and relevant problems for her classes was often something she grappled with.

### **3.3 Research Design**

The research is an evaluation of a curriculum intervention and involves the collection and analysis of pre and post-test data pertaining to the intervention. The pre and post-tests consisted of the administration of a questionnaire to evaluate participants' understanding of what constitutes a mathematical problem and their ability to construct a mathematical problem for primary students (See Appendix A). The questionnaire was similar to that used by Chapman (2008) in her study of prospective teachers' abilities in relation to the pedagogical and mathematical aspects of problem solving. Her goal was to develop the prospective teachers' understanding of problems, the problem solving process, problem solving pedagogy and problem solving as inquiry-based teaching. The questionnaire consisted of the following five questions:

1. What is a problem?
2. Choose a class from 1<sup>st</sup> to 4<sup>th</sup> and make a maths problem that would be a problem for those children.
3. What did you think of to make the problem?
4. Why is it a problem?
5. Is it a 'good' problem? Why?

The questionnaire was administered to participants prior to the intervention. Their responses to this questionnaire were analysed and coded. The study intervention then followed. Briefly, this consisted of three weeks of input from both lecturers and tutors on the topic of problems, problem solving and problem posing. Lecture/focus sessions took place early in the week followed by tutorial/workshop sessions addressing the same/similar facet later the same week. Contact time between lecturers/tutors and student teachers amounted to six hours. Following the three week intervention the participants were presented with the original questionnaire again. Their responses were coded and analysed using the original coding system. The differences in the participant's response form the basis of this study and will serve as data to inform recommendations for the design of problem posing experiences in initial teacher education.

### **3.4 The Participants.**

The research focuses on the analysis of a data set generated as a result of the implementation of a new module (EDU 153) in Mathematics Education in Mary Immaculate College following the introduction of a new four year undergraduate initial teacher education programme (Bachelor of Education (B.Ed.)). The participants in this study were a group of 415 first year pre-service primary teachers. Participants were in the second semester of their four

year degree programme, had completed one school placement and were involved in their second school placement while taking part in this study. Good practice protocols were used at all stages on the study (See Section 3.6). The gender breakdown in the study reflects the gender breakdown in teaching as a profession in general (See Figure 3.1).

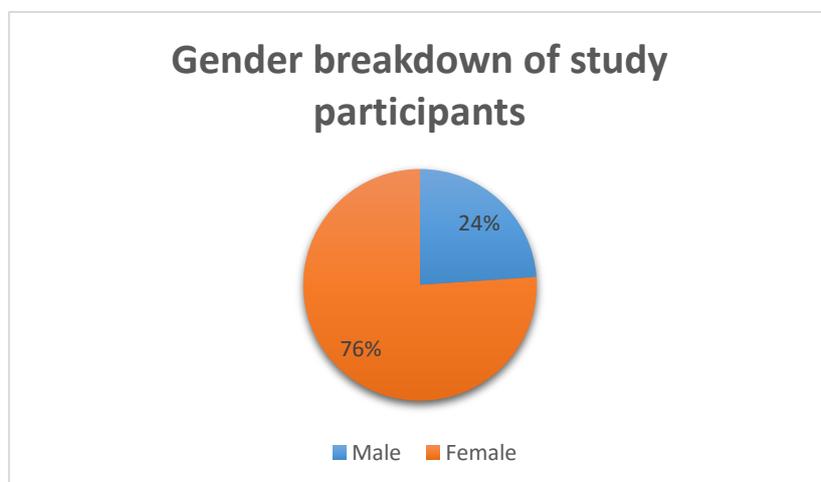


Figure 3.1. Proportion of male and female participants in the study

### 3.5 The Study Intervention

The three week study intervention was a mixture of lectures and tutorials (akin to workshops) on problem solving and problem posing. This approach ensured that the participants engaged with the theories underpinning problem posing and problem solving and also had an opportunity to engage in problem solving and problem posing themselves. During the study intervention the participants were given ample opportunity to explore various problem types (See Appendix C). Problem solving phases and problem solving strategies were explored through the use of these various problems. All these aspects of problems and problem solving were designed with the intent to develop and nurture the participants' talents in posing and selecting problems for their own classrooms when the opportunity would arise. The intervention consisted of a number of elements and foci.

*Element 1: The value of problem solving.* The expansive nature of problem solving was explored reflecting the viewpoint that problem solving is a mathematical skill which permeates right across the mathematics curriculum-no one strand or strand unit has exclusive rights to this skill. To emphasise this, the problems presented to the participants were taken from various strands and strand units of the Primary School Mathematics Curriculum. NCCA (1999). (See Appendix C).

The NCTM (2000) asserts that problem-solving is a vehicle for learning new mathematical ideas and skills. The problem-solving classroom was introduced and explored as one in which concepts and procedures are learned through the vehicle of problem solving. It was emphasised that teaching through problem-solving focuses pupils' attention on ideas and on making sense and helps pupils develop confidence in doing and understanding mathematics. It was argued that problem-solving, especially when set in an everyday or real-life context, engages pupils more effectively and clarifies the use of mathematics in their lives.

The potential that problem solving provides for opening up the opportunity for discussion, mathematical argument and collaborative work in the classroom were also discussed and demonstrated. Moreover, the role of problem solving in developing higher order thinking skills and in promoting confidence and curiosity through trial and error and risk-taking was stressed.

The power of the 'light bulb' moment when solving problems cannot be underestimated when it comes to building a pupil's confidence and positive attitude towards mathematics. During the workshop sessions when the participants were involved in problem solving the satisfaction which the participants and the researchers themselves achieved from a successful outcome was palpable in the room and provided a sense of what success feels like for children.

Francisco and Maher (2005) state that providing pupils with the opportunity to work on complex tasks as opposed to simple tasks is crucial for stimulating their mathematical reasoning. The problem-solving classroom can also be a hub of mathematical creativity. As pupils, and indeed teachers, become more confident and adept at problem solving they will become equally more adept and confident at problem posing. They will have gained an understanding of what constitutes a good mathematical problem and so embark upon making their own problems. This would be a great achievement for pupil and teacher alike. Therefore, if we subscribe to the value of the problem solving approach to the teaching of mathematics, then the question is how do we present pupils with worthwhile problems?

***Element 2: What is a problem?*** Participants explored what it means for a mathematical task to be a problem. The researchers drew from the perspective of Van de Walle (2014) that a problem is something for which we do not know the answer and for which a strategy is not immediately obvious. The implications for this perspective on the experiences of classroom pupils was also explored. For example, pupils will be uncertain as to how to approach the solution to the problem. Moreover, pupils have no learned algorithm to which to resort, hence they really don't know what to do so they must figure out the strategy to arrive at the answer. Links to the primary school mathematics curriculum and constructivist theory were made - pupils construct their own understanding using their prior mathematical knowledge, "learners....construct meaning by making links between new and existing knowledge". (NCCA, 1999, p.5) They draw on concepts and procedures already learned in exploring a solution to a problem. The importance of prior mathematical knowledge in arriving at the solution to a problem was discussed at length. Obviously this brought in the aspect of laying firm foundations in the teaching of mathematics.

***Element 3: The types of mathematics problems.*** Different problem types were explored using the Irish Primary Mathematics Curriculum Teacher Guidelines as a reference.

(NCCA, 1999, p.35) The curriculum states that there are seven different types of problems: word problems, practical tasks, open-ended investigations, puzzles, games, projects and mathematics trails. These seven categories were discussed with the participants and examples of different problem types, with the exception of projects and mathematical trails, were explored during the study intervention. The representation of problems in common mathematics textbooks was also discussed i.e. one-step, one right answer type problems at the end of a chapter on a particular top. During the tutorial/workshop sessions participants were asked for examples of problems they had encountered in primary school. The examples were invariably of the format - “Mary went to the shop. She bought 10 sweets. She ate two. How many had she left?” This is a typical example of a word problem attached to the end of a textbook chapter on subtraction. The use of textbooks was acknowledged as having its place in the mathematics classroom but the construction of real-life problems which have a bearing on the pupils everyday life was stressed as being the ideal approach whenever possible. For example when presented with the ‘value-for-money crisp’ problem and the ‘school bus’ problem the use of similar tasks in the classroom when planning a school party or school tour was discussed (See Appendix C). Participants were particularly intrigued with the insertion of extra or superfluous information in problems to make them more challenging and encourage higher order thinking. The pupils have to extract the relevant information to solve the problem. Using the example already quoted above- “Mary went to the shop. She bought 10 sweets. She ate two. How many had she left?” could be modified to “Mary is eight years old, she has red hair and blue eyes. She went to the shop. She bought 10 sweets. She ate two. How many had she left?” Participants were given the opportunity to share their own problems incorporating superfluous information.

***Element 4: Exploring problem structures.*** The Vacc (1993) framework for categorising problems was then introduced. According to Vacc, problems can be categorised

as factual, reasoning or open. Factual problems are those which provide little information as to whether the pupils understand the concept or not. Reasoning problems are not immediately solvable and require higher order thinking and figuring out. Open problems have a wide range of acceptable answers. As an example, one of the tutorial/workshop activities was the posing of problems using a set of tangrams (See Appendix C). The participants were asked to write three/four problems based on the set of tangrams. When they had written their problems they were asked to categorise them according to the Vacc framework. Most of the problems written were factual i.e. how many triangles can you find? The participants were asked to look at their problems through the Vacc lens and re-work them so that there was a spread across factual, reasoning and open. They could work on this in pairs or small groups. This was very worthwhile as the participants now saw how a problem could be up-graded with just a little more effort. This re-formulation of problems to make them more mathematically interesting was stressed as a great weapon in their arsenal as teachers. The ability to make simpler, existing problems more complex and worthwhile is a great talent to possess in a busy Primary school classroom. This new terminology made perfect sense to them and the new problems were much more challenging and worthwhile. As stated previously, in 2008 Crespo and Sinclair; using a food metaphor; proposed that problems could be considered as either nutritious or tasty. Nutritious problems being those that are factual and in which we practice operations or procedures with which we are familiar. Tasty problems are those which have aesthetic criteria such as surprise, novelty or fruitfulness. Surprise arises when a problem throws up things which were unexpected such as a pattern emerging, novelty can relate to the way in which the problem is stated, while fruitfulness is when a problem leads on to more problems and maybe answers other questions. For instance in the tangram problem posing session, a nutritious problem would be how many triangles can you see? A tasty tangram problem would be - can you make a shape using all the tangram pieces? Crespo and Sinclair (2008) state that problems can be both nutritious and tasty, each being equally valuable but teachers should strive for a mixture

of both when setting problems for their pupils. Participants were also introduced to the nutritious/tasty categorisation and found this a very useful and visual way of categorising problems.

*Element 5: Problem-solving strategies.* The intervention emphasised that problem solving can be a big obstacle for many pupils and the teacher must approach it in a very systematic, structured way. Teachers should consider very carefully the problems they present to their pupils. The type of problem and the context in which it is presented must engage the pupils' interest so that they see the value of working on the problem to reach a solution. The tutorial/workshop sessions gave the participants the chance to put the theory into action. They were presented with problems to solve which were varied, interesting and engaging (See Appendix C). They were constantly reminded of the problem solving strategies (See Figure 3.2) which had been covered in the lecture/focus sessions and encouraged to use these strategies and see which they found the most useful. These sessions were very lively with lots of discussion ensuing regarding the solutions found and the various paths or strategies used to arrive at these solutions.

Participants were introduced to the various strategies which children can use when solving problems as illustrated in Figure 3.2. Participants were made aware that these strategies are all very valid and the solution to a problem may involve the use of one or more of these strategies.

# Problem solving strategies

- Guess and check
- Look for a pattern
- Make an orderly list
- Draw a picture
- Eliminate possibilities
- Solve a simpler problem
- Use direct reasoning
- Use a model
- Use a formula
- Solve an equation
- Make a table or chart
- Work backwards

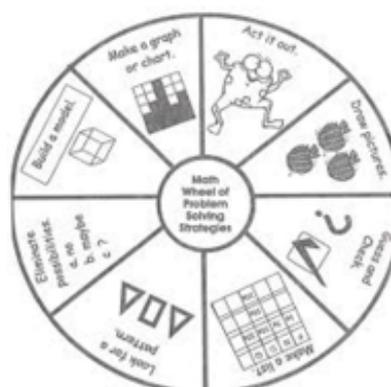


Figure 3.2 Problem solving strategies explored in the study intervention (Adapted from Van de Walle, 2014, p. 56)

The various strategies in the strategy wheel were explained and discussed with examples of each being thoroughly explored. It was emphasised that some of these strategies work for some problems and for some pupils and that finding the best strategy is often the most difficult task of all. However, if pupils are introduced to these strategies they will, with time, become better problem solvers. Participants were encouraged to display this strategy wheel in their classrooms when on School Placement for all mathematics classes.

Many of the strategies on the Strategy Wheel can be traced back to the work of George Polya (1961) and his four step plan in solving a problem.

1. Understand the problem- identify what needs to be done, identify the relevant/irrelevant information, identify what is being asked, re-formulate the problem by drawing a diagram, making a list etc.

2. Devise a plan- what strategies could be used, solve a simpler problem, draw a picture, find a pattern.
3. Carry out the plan-follow through on strategy selected, look at the problem again if this plan is not working out find a new strategy.
4. Look back-look carefully at the solution, does it make sense, what was the pathway to this solution, could this strategy be used for another problem, if you were solving this problem again would you use the same strategy.

During the tutorial/workshop sessions, emphasis was placed on the actions of the teacher to support pupils before, during and after problem solving. *Before activities* were outlined as involving brainstorming approaches to solving the problem, clarification of the task involved, and providing a simpler version of the problem. *During activities* are teacher practices to help pupils plan before they act such as listening and observing carefully to see how pupils are thinking, interacting appropriately and ensuring that the teacher does not interfere too much. *After activities* occur when the problem has been solved and consist of asking questions such as ‘did anyone find a different way to solve this problem?’, ‘can you explain your strategy to the class?’, and ‘what have you learned that may help you in another situation?’ These *before, during and after activities* were adapted from Van de Walle (2014) and modelled by the researchers while the participants were engaging in problem solving.

Other off-shoots of Polya’s four point plan were presented in the form of anagrams i.e. RUDE –Read the problem carefully, Understand what is being asked, Draw a picture/diagram to help you, Estimate what you think the answer will be. PDST (2010). THINK- Talk about the problem, How can it be solved, Identify a strategy, Notice did /how this worked, Keep thinking as you work. Van de Walle (2014). These more child-friendly approaches appealed to the participants who could see their relevance to the primary school classroom.

When presented with a problem, participants were asked to ‘think, pair, share’. They participants were encouraged to look at the problem individually, think how they would solve it, try solving it and then discuss their strategy with another participant. Then this pair would share with another pair. This collaborative approach to learning gave participants the opportunity to formulate their own strategies before sharing them, discussing them and justifying them. Peer learning may be a valuable result of this approach. This aspect of sharing was stressed in all the tutorial/workshop sessions and sometimes resulted in lively exchanges.

**Element 6: Considerations when designing and selecting problems.** In the tutorial/workshops on problem posing, the participants were alerted to particular design features or considerations when designing or selecting problems. These were: Problem type (word, computational, exploratory, puzzle etc.), Curriculum strand, Single or multiple entry points, Single or multiple steps, Single or multiple solutions, Type of understanding required to solve the problem (procedure or conceptual), Level of cognitive demand (low, medium, high) and computational v puzzle and exploratory type problems. Figure 3.3 provides insight into how participants were supported in examining particular design features of the problem presented through posing questions that focused their attention on these design features.

## Categorise the Problem

Mike is mixing cement to fill a hole in his footpath. The mixture he needs to fill the hole is 2 buckets of cement, 6 buckets of sand and exactly 4 litres of water. His bucket holds exactly 5 litres of water and he has a container that holds exactly 3 litres. How will Mike get exactly 4 litres of water to mix the cement?



Checklist

- Type of Problem?
- Curriculum strand(s)?
- Single or multiple entry points?
- Number of steps?
- Single or multiple correct solutions?
- Focus on procedures or concepts?
- Level of cognitive demand?

Figure 3.3 Analysing the design features of a mathematical problem.

Participants were presented with many problems to solve and categorize; where categorization was supported through the presentation of questions similar to those in Figure 3.3 above. This approach made participants very aware of the various aspects of problems. It was a very worthwhile exercise in that they began to see problems under many different headings which they had been heretofore unaware of. This tutorial/workshop session was followed by a session on posing and evaluating problems. In this session the students were given the task of posing problems using tangram pieces. They were reminded of all the considerations to be taken into account when designing or selecting problems. The students again worked individually, in pairs and then in groups for this task. Sharing of problems posed was followed by discussion. The points covered in this checklist were all illustrated in the problems presented during the lecture and tutorial sessions and discussed at length. For example the Daedalus and Icarus problem (See Appendix C) is a good example of a problem with multiple entry points, multiple steps and multiple answers. Multiple entry points refers to the situation in which every child in the class will be able to make a start at this problem. There are many steps to the conclusion in the Daedalus and Icarus problem and, depending on the initial number chosen, there are multiple answers. Thinking of it in terms of the Vacc (1993) framework it is an open-ended investigative problem and it is both nutritious and tasty using the Sinclair and Crespo (2008) terminology. This particular problem would be perfect for a Third/Fourth class learning multiplication and its interesting back story adds to its usefulness and value.

### **3.6 Ethical Issues**

Throughout the study, the necessary ethical obligations were fulfilled. The faculty members involved in the administration of the questionnaires were granted Mary Immaculate Research Ethics Committee (MIREC) approval. Best practice protocols were implemented at

all stages of the project. For example, prior to the study participants were informed about the purpose and nature of the study. They were informed that participation in the study on a purely voluntary basis and consent forms were distributed (See Appendix B). It was also communicated that a decision regarding participation in the study had no relationship with their participation in the programme. They were assured of confidentiality and of the usefulness of the study in the design of future pre-teacher education modules. Names of the participants were not be used in the research and their identity will be protected at all times. All data were stored in a secure location in a locked cabinet.

### **3.7 Data Collection and Analysis.**

The research is based on the collection of the pre and post-test questionnaires from the participants. The pre-test data was collected from all participants prior to the start of the intervention. Subsequently, all pre-test data were coded and analysed. Following the three week intervention the original questionnaire was administered to the same participants again and the results coded, analysed and compared with the results of the original questionnaire. This method of research is designed to show the effectiveness, if any, of the intervention. The qualitative data were hand-coded to ascertain the results from the collected data. Themes that appear regularly were identified and within these themes common beliefs or ideas were recorded. Whilst the researcher was not involved in the collection of the pre or post intervention data, she was involved in the study intervention. The researcher was part of the design team that devised the content of the study intervention and was also involved in the delivery of the tutorial/workshop sessions following on from this.

The pre-intervention data were analysed first on a question-by-question basis. For example, the responses to question 1 were examined for each of the 415 participants. This

involved taking a piece of data, applying a code to it and providing a description for that code. The data were then clustered into categories in an effort to identify themes or patterns. For example, in relation to Question 1, 26 codes were established and 7 categories were constructed from these codes. The fit between the data and categories was a process of continual refinement. This process was carried out for each of the five questions. The author made an initial 'first pass' on the pre-intervention data to establish codes, clustered these codes into categories and identified emerging themes. As the findings emerging from qualitative analysis may be influenced by the researcher's personal biases (Suter, 2012). The researcher then presented the original data and codes to a second researcher (who was involved in the study and understood the context and data). This researcher completed a 'first pass' through the established codes and examined and critiqued the categories and emergent themes established by the author. Both researchers discussed these codes, categories and emerging themes, and where necessary, the initial data were revisited to assess the evidence for the claims. This process was carried out for each of the five questions posed on the questionnaire. At the completion of this process dominant themes were identified. (Merriam, 1988).

A similar strategy was followed for analysis of the post-intervention data. The same criteria used to establish codes, categories and themes for the analysis of pre-intervention data were used. However, the post-intervention responses and hence data were more complex resulting in the establishment of new codes and hence categories and themes. For example, in relation to Question 2, 34 codes were established and 10 categories were constructed from these codes. This compares to the 26 codes and 7 categories established from examination of the pre-intervention data for the same question.

### **3.8 The limitations of this type of study.**

The main limitation of this type of study may lie in that in analysing the data the researcher may naturally set out to find the responses that they wish to find or originally hypothesize. However, in analysing any data the researcher must strive to be completely objective and faithfully record the responses given.

This study examines a single cohort of student teachers and has no control group. A control group was not used as the researcher believed that all pre-service teachers within the cohort were entitled to access the best programme on offer. Given this, it was not possible to find a matched control group of pre-service primary teachers.

The time available to invest in this intervention is also a very valid limitation. By necessity, with so many other aspects of the curriculum to be covered, the intervention was slotted into a three week period comprising of just six contact hours between lecturers, tutors and student teachers.

### **3.9 Conclusion**

There is no doubt but that in this ever-changing world in which we live that we need to nurture people who can reason and flexibly solve problems they have not encountered before. Our children's prowess in this area should and can be encouraged and developed by their teachers. However, these same teachers need to be nurtured in turn and their problem posing skills developed before and during their teaching careers. This is the aim underpinning this research. How best to train our teachers to pose good, nutritious, tasty and varied problems in their classrooms. The results of this study will be outlined in the following chapter.

## **Chapter Four**

### **Results of the Study**

#### **4.1 Introduction**

This chapter analyses the findings of the research. As outlined in the previous chapter, the data were collected through the administration of a questionnaire pre and post-intervention (See Section 3.3).

#### **4.2 Analysis of the Responses to the Questionnaire Pre- intervention**

In order to analyse the responses of the participants to the questionnaire, the author initially read through all the questionnaires to establish the most frequently occurring responses; codes were assigned, categories constructed and dominant themes identified (See Section 3.7). This approach was followed for the five questions on the questionnaire.

#### **4.3 Question One - What is a problem?**

The first question was designed to provide insights into what participants believed to be the main attributes of a problem. The question posed was ‘What is a problem?’ Most of the participants provided more than one response to the question; hence several codes were assigned to the majority of responses. The responses to question one both pre and post intervention responses are presented on Table 4.1.

<b>Responses to “What is a problem?”</b>	<b>Pre – intervention frequencies</b>	<b>Post – intervention frequencies</b>
Contains words, numbers, is a story	100	66
You have to find a missing part or missing information	32	13
Is a question	136	137
You have to use various strategies, take steps	122	190
Has various solutions	16	76
Is an equation	18	6
You have to find an answer or solution	77	3
Needs higher order or critical thinking	140	94
The strategy is unclear	20	44
The answer is unclear	47	38
You must use prior math knowledge.	53	34
It must be solved	105	26
Is difficult, is a challenge, is a struggle	86	56
Contains every day, real-life scenario	7	4
It should be interesting	2	1
It should be age/ability appropriate	4	0
Is not straight forward or obvious	38	58
Is not a simple algorithm, sum or calculation	2	13
Can be of various types or from various strands	15	72
You have to plan step by step	20	28
It must be worked on or figured out	44	37
Children must want to solve it	1	1
It develops children’s understanding	4	1
The answer is not as important as the process	3	2
It could be collaborative/group work	2	1
You could use concrete material to solve it	7	7

Table 4.1 Frequency of responses, pre and post-intervention, to Question One.

In the pre-intervention data, the most popular responses were: a problem needs higher order or critical thinking; problems are seen as containing words, numbers, are a story; they are questions that have to be solved; you solve them by using various strategies. From these responses participants obviously see problems as word problems that are difficult to solve. For example Participant 19, pre-intervention, stated that: *“A problem is a mathematical question that requires an answer. This answer must be achieved by taking a number of steps. A problem requires higher order thinking.”*

When comparing and analysing the findings from the pre and post-intervention data, some interesting differences became apparent. The number of participants who mentioned problems as ‘containing words, numbers or being a story’ dropped appreciably in the post-intervention data. Another category in which there was a considerable difference (pre-intervention, n= 77; post-intervention, n=3) was ‘you have to find an answer to a problem’. This suggests that students now realise that the process of problem solving is more important than the product. This was also supported by the outcome in the response to ‘a problem must be solved’ (pre-intervention, n= 105; post-intervention, n=36). The response to ‘Can be of various types or from various strands’ (pre-intervention, n= 15; post-intervention, n=72) also showed a great difference in the pre and post-intervention findings. One can surmise from this data that the participants’ idea of what constituted a problem had changed or broadened. This finding was corroborated by the extra responses which arose in the post-intervention data (see Table 4.2) such as - a problem can be a puzzle and also in the types of problems which the participants wrote. For example Participant 342, post-intervention, wrote; *“A problem is a question which can be solved in multiple ways and can consist of a word problem, puzzles, activities etc. and the use of concrete materials may be useful. A problem tends to make children stop and think and figure out what to do rather than giving them a step by step method.”*

<b>Extra Responses</b>	<b>Frequency</b>
Can be open or closed	53
Have no set way or known way of solving	88
Can contain relevant and irrelevant material	29
Can be solved by trial and error	16
Should be nutritious and tasty	4
Polya's heuristics	2
Can be puzzle-type questions	36
Can have multiple entry points	6

Table 4.2 Additional responses from the post intervention data in response to Question 1

These extra responses show us quite clearly that post-intervention some of the participants now see problems more broadly. Their eyes have been opened to other aspects that they had previously not been aware of. These responses were reflected in the types of problems they set when answering Question 2 'Choose a class between 1<sup>st</sup> and 4<sup>th</sup> and make a maths problem that would be a problem for those children' (See Section 4.4).

#### **4.3.1 Themes Relating to Participants' Understandings of what Constitutes a Problem**

Having closely examined the responses, some themes seemed to emerge. Therefore the author decided to collapse the raw scores into categories where they would fit more succinctly. Seven themes emerged from the responses to Question 1 (See Table 4.3); each theme consist of an amalgamation of codes established from the initial analysis of the data (see Table 4.1).

The first theme, *Format of Problems*, is a throwback to the participants' experience of problems- i.e. traditional word problems appearing in school textbooks. These were usually cover stories in which the pupils were asked to practice the operation that had been recently taught. The second theme, *Layout of Problems*, shows that students see problems as being more than a simple "sum". The third theme, *Learner Activity*, sets out the activities a pupil must engage in order to solve a problem. The fourth theme, *Path to a solution*, reveals that participants viewed problems as not being straightforward, as being a struggle, as being difficult. Again participants know that maths problems are a problem for many people. The solution is the goal. Interestingly a few participants saw that a problem could have more than one solution. The fifth theme, *Motivating factors*, recognises the importance of the children wanting to solve the problem which should be based on interesting, everyday real-life situations. The final coherent theme, *Learner outcomes*, indicates that participants recognise that a problem will require a higher level of thinking in order to be solved. They see problems as a tool for developing children's understandings i.e. conceptual knowledge.

There were some other aspects of problems that were referred to and, while mentioned by relatively few participants, they are very valid considerations in mathematical problems (See Table 4.2).

Collating the pre and post-intervention responses under the themes mentioned, the results are now presented in Table 4.3. This highlights some interesting differences in the categories Learner Outcomes and Other Aspects of Problems particularly. The author accounts for the change in outcome in Learner Outcomes in that, post-intervention, the participants now saw the usefulness of the strategies in tackling a problem. (See Section 3.5) Maybe the use of the strategies has now cleared the path to a solution and in so doing has developed the students own higher order thinking. Checking back to Table 4.1 the response – Can be of various types or from various strands- accounts for the difference in outcome in

		<b>Pre-intervention</b>	<b>Post-intervention</b>
<b>Themes</b>	<b>Codes</b>	<b>Frequency</b>	<b>Frequency</b>
Format of Problems	Contains words, numbers, is a story	100	66
Lay-out of Problems	The problem is: <ul style="list-style-type: none"> <li>• a question</li> <li>• an equation</li> <li>• not a simple algorithm, sum or calculation</li> </ul>	156	156
Learner Activity	The learner has to: <ul style="list-style-type: none"> <li>• find a missing part or information</li> <li>• use various strategies, take steps</li> <li>• worked on or figured it out</li> <li>• use prior maths knowledge</li> <li>• plan step by step</li> </ul>	271	302
Path to a Solution	The problem: <ul style="list-style-type: none"> <li>• has an answer that must be found</li> <li>• must be solved</li> <li>• has various solutions</li> <li>• is difficult, is a challenge, is a struggle</li> <li>• strategy is unclear</li> <li>• answer is unclear</li> <li>• is not straight forward or obvious</li> <li>• answer is not as important as the process to find the answer</li> </ul>	391	304
Motivating Factors	Uses an everyday, real-life scenario  It should be interesting  Children must want to solve it	10	6
Learner Outcomes	Needs higher order or critical thinking  It develops children's understanding	144	95

Other Aspects of Problems	<p>It could be collaborative/group work</p> <p>You could use concrete materials to solve it</p> <p>It can be of various types or from various strands</p> <p>It should be age/ability appropriate</p>	28	80
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Table 4.3. Themes constructed in response to Question One from pre and post-intervention results

Other Aspects of Problems theme. The participants had been introduced to many different types of problems during the intervention moving them away from their pre-conception of problems as being just word problems. (See Appendix C)

#### **4.4 Question Two - Choose a Class between 1<sup>st</sup> and 4<sup>th</sup> and Make a Problem that would be a Problem for that Class**

The second question asked the participants to choose a class and make a maths problem that would be a problem for that class (See Section 3.3).

The spread of classes was fairly even on both occasions (See Table 4.4). The most popular classes were Second and Third in both pre and post-intervention data. Post-intervention, First Class was chosen by just 64 participants in contrast to 91 in the pre-intervention results. The fact that many participants again failed to specify a class was surprising as this is clearly stated in the question.

	<b>Pre-intervention.</b>	<b>Post-intervention.</b>
<b>Grade</b>	<b>Frequency</b>	<b>Frequency</b>
First Class	91	64
Second Class	103	99
Third Class	95	95
Fourth Class	76	82
Not specified	46	53

Table 4.4 Class levels identified pre and post-intervention for Question 2

When writing their problems the students chose contexts such as Moshi Monsters, Match Attax Cards, sweets, pizza etc. which they obviously thought would appeal to the children. They also situated many of the problems in school, at birthday parties or in the shop. This shows that they are aware of the importance of engaging the children's interest in the problem solving process.

The majority of the problems in the pre-intervention data involved just one step and were fairly typical of the problems one would encounter in a mathematics textbook: one step, one right answer, very formulaic problems with which the participants would be familiar with from their own Primary school days. For example: *If there are 20 books on the bookshelf and Sally took 6 of them. How many books are left on the bookshelf?* (First Class)

[Participant 68, pre-intervention]

In contrast, the majority of the post-intervention problems involved two or more steps (See Table 4.5). This shows that when the participants were made aware of the different facets of problems, they can come up with better and more challenging work for their pupils.

Participant 62, post intervention, wrote this problem for an unspecified class: *Sean surveyed his class about favourite football teams. There was 30 children in the class.  $\frac{1}{3}$  supported Man United, 6 supported Liverpool and the remainder was evenly divided between children who supported Chelsea and Tottenham. How many children supported Chelsea?*

[Participant 62, post-intervention]

	<b>Pre-intervention.</b>	<b>Post-intervention.</b>
<b>Number of steps in problem</b>	<b>Frequency</b>	<b>Frequency</b>
One step problem	282	164
Two or more steps problem	121	220

Table 4.5 Number of steps in pre and post-intervention problems posed for Question 2

In both the pre and post-intervention, addition and subtraction were the most popular operations required to solve the problems (See Table 4.6). The author was surprised that division held sway over multiplication, it would be expected to have been the other way round.

	<b>Pre-intervention</b>	<b>Post-intervention</b>
<b>Operations in problem</b>	<b>Frequency</b>	<b>Frequency</b>
Addition	116	104
Subtraction	131	114
Multiplication	57	75
Division	94	78

Table 4.6 Operations involved in pre-intervention and post-intervention problems posed in Question 2

An example of a division problem written for 3<sup>rd</sup> class is presented below from Participant 6, pre-intervention.

*The Easter Bunny has 125 eggs. He gives 5 eggs to each child he visits. How many children will he be able to visit with 125 eggs?*

[Participant 6, pre-intervention]

Many of the two step problems, such as that written by Participant 261 for Fourth Class, post-intervention, involved addition and subtraction.

*There are 10 screens in the Omniplex Cinema. If there were 21 people in screen 1, 35 people in Screen 2, 12 in Screen 3 , 56 in Screen 4, 7 in Screen 5, 0 in Screen 6, 89 in Screen 7, 44 in Screen 8, 51 in Screen 9 and 20 in Screen 10. How many are in the cinema altogether? If 385 tickets were sold earlier how many people did not show up?*

		<b>Pre-intervention</b>	<b>Post-intervention</b>
<b>Strand of Mathematics Curriculum</b>	<b>Strand Unit</b>	<b>Frequency</b>	<b>Frequency</b>
<i><b>Number</b></i>	Operations	283	215
	Place Value	3	0
	Fractions	54	39
	Decimals	1	4
	Counting/ Numeration	8	26
<i><b>Measures</b></i>	Money	28	25
	Length	6	4
	Weight	4	3
	Time	6	4

Table 4.7 Results for Strand/Strand Unit breakdown for Question 2

The vast majority of the problems fell into the Operations Strand Unit of the Number Strand (See Table 4.7). This is probably due to the fact that at this stage of their college programme the student teachers had just covered the Number Strand. These problems were word problems based on the four operations. Most were straight forward problems such as Participant 2's, pre-intervention, problem written for 1<sup>st</sup> class:-

*George had 17 Moshi Monsters. He gave 9 of them to Joanne. How many did he have left?*

Those problems involving Fractions were also a combination of operations and fraction knowledge. An example is the problem written by Participant 38, pre-intervention, for 2<sup>nd</sup>. Class:

*There are 20 students in a class who want to go swimming. If  $\frac{1}{4}$  of them forget their swimming hats, how many hats will the teacher have to buy?*

A 'purer' fraction example from Participant 3, post-intervention, written for an unspecified class is:

*$\frac{2}{8}$  of a pizza has chicken on it. Another  $\frac{3}{8}$  of the same pizza has sausage on it. If Paul doesn't like any of these toppings how much pizza is left for him?*

The majority of the problems in the Measures Strand were from the Strand Unit Money. These were mostly based on the four operations. For example Participant 1, pre-intervention, wrote the following problem for Third Class:

*There are 17 children in Third Class. Each Wednesday they go swimming. The cost of the swimming is €8. How much in total does the swimming cost for all the children?*

A smaller amount of problems involved Length, Weight and Time. (See Table 4.7). A weight example from Participant 387, post-intervention, for Third/Fourth Class is:-

*If John had €18 and he wanted to buy sweets in the sweetshop which cost €1 per 1.4kg.*

*How many grammes of sweets can he afford?*

This time problem written by Participant 393, pre-intervention, is for an unspecified class and involves two or more steps:-

*If Train 1 left the station at 12.45 a.m. and arrived at its destination at 2.30 p.m.*

*Train 2 left at the same time as Train 1 but did not arrive at its destination until 3.15*

*p.m. What was the difference in time between the two journeys? (Unspecified Class)*

The mathematical skill most common in the posed problems was that of estimation. The students had received input in this skill in their lectures and tutorials this may account for the popularity of this aspect of the problems. For example Participant 27, pre-intervention, wrote the following problem for Fourth Class:

*John has 47,063 sheep in one field and 53,984 sheep in another field. Using your best estimate figure out roughly how many sheep John has altogether.*

Other aspects of some of the problems posed are presented in Table 4.8. In the pre-intervention, a few of the problems (n=4) were inappropriate for the specified class in that they were either too difficult or too easy. Some students (n=12) posed puzzle-type problems and only two problems had more than one right answer. Participant 80, pre-intervention wrote this puzzle-type problem for Fourth Class:-

*The number is less than 6000. The hundreds number is the same as the number of days in the week. The tens number is a multiple of 3 but is greater than 4 but less than 8. The units digit is two less than the tens digit. What is the number?*

When comparing pre and post-instruction results Table 4.8 clearly shows that the participants have been made aware of and taken into account aspects of problems which they may have heretofore not considered. The jump in puzzle-type problems, problems with

multiple right answers and those containing irrelevant/extraneous information was quite heartening for those who had designed and delivered the intervention sessions.

	<b>Pre-intervention</b>	<b>Post-intervention</b>
<b>Other aspects of posed problems</b>	<b>Frequency</b>	<b>Frequency</b>
Inappropriate for Class	4	3
Puzzle-type problems	12	64
More than one right answer	2	47
Irrelevant/Extraneous information	0	45

Table 4.8 Other aspects of posed problems for Question 2

The table above clearly shows the benefits of the study intervention in making participants aware of the various types and aspects of a mathematical problem. If you compare the pre and post-intervention data results, you will see immediately that many more participants selected a puzzle-type problem when posing and also that more problems contained more than one right answer. These are very welcome developments as it suggests that participants had learned the value of the open-ended problem in the Primary classroom. Two examples of puzzle-type problems were:

*If it takes 3 glasses of water to fill a jug and two jugs to fill a kettle, how many glasses of water does it take to fill four kettles? (Second Class)*

[Participant 168, post-intervention]

*The children in First Class are making Play-Doh caterpillars. They have four colours to choose from. How many different ways can they make a caterpillar that is four sections long? Remember they can only use each colour once. (Unspecified Class)*

[Participant 216, post-intervention]

The inclusion of extraneous information was also a new development following on from the intervention. It may be that the participants saw the novelty value of this approach or

alternatively the additional challenge it provided. One can imagine them modelling this type of problem for their pupils and also engaging their pupils in problem posing incorporating this aspect within problems. Two examples of problems with extraneous information follow. It is noteworthy that these problem also have more than one right answer:

*Jane is 9 years old and has blue eyes. On Saturday morning she goes cycling on her bike to the local park. When she gets there all of her friends are there too. They are all aged 9 or 10 and their younger brothers and sisters are with them riding their tricycles. Jane can count 18 wheels altogether. How many bicycles and tricycles are in the park? (Fourth Class)* [Participant 285, post-intervention]

*John has a farm. He drives a John Deere tractor. John has cows and hens on his farm. Overall there are 30 legs on his farm and there is at least 4 of each animal. Find out the possible number of cows and hens on the farm. (Fourth Class)*

[Participant 169, post-intervention]

#### **4.5 Question Three - What did you think of to make the problem?**

It became apparent from the responses to ‘What did you think of to make the problem?’-question three, that many aspects were considered by participants when making a problem. The list presented in Table 4.9 outlines the particular aspects.

	<b>Pre-instruction</b>	<b>Post-instruction</b>
<b>What did you think of to make a problem?</b>	<b>Frequency</b>	<b>Frequency</b>
Strand Unit	128	77
Children's prior knowledge	84	77
Children's ability	184	150
Interesting context	182	143
Strategy children would use	61	35
A challenge for the children	56	79
To test/assess the children.	10	16
Language used in the problem	60	58

Table 4.9 Aspects of problems which participants considered when making problems Question 3

The children's ability and an interesting context were the most important aspects both pre and post-intervention according to this data set. Participants also considered the strand unit that the problem was based on as an important aspect in choosing a problem. Participant 115, pre-intervention, wrote:

- *The children's level in Maths*
- *If the children have covered this material.*
- *To make it a bit challenging for the children*
- *To test their knowledge.*

Overall all of the aspects chosen by participants are important when choosing or posing a problem for pupils to solve. Comparison of the pre and post-intervention responses don't show a huge difference in frequency of responses except in the area of strand unit (See Table 4.9). However, many more new aspects were mentioned in the participants' responses post-intervention as shown in Table 4.10. These extra responses show that the participants' knowledge of what constitutes a good problem has now been extended. The inclusion of

irrelevant/extraneous material has obviously struck a chord with them as being useful in helping children to improve their problem solving skills (See Table 4.10). The fact that they also considered that problems with more than one step are more valuable in the Primary classroom is also a welcome development. Multiple strategies and multiple answers were also new aspects reported in the post-intervention question. Mentioning the use of drawings/diagrams and concrete materials also suggests that the participants now value the process as opposed to the answer and that they are now aware of the use of visual aspects in problem solving.

<b>Extra Responses</b>	<b>Frequency</b>
Irrelevant/extraneous information	52
Lecture/tutorial instruction	9
More than one step	26
More than one strategy	55
Multiple answers	45
Multiple entry points	8
Group work	3
Use of drawing/diagram	22
Use of concrete materials	26

Table 4.10 Additional aspects of problems which participants considered when making problems in the post-intervention phase (Question 3)

#### **4.6 Question Four-Why is it a problem?**

Question 4 asked participants to justify ‘why is...’ the problem they wrote in question 2 ... ‘a problem?’ As can be seen from the Table 4.11, the pre-intervention responses were relatively well-informed considering that at the time of the study the participants had just completed one Semester of their initial teacher education.

	<b>Pre-intervention</b>	<b>Post-intervention</b>
<b>Why is it a problem?</b>	<b>Frequency</b>	<b>Frequency</b>
Missing piece of information	41	35
Various/multiple steps to solution	38	50
Relates to real life	13	5
Various strategies could be used	50	125
Requires thinking/figuring out	175	164
Challenges pupils	27	29
Language used must be deciphered	62	63
Children must use prior knowledge	58	28

Table 4.11 Participants' rationale in response to 'why is it a problem'- pre and post-intervention results

Participants reported that the problem they had posed could be categorised as a problem for the pupils because it required thinking, working or figuring out. This was by far the most frequent aspect mentioned in their responses in both the pre and post-intervention data (See Table 4.11). For example Participant 208, pre-intervention, wrote this problem for Third Class “*If Mary was hosting a party and she had 6 pizza and 8 guests. What fraction of pizza would each person get?*” When explaining why it is a problem, the participant’s reply was:

*It can be solved in more than one way. It requires thought to be solved. Diagram and algorithms can be used to solve it. Children can explore different ways of solving it.*

Comparison of the pre and post-intervention data reveals that the intervention brought about the greatest change in the category ‘various strategies could be used’ (See Table 4.11). It is possible that the participants could now relate to the variety of possible strategies children could use following their own experiences in the lecture/tutorial sessions (See Section 3.5).

#### 4.7 Question Five-Is it a ‘good’ maths problem? Why?

Question 5 asked participants ‘Is it a ‘good’ maths problem? Why?’ In the pre-intervention, in response to this question 23 students were very honest and rated their own problem as not being a ‘good’ problem. Some said their problem was only ok (n=33) and some rated their problem as average (n=10). Those that rated their problem as “good” gave a variety of reasons (See Table 4.12).

	<b>Pre-intervention.</b>	<b>Post-intervention.</b>
<b>Why it is a good maths problem?</b>	<b>Frequency</b>	<b>Frequency</b>
Relates to children’s lives	109	54
A challenge for the pupils	65	79
Makes the children think	85	125
Engages the children’s interest	51	49
Chn can use prior knowledge	18	40
Relates to Maths Curriculum	18	10
Appropriate for chosen class.	51	52
Various strategies can be used to solve the problem	72	127
Suitable for Pair/Group work	7	10

Table 4.12 Participants’ rationale for rating their problems as ‘good’ pre and post-intervention results

Interestingly, the response “Relates to children’s lives” decreased considerably between pre and post-intervention (See Table 4.12). The author questions if this signals that the participants now see problems as something that can stretch the children’s interests? Maybe they see problems as being more than just stories that everyday experiences can be incorporated into and realise now that problems can be puzzle-type, project based, games etc. as explored in the lecture/tutorial sessions. This wider focus may also be reflected in the change to the response “Makes the children think” suggesting that more participants see the benefit of providing a challenge for children. The responses to “Various strategies can be

used” also points to the new awareness of the participants of the many strategies that can be employed when solving problems (See Table 4.12 and Section 3.5). Strangely pair/group work was down the list of priorities. An example of one such response from Participant 195, pre-intervention, referring to this problem for First Class, *There are 29 boys and 15 girls in school today. How many pupils are there in school today altogether?* was:-

*Yes it is a good problem. It follows the curriculum guidelines for the class group. It assesses their knowledge of place value as they have to regroup and it is set in a context relevant to children.*

Question 5 also threw up new responses post-intervention that had not appeared in the pre-instruction data results (See Table 4.13). The fact that these new responses occurred in the post-instruction data again indicate that the participants have learned a huge amount as to what constitutes a good problem. In addition to the responses in Table 4.13 some participants also alluded to topics such as differentiation, and peer learning as contributing to making a good problem.

<b>Extra Responses</b>	<b>Frequency</b>
Assess/test children	37
Use diagrams/concrete materials	41
Multiple entry points	14
Irrelevant/extraneous information	55
Multiple answers	59
Multiple steps	48

Table 4.13 Additional participants’ rationale for rating problems as ‘good’ post-intervention

Illustrating some of these responses, Participant 47, post-intervention, wrote the following problem: *Share 10 sweets among three children in three different ways.* (Third Class) When answering Question 5-Is it a good problem? Why? Participant 47, post-intervention, wrote:-

*In my opinion it is a good problem as it is open-ended and it encourages good in-class discussion on the answers It would highlight to the pupils that maths problems can have more than one/potential answer.*

#### **4.8 Conclusion.**

The findings of this research will be discussed and analysed in detail in the next chapter. However, this chapter reveals the usefulness and merit of the three week intervention with the participants. It points the way for further study in this area with both pre-service and practising primary teachers. The findings suggest that participants benefitted from exposure to the various types of problems and the various methods/strategies appropriate to problem solving and problem posing. These two areas resulted in the participants' post-intervention problems in general being better. The opportunity to engage in problem solving and in problem posing during the study intervention paid dividends in the quality of the problems posed.

## **Chapter Five**

### **Conclusions and Recommendations**

#### **5.1 Introduction**

In undertaking this study the author set out to ascertain pre-service primary teachers' understanding of what mathematical problems were early in their initial teacher education programme and to research if a study intervention could improve and or change these perceptions. This chapter seeks to closely examine the results of this research as laid out in chapter four and draw conclusions from the results. This in turn will lead to recommendations for further research in the area of problem posing in initial teacher education.

#### **5.2 Impetus for Study**

The interest and impetus for this study came from the author's experience as a primary teacher, who when faced with the task of setting or posing problems for her class was often at a loss as to how to go about this. The textbook, and subsequently websites, supplied some problems but these did not always suit the environment in which the author taught. Therefore, the task of setting or posing worthwhile, challenging and interesting problems was one which occupied and worried the author over many periods in her teaching career. On speaking to other teachers, the author realised that her experience was not unique and that they too faced the same dilemma. Various research studies report the important role that problem posing plays in mathematics education (See Section 2.2). Therefore, if this issue is addressed at the initial teacher education stage, primary teachers of the future will be more skilled in this very important area of mathematics education.

## **5.3 Summary of Study Findings.**

### **5.3.1 Responses to Question One-What is a Problem?**

In the pre-intervention phase in answering the question: “What is a problem?” (See Section 4.3) participants’ most popular answer was that a problem needs higher order or critical thinking. Problems were seen as containing words, numbers, is a story. Participants saw them as questions that have to be solved; these problems can be solved by using various strategies. This finding reflects the findings of Chapman in her studies of 1999 and 2005 with pre-service teachers (See Section 2.1.3)

Comparing this to the post-intervention data, the number of participants who mentioned problems as containing words, numbers or being a story had dropped considerably (See Table 4.1). One can surmise from this that participants’ understandings of what constituted a problem had changed or broadened. Again Chapman would have found similar changes in her studies. This finding was corroborated in the extra categories which arose in the post- intervention data such as ‘a problem can be a puzzle’ (See Table 4.2) and also in the types of problems which the participants wrote (See Section 4.4). The increased frequency of responses following the intervention identifying a problem as being ‘of various types or from various strands’ also showed a marked increase.

Another category in which there was an obvious drop in mentions following the intervention was ‘you have to find an answer to a problem’. This leads to the conclusion that now participants realise that the process of problem solving is more important than the product (See Table 4.1). This was also supported by the outcome in the response to - a problem must be solved. When comparing the collapsed themes (See Table 4.3) differences are also noticeable.

However, the most interesting aspect when analysing the post-instruction results was that a whole set of new responses turned up that was not mentioned in the pre-instruction data (See Table 4.2). These extra responses clearly indicate that the study intervention had borne fruit. The participants now see problems more broadly. Their eyes have been opened to other very important aspects of problems that they had previously not been aware of. These responses were reflected in the types of problems they set when answering question two post-instruction on the questionnaire. (See Section 4.4).

### **5.3.2. Responses to Question Two-Choose a class between 1<sup>st</sup> and 4<sup>th</sup> and Make a Problem that would be a Problem for that Class**

Post-intervention, the majority of the problems involved two or more steps. This was a big change from the pre-intervention data when one step problems were in the majority. (See Table 4.5). In 2003 Sandra Crespo undertook a study with pre-service teachers. The results of her study are very much mirrored in the findings here. Her study participants went from designing short, one-step, one answer problems to designing more adventurous and ambitious problems (See Section 2.2.5). This suggests that when the participants were more aware of the different facets of problems, they can come up with better and more challenging work for their pupils. When comparing pre and post- intervention results it is clear that the participants have been made aware of and taken into account aspects of problems which they may have heretofore not considered. The increase in puzzle-type problems, problems with multiple right answers and those containing extraneous information was quite heartening for those who had designed and delivered the intervention sessions. This shows that the participants had moved away from over-reliance on word problems (See Table 4.7).

### **5.3.3 Question Three - What did you Think of to Make a Problem?**

The initial responses to this question did not show any big differences between the pre and post-instruction (See Table 4.8). However, the extra responses presented in the post-intervention were very interesting (See Table 4.9). These extra categories show that the participants' knowledge of what constitutes a good problem has now been extended. Crespo (2003) found a similar change in the views of her study participants (See Section 2.2.6). They were able to generate better problems when they gained personal experience of problem solving. The inclusion of irrelevant information seems to have made an impact on participants as being useful in helping children to improve their problem solving and posing skills. The fact that they also considered that problems with more than one step are more valuable in the primary classroom is also a welcome development. Multiple strategies and multiple answers were also new considerations post-instruction. The use of drawings/diagrams and concrete materials would also show that the participants are now more aware of benefit of focusing on the process of problem solving as opposed to the answer.

### **5.3.4 Question Four-Why is it a Problem?**

The greatest change between the pre and post-instruction here was the response by participants that 'various strategies could be used' (See Table 4.10). This indicates that the participants could now relate to the children using the various strategies that they themselves had been introduced to in the lecture/tutorial sessions. Again this points to the usefulness of the intervention study in introducing the pre-service teachers to these strategies, they now had a variety of methods or approaches in tackling a problem (See Section 3.5). Working through the various problem solving approaches on the Problem Solving Wheel and Polya's four step heuristic gave them a way into solving problems. These strategies made problems less 'problematic'.

### 5.3.5 Question Five-Is it a 'good' Problem? Why?

The greatest change in responses here was that now participants did not see 'relating to children's lives' as vital as in the pre-intervention data (See Table 4.11). These additional responses that appeared following the intervention points to participants' understanding of problems as being broader than previously. The author accounts for this finding in that the participants now saw problems as more than words or stories incorporating settings or items from pupils' everyday lives. Now they had been introduced to the idea that problems were more than just word problems and could also be puzzles, games, projects, practical tasks, open-ended investigations, maths trails as laid out in the Primary School Mathematics Curriculum by the DES (1999). The author sees this a positive development. Anything that broadens a person's understanding of an issue is surely a good thing.

The fact that these new responses occurred in the post-intervention data again is an indication that the participants have learned a huge amount as to what constitutes a good problem. The usefulness of introducing the participants to the Vacc (1993) framework for categorising problems was clear in these findings. The participants could now think of the various components of what makes a good problem (See Section 2.2.6). In addition to the responses presented in Table 4.11, some participants also alluded to new considerations such as differentiation and peer learning as contributing to making a good problem (See Table 4.12). They have been made aware of new facets of problem-posing and have used this information when constructing problems for their chosen class.

## 5.4 Drawing Conclusions

On first checking out the pre-intervention data the author was immediately struck by the grasp these very young pre-service primary teachers had of the subject. They were only in their second semester of initial teacher education yet had already picked up much academic mathematical language such as critical thinking, strategies etc. In setting the problems they used settings and items that would appeal to children such as birthday parties, pizza and so on. This demonstrated to the author the participants' genuine engagement with their subject and their chosen career. This bodes well for their continued progression and success as student teachers and indeed as teachers. In examining the pre and post-intervention data, there can be no doubt but that the participants benefited greatly from the intervention. In every area of problem solving/posing their skills had improved. This is borne out in analysing the results of the questionnaires. Therefore, the conclusion must be drawn that this approach to problem solving and problem posing is very worthwhile in initial teacher education.

## 5.5. Recommendations

In recommending the inclusion of a module on problem solving/posing in initial teacher education mathematics courses one must be cognisant of the difficulties facing the initial teacher educators:

- ***The Pre-Service Teachers' Mathematical Ability.*** Hourigan and O'Donoghue (2007), Leavy and Sloane (2010) and NCCA (2005) all found that post-primary graduates perform best at lower order mathematical skills such as memorization of procedures and formulae as opposed to thinking creatively, providing reasons for solutions or engaging in mathematical problem solving. These short-comings must be acknowledged and addressed in initial teacher mathematics education courses. If these

pre-service teachers are not comfortable and familiar with these skills they will not promote them in their classrooms. (See Sections 2.2.4 and 2.2.5). As previously stated the implementation of the intended syllabus of Project Maths (2010) will negate this findings in future student teacher intakes. Research carried out by Jeffes et al (2013) on evaluating recent reform at Secondary level, and cited in Hourigan, Leavy and Carroll (2016), states that “emerging evidence of positive impacts on student experiences of, and attitudes towards, mathematics”.

- ***The Challenge of Change.*** The pre-service teachers have spent sixteen years in education prior to entering initial teacher education. This “apprenticeship of observation” must be challenged (Crespo, 2003, p.264). The influence of this “apprenticeship” was seen when most participants saw problems as containing words, numbers, a story. Their experience of problems had mostly been word problems set at the end of a textbook chapter on a particular topic. Therefore, when asked what a problem was they naturally reverted to what they knew. Changing this perception is not easy but it can be informed by research in this area. Changing the “what” and the “how” of teaching Mathematics are equally important. The author attended a seminar addressed by Magdalene Lampert in Marino Institute of Education in May, 2016. In her address titled “How does someone learn to teach responsibly and responsively” Lampert asserts that pre-service teachers need to be “coached” in the practices, principles and knowledge of ambitious teaching. This coaching, she asserts, should be tightly structured and constantly monitored and reflected upon. This approach, which somewhat mirrors Japanese lesson Study may be worth considering in future ITE mathematics courses.
- ***The Problem Types.*** Quite a lot of the intervention was dedicated to the study of what makes a good problem. The pre-service primary teachers were introduced to the very many facets of what makes a worthwhile problem. They were given the chance to write

and critically analyse their own problems. The usefulness of this approach was clear to be seen in the quality of some of post-intervention problems. This is an area that the author would recommend spending more time on. Problems as newly generated or reformulated from a given problem was also a very interesting aspect which the author came across in her research. This is an area that would be worthwhile including in an initial teacher education programme. The Brown and Walter (1983) what-if-not approach to changing the attributes of a problem thereby coming up with a new question (See Section 2.2.2). The issue of pupil generated problems was also a very interesting topic which we did not address in the study intervention (See Section 2.2.8). This aspect is stressed in NCTM (2000) and is one which the researcher would recommend including in future ITE mathematics courses.

- ***Methodologies.*** In the intervention phase of this study the pre-service teachers were given the chance to ‘think-pair-share’ when solving/posing problems. This approach, which allows the individual to first think their own thoughts, then share them with another and then with a bigger group has been found to be very effective in many areas of the curriculum. It allows someone to formulate their own thoughts before discussing them with others. Malaspina, Mallart and Font (2012) found that this socialization phase to be very effective in problem solving/posing (See Section 2.2.5). It resulted in the enhancement of the skills of the participants and in the quality of the problems posed. The benefits of group work was also outlined in the research of O’Shea (2003). His study also emphasised the value of the use of Polya’s heuristics when problem solving. This four step plan was used in the intervention study when tackling problem solving (See Section 2.1.4). This active and indeed interactive approach is to be recommended as corroborated by research in the field.
- ***Primary Pupils’ Mathematical Readiness.*** Of course, in mathematical problem posing, as in every area of the curriculum, pre-service primary teachers need to know the

developmental stage of their pupils. Leavy and O'Shea (2011) emphasise the importance of pupils understanding the work presented to them. "The relationship between understanding and problem solving is symbiotic. The tasks must be accessible to the learners in that they build on knowledge that the learners already have while at the same time being engaging and drawing on contexts and situations that are new to them" (Leavy and O'Shea, 2011, p.9). Therefore, the pre-service primary teachers must be fully aware of the level of ability and maturation of their pupils. In the data set, some of the problems posed were inappropriate for the class suggested (See Section 4.7). This finding indicates that more work is needed in this area within future ITE mathematics courses.

## **5.6 Limitations.**

Researcher bias is the most limiting factor of many academic studies. The researcher obviously sets out with pre-conceived ideas and hopes as to what the study will reveal. One can only hope that integrity will prevail and the results will be recorded faithfully.

While the results were heartening to all those involved in the study intervention the lack of a control group for comparison makes one wonder if these changes in participants' perceptions would have come about anyway in the course of their Mathematics Education studies.

The author would have wished for more time in helping to deliver the tutorial lessons of the study intervention. The three week intervention of three lecture/focus sessions and three tutorial sessions necessarily curtailed what could be achieved in the study intervention. The tutorial sessions were rushed as so much material had to be covered. The 'socialization' or discussion phase could have been more beneficial if time allowed. Time for writing problems

likewise was at a premium with pre-service teachers not having enough time to sort their thoughts.

Are the recorded changes in perception lasting and enduring? It would have been beneficial to see how this same cohort of pre-service teachers tackled problem solving/posing six months after the intervention but time did not allow for this to happen.

However, all things considered, the participants gained a lot from the intervention as shown from the results. All one can hope for is that this will stay with them as they progress in their teaching career.

### **5.7 Closing Statement.**

Undertaking this research study was a privilege and indeed a labour of love for the author as it is an area of special interest to her. Many of the things which came to light in the course of the research made perfect sense when viewed through the lens of many years of teaching experience. As in any study the research posed as many questions as it answered and so as the work progressed the author considered the many other areas of study which could be undertaken into the future. For example, it would be very interesting to observe how pre-service primary teachers would deal with a problem solving/posing lesson in the classroom pre and post- intervention. However, this study achieved what it set out to do and as such the author hopes that it will add to the body of knowledge on this very important aspect of initial teacher mathematics education.

## References

Abu-Elwan, R. (2009) *The Development of Mathematical Problem Posing Skills for Prospective Teachers*. Available:

[https://www.researchgate.net/publication/268011805\\_The\\_development\\_of\\_Mathematical\\_Problem\\_Posing\\_Skills\\_for\\_Prospective\\_Middle\\_School\\_Teachers](https://www.researchgate.net/publication/268011805_The_development_of_Mathematical_Problem_Posing_Skills_for_Prospective_Middle_School_Teachers)

Aguirre, J. and Speer, N. M. (1999) 'Examining the relationship between beliefs and goals in teacher practice'. *The Journal of Mathematical Behavior*, 18(3), 327–356.

Anderson, J., Sullivan, P. and White, P. (2005) 'Using a Schematic Model to Represent Influences on and Relationships between Teachers' Problem Solving Beliefs and Practices', *Mathematics Education Research Journal*, 17(2), 9-38.

Andreescu, T., Gallian, J., Kane, J. and Mertz, J. (2008) 'Cross-cultural analysis of students with exceptional talent in mathematical problem solving'. *Notices of the American Mathematical Society*, 55(10), 1248–1260.

Arbaugh, F. and Brown, C.A. (2004) *What makes a mathematical task worthwhile? Designing a learning tool for high school mathematics teachers*. National Council of Teachers of Mathematics.

Australian Education Council (1991), *A National Statement on Mathematics for Australian Schools*. Curriculum Corporation, Australia.

Balka, D. S. (1974) 'Creative Ability in Mathematics', *Arithmetic Teacher*, 21, 633–636.

Ball, D.L. (1990) 'The mathematical understandings that prospective teachers bring to teacher education', *The Elementary School Journal*, 90, 449-466.

Blum, W., & Niss, M. (1991) 'Applied mathematical problem solving, modelling, applications, and links to other subjects—State, trends and issues in mathematics instruction', *Educational Studies in Mathematics*, 22(1), 37–68.

Bonotto, C. (2013) 'Artefacts as sources for problem posing activities', *Educational Studies in Mathematics*, 83(1), 37-55.

Borwein, J.M. (2006) Aesthetics for the Working Mathematician. In Sinclair, N., Pimm, D. and Higginson, W. eds. *Mathematics and the Aesthetic. New approaches to an Ancient Affinity*. 21-40. New York Springer.

Brown, S.L. and Walter, M.I. (1983) *The Art of Problem Posing*. Hillsdale, NJ, Erlbaum.

Cai, J. and Brook, M (2006) 'Looking Back in Problem Solving', *Mathematics Teaching Incorporating Micromath*, (196), 42-45

Cai, J. and Hwang, S. (2002) 'Generalized and generative thinking in US and Chinese students' mathematical problem solving and problem posing'. *Journal of Mathematical Behaviour*, 21, 401-421.

Cai, J., Hwang, S., Jiang, C. and Silber, S. (2015) Problem Posing in Mathematics Education: Some answered and unanswered questions, in Singer, F.M., Ellerton, N. and Cai., J (eds.), *Mathematical Problem Posing, From Research to Effective Practice*. NY: Springer, 3-34.

Close, S. (2013). 'TIMSS 2011 and the Irish mathematics curriculum – what can we learn from curriculum matching and item analyses?', in Dooley, T, Nic Mhuirí, S., O'Reilly R. and Ward, R., eds., *Mathematics education: Crossing boundaries. Proceedings of the Fifth Conference on Research in Mathematics Education*, Dublin: St. Patrick's College, 64-76.

Conference Board of the Mathematical Sciences. (2001) *The Mathematical Education of Teachers*, Providence RI and Washington DC: American Mathematical Society and Mathematical Association of America.

Available: [http://www.cbmsweb.org/MET\\_Document/](http://www.cbmsweb.org/MET_Document/)

Chapman, O. (1999) 'In-service teacher development in Mathematical problem-solving' *Journal of Mathematics Teacher Education*, 2, 121-142.

Chapman, O. (2008) 'Instructional Practices to Facilitate Prospective Mathematics Teachers' Learning of Problem Solving for Teaching'. *Paper presented at ICME, Monterrey, Mexico*.

Corcoran, D. (2005) 'Mathematics Subject Knowledge of Irish Primary Pre-service teachers'. Paper presented at *European Conference on Educational Research*, University College Dublin.

Crespo, S. (2003) 'Learning to pose mathematical problems: exploring changes in pre-service teachers' practices' *Educational Studies in Mathematics*, 52, 243-270.

Crespo, S. and Sinclair, N. (2008) 'What makes a problem mathematically interesting: Inviting prospective teachers to pose better problems' *Journal of Mathematics Teacher Education*, 11, 295-415.

Cunningham, R. (2004) 'Problem posing: An opportunity for increasing student responsibility'. *Mathematics and Computer Education*, 38, 83-89.

Curran, T. (2014) 'We need to talk about mathematical problem solving', *InTouch*, May & June.

Davis, Robert B., Maher, Carolyn A. and Noddings, N. (eds.) (1990) *Journal for Research in Mathematics Education, Monograph No. 4, Constructivist views on the teaching and learning of mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Delaney, S. (2014) *Mathematics Teaching of High–Leverage Practices*. Dublin: Marino Institute of Education.

Delaney, S. (2010) 'The absent voice of teachers', *InTouch*, October.

Department of Education and Skills. (2011). *The National Strategy to Improve Literacy and Numeracy among Children and Young People. 2011-2020*.

Duncker, K. (1945) 'On Problem Solving' *Psychological Monographs*, 58 (5), 270.

Ebby, C.B. (2000) 'Learning to teach mathematics differently. The interaction between coursework and fieldwork for pre-service teachers', *Journal of Mathematics Teacher Education*, 3(1) 69-97.

Educational Research Centre (2015) *Improved performance among Primary School Pupils in the National Assessment of English Reading and Mathematics*. Press Release

Edwards, D. and Mercer, N. (1987) *Common Knowledge: The Development of Understanding in the Classroom*, Routledge, London.

Ellerton, N.F. (1986) 'Children's made-up mathematics problems-a new perspective on talented mathematicians'. *Educational Studies in Mathematics*, 17(3), 261-271.

English, L.D. (1998) 'Children's Problem Posing within formal and informal contexts', *Journal of Mathematics Teacher Education*, 29, 83-106

Ernest, P. (1989) 'The impact of beliefs on the teaching of mathematics'. In P. Ernest (ed) *Mathematics Teaching: The State of the Art*, London: The Falmer Press, 249-254.

Fan, L. and Zhu, Y. (2007) 'From convergence to divergence: the development of mathematical problem solving in research, curriculum, and classroom practice in Singapore', *ZDM*, 39(5), 491-501.

Francisco, J.M. and Maher, C.A. (2005). *International Group for the Psychology of Mathematics Education*, Paper presented at the 29<sup>th</sup> Conference of the International Group for the Psychology of Mathematics Education (Melbourne, Australia, Jul 10-15, 2005), 329-336.

Freire, P. (1970). *Pedagogy of the Oppressed*. New York: Herder and Herder.

Gerdes, P. (1985). 'Conditions and strategies for emancipatory mathematics education in undeveloped countries, *For the Learning of Mathematics*, 5(1), 15-20.

Getzels, J. W. and Jackson, P. W. (1962) *Creativity and intelligence: Exploration with gifted children*. New York: Wiley.

Ghasempour, Z, Baker, M.N. and Jahanshahloo, G.R. (2013) 'Innovation in Teaching and Learning through Problem-Posing Tasks and Metacognition Strategies', *International Journal of Pedagogical Innovations*, 1(1), 57-66.

Goldin, G.A. (1997) 'Observing Mathematical Problem Solving through Task Based Interviews'. *Journal of Research in Mathematics Education*, 9(4), 40-62.

Gonzales, N.A. (1994) 'Problem Posing: A neglected component in Mathematics courses for prospective elementary and middle school teachers', *School Science and Mathematics*, 94(2), 78-84.

Gubermann, R. and Leikin, R. (2013) 'Interesting and different Mathematical problems: Changing teachers' views by employing multiple-solution tasks', *Journal of Mathematics Teacher Education*, 16, 33-56.

Hadamard, J. (1945) *The Mathematician's Mind. The psychology of invention in the mathematics field*. Princeton NJ. Princeton University Press.

Hashimoto, Y. (1987) *Classroom practice of problem solving in Japanese elementary schools*. In J.P. Becker and T. Miwa (Eds) *Proceedings of the Us\_Japan seminar on Mathematical Problem Solving*. pp. 94-19 Columbus. OH:ERIC/SMEAC. Clearinghouse (ED 304-315)

Hashimoto, Y. and Sawada, T. (1984) 'Research on the mathematics teaching by developmental treatment of mathematical problems', in Kawaguchi, T., ed., *Proceedings of ICMI-JSME Regional Conference on Mathematical Education*, Tokyo: Japan Society of Mathematical Education, 309–313.

Healy, C.C. (1993) *Creating miracles. A story of student discovery*. Berkeley. CA. Key Curriculum Press.

Henningsten, M. and Stein, M.K. (1997) 'Mathematical tasks and student cognition: Classroom based factors that support or inhibit high level mathematical thinking and reasoning', *Journal of Research for Mathematics Education*, 28(5), 524-549.

Hensberry, K. K. R., & Jacobbe, T. (2012) 'The effects of Polya's heuristics and diary writing on children's problem solving', *Mathematics Education Research Journal*, 24(1), 59-85.

Hiebert, J., Carpenter, T.P., Fennema, E., Fuson, K.C., Wearne, D. and Murray, H. (1997) *Making Sense: Teaching and Learning Mathematics with Understanding*. Portsmouth. NH. Heinemann.

Hourigan, M. (2010) *Mathematical Subject Matter Knowledge*. NCE-MSTL Resource and research Guides. Vol. 2.

Hourigan, M. and O'Donoghue, J. (2007) 'Mathematical under-preparedness: the influence of the pre-tertiary mathematics experience on students' ability to make a successful transition to tertiary level mathematics courses in Ireland', *International Journal of Mathematics Education in Science and Technology*, 38(4), 461–476.

Hourigan, M., Leavy, A.M. and Carroll, C. (2016) 'Come in with an open mind': Changing attitudes towards mathematics in Primary teacher education. *Educational Research*, DOI: 10.1080/00131881.2016.1200340

Hourigan, M. and O'Donoghue, J. (2013). 'The Challenges facing initial teacher education: Irish prospective teachers' mathematics subject matter knowledge', *International Journal of Mathematics Education in Science and Technology*, 44(1), 36-58.

Karp, A. (2010) 'Analysing and attempting to overcome prospective teachers' difficulties during problem-solving instruction', *Journal of Mathematics Teacher Education*, 13, 121-139.

Keil, G. E. (1965) *Writing and solving original problems as a means of improving verbal arithmetic problem solving ability*. Doctoral dissertation, Indianan University.

Kilpatrick, J. (1987) *Problem formulation: Where do good problems come from?* In A.H. Schoenfeld (ed) *Cognitive Science and mathematics Education*. pp 123-147. Hillsdale. Erlbaum.

Kilpatrick, J., Swafford, J. and Findell, B. (Eds.) (2001) *Adding it up. Helping Children Learn Mathematics*. Washington DC National Academic Press.

Knapp, N. and Peterson, P. (1995) 'Teachers' interpretations of "CGI" after four years: Meanings and practices', *Journal of Research in Mathematics Education*, 26, 40-65.

Koenker, R.H. (1958) 'Twenty Methods for Improving Problem Solving', *The Arithmetic Teacher*, 5(2), 74-78.

Kontorovich, I., Koichu, B., Leikin, R. and Berman, A. (2011) 'An exploratory framework for handling the complexity of mathematical problem-posing in small groups', *Journal of Mathematical Behaviour*, 39(1), 149-161. .

Krutetskii, V. A. (1976) *The psychology of mathematical abilities in school children*. Chicago: University of Chicago Press.

Kwek, M.L. 2015. Using Problem Posing as a Formative Assessment Tool, in Singer, F.M., Ellerton, N. and Cai., J, eds., *Mathematical Problem Posing, From Research to Effective Practice*. NY: Springer, 273-292.

Lampert, M. (1990) 'When the problem is not the question and the solution is not the answer: Mathematical knowing and teaching', *American Educational Research Journal*, 27(1), 26-63.

Lampert, M. (2001) *Teaching Problems and the Problems of Teaching*. Yale University Press.

Lave, J. (1992) 'Word problems: a microcosm of theories of learning', in Light P and Butterworth G., eds, *Context and cognition: ways of learning and knowing*. Harvester Wheatsheaf, New York, 74–92.

Lavy, I. and Bershadsky, I. (2003) 'Problem posing via "what if not?" strategy in solid geometry-a case study', *Journal of Mathematical Behavior*, 22, 369-387.

Lavy, I. and Shriki, A. (2007) 'Problem Posing as a means for developing Mathematical Knowledge of Prospective Teachers', in Jeong-Ho, W., Kyo-Sik, P., Hee-Chan, L. and Dong-Yeop. S., eds., *Proceedings of 31<sup>st</sup> Conference of International Group for the Psychology of Mathematics Education (PME)*, Seoul, South Korea: PME, 129-136.

Leavy, A.M. and O'Shea, J.P. (2011) *Problem Posing and Pre-Service Primary Teachers: An Initial Study*. The Third National Conference on Research on Mathematics Education in Ireland. MEI IV, September, 22-23, Dublin.

Leavy, A. M. and Sloane, F. (2010) *Developing statistical literacy in primary level mathematics*. Dublin: Stationery Office.

Lee, J. (2012) 'Prospective elementary teachers' perceptions of real-life connections reflected in posing and evaluating story problems', *Journal of Mathematics Teacher Education*, 15, 429-452.

Leikin, R. (2003) 'Problem solving preferences of mathematics teachers: Focusing on symmetry', *Journal of Mathematics Teacher Education*, 6, 297-329.

Leung, S.S. (1993) *The relation of mathematics knowledge and creative things to the mathematical problem posing of prospective elementary school teachers on tasks differing in numerical information content*. Doctoral Dissertation: University of Pittsburg.

Leung, S. S. and Silver, E. A. (1997) 'The role of task format, mathematics knowledge, and creative thinking on the arithmetic problem posing of prospective elementary school teachers', *Mathematics Education Research Journal*, 9(1), 5-24.

Malaspina, U., Mallart, A. and Font, V. (2015) *Development of teachers' mathematical and didactic competencies by means of problem posing*. CERME 9

Marshall, S. (1995) *Schemas in Problem Solving*. Cambridge: University Press.

Merriam, S. (1988) *Case Study Research in Education: A Qualitative Approach*. San Francisco: Jossey-Bass Publications.

Morselli, F. and Sabena, C., (2014) *Let's play! Let's try with numbers! Pre-service teachers' affective pathways in problem solving*. University of Turin.

Muller-Hill, E. and Spies, S. (2014) *On the role of affect for sense making in learning mathematics-Aesthetic experiments in problem solving processes*. University of Cologne and Siegen University.

National Council for Curriculum and Assessment (NCCA) (1999) *Mathematics Primary School Curriculum*. Dublin: NCCA.

National Research Council (NRC) (2010) *Preparing Teachers. Building Sound Evidence for Sound Policy*. USA: National Academy of Sciences.

Ng, L. E. (2002) *Representation of problem solving in Singaporean primary mathematics textbooks with respect to types, Polya's model and heuristics*. Unpublished Master thesis. National Institute of Education, Singapore.

Norton, A. and Kastberg, S. (2012) 'Learning to pose cognitively demanding tasks through letter writing'. *Journal of Mathematics Teacher Education*, 15, 109-130.

Olson, J.C. and Knott, L. (2013) 'When a problem is more than a teacher's question', *Educational Studies in Mathematics*, 83(1), 27-36.

O'Shea, J.P. (2003) *Mathematical Problem Solving in the Senior Primary School: An Investigation into the Beliefs and Experiences of Class Teachers*. Unpublished M.Ed. Thesis: Mary Immaculate College, University of Limerick.

Penrose, R. (1974) 'The role of aesthetic in pure and applied mathematical research', *The Institute of Mathematics and its Applications*, 7/8(10), 266-271.

Perez, K.D. (1985) *Word Problems. Math in Action. Teacher's Guide & Resource*, Hayward, CA: Janus Book Publishers.

Perkins, R., Shiel, G., Merriman, B., Cosgrove, J. and Moran, G. (2013) *Learning for life: The achievements of 15-year-olds in Ireland on mathematics, reading literacy and science in PISA 2012*. Dublin: Educational Research Centre.

Polya, G. (1961) *Mathematical Discovery-On Understanding, Learning and Teaching Problem Solving*. Princeton University Press.

Polya, G. (1973) *How to Solve it. A new aspect of mathematical method*. Princeton University Press.

Putnam, R. T. and Borko, H. (2000) 'What do new views of knowledge and thinking have to say about research on teacher learning?', *Educational Researcher*, 29(1), 4-15

Roddick, C. Becker, J.R. and Pence, B.J. (2000) 'Capstone courses in Problem Solving for Prospective Secondary Teachers. Effects of beliefs and teaching practices', in *Proceedings of the 24<sup>th</sup> PME Conferences*, 4, 97-104.

Rosli, R., Goldsby, D., and Capraro, M.N. (2013) 'Assessing students' mathematical problem-solving and problem-posing skills'. *Asian Social Science*, 9(16), 54-60.

Rubio, N. (2012) *Competencies of professionals in didactic analysis of mathematical processes*. Unpublished Doctoral Thesis. University of Barcelona, Spain.

Schleppegrell, M.J. and Bowman, B. (1995) 'Problem-posing: a tool for curriculum renewal', *ELT Journal*, 49(4), Oxford: University Press.

Schoenfeld, A.H. (1983) *Problem solving in Mathematics Curriculums. A report, recommendations and an annotated bibliography*. Washington DC: The Mathematical Association of America.

Schoenfeld, A.H. (1985). *Mathematical Problem Solving*. Orlando: Academic Press.

Schoenfeld, A.H. (1989) 'Exploration of students' mathematical beliefs and behaviours', *Journal of Research Mathematics Education*, 20(4), 338-355.

Schoenfeld, A.H. (1992) Learning to think mathematically. Problem solving, metacognition and sense making in mathematics, in Grouws, D.A., ed, *Handbook of Research on Mathematics Teaching and Learning*. New York: McMillian, 334-370.

Seo, H.S. (1998) 'An important component on using the what-if-not strategy', *Research in Mathematical Education*, 3(1), 13-19.

Shiel, G., Kavanagh, L. and Millar, D. (2014) *The 2014 National Assessment in English Reading and Mathematics Performance Report*. Dublin: Educational Research Centre.

Shimada, K. (1997) *The significance of an open-ended approach*. In T.P Becker and S. Shimada (Eds.) *The open-ended approach: A new proposal for teaching Mathematics*. Reston VA National Council of teachers of Mathematics.

Silver, E.A. (1994) 'On mathematical problem posing', *For the Learning of Mathematics*, 14(1), 19-28.

Silver, E.A. (2013) 'Problem posing research in Mathematics education: looking back, looking around, looking ahead', *Educational Studies in Mathematics*, 83, 157-163.

Silver, E.A. and Cai, J. (1996) 'An analysis of arithmetic problem posing by middle school students', *Journal of Research in Mathematics Education*, 27, 521-539.

Silver, E.A., Kilpatrick, J. and Schlesinger, B. (1990) *Thinking through Mathematics*. New York. The College Entrance Examination Board.

Silver, E. A., Smith, M. S., & Nelson, B. S. (1995) 'The QUASAR project: Equity concerns meet mathematics education reform in the middle school' in Secada, W., Fennema, E. and Byrd Adajian, L., eds., *New directions in equity in mathematics education*, New York: Cambridge University Press, 9-56.

Singer, F.M, and Voica, C. (2013) 'A problem-solving conceptual framework and its implications in designing problem-posing tasks', *Educational Studies in mathematics*, 83(1), 9-26.

Skinner, P. (1991) *What's your problem?: Posing and solving mathematical problems, K-2*. Portsmouth, NH: Heinemann.

Song, S., Yim, J., Shin., E.J., and Lee., H. (2007) 'Problem Posing with use of the What if not? strategy in NIM game' in Woo, J. H., Lew, H. C., Park, K. S. and Seo, D. Y., eds., *Proceedings of 31<sup>st</sup> Conference of International Group for the Psychology of Mathematics Education*, 4, 193-200.

Southwell, B. (1998) *Problem solving through problem posing. The experience of two teacher education students*. In C. Kanes, M.Goos and E. Warren. (Eds) *Teaching mathematics in new times. Proceedings of 21<sup>st</sup> Annual Conference of the Mathematics Education Research Group of Australia. (MERGA-21) Gold Coast.*

Stigler, J.W., and Hiebert, J. (1999) *The Teaching Gap: Best ideas from the world's teachers for improving education in the classroom*. New York. The Free Press.

Streefland, L. (1987) *How to Teach Fractions So As to Be Useful*, OW & OC, The State University of Utrecht, Utrecht, The Netherlands.

Streefland, L. (1991) *Fractions in Realistic Mathematics Education: A Paradigm of Developmental Research* (R. Rainero et al., Trans.), Kluwer Academic Publishers, The Netherlands.

Sullivan, P. and Mornane, A. (2013) 'Exploring teachers' use of, and students' reactions to challenging mathematics tasks'. *Mathematics Education Research Journal*, 26, 193-213.

Surgenor, P. Shiel, G. Close, S. and Millar, D. Counting on Success. Prepared for the DES by the E.R.C. (2006)

Suter, W.N., 2012. *Introduction to Educational Research: A Critical Thinking Approach*. 2<sup>nd</sup> Edition. UK: Sage Publications.

Sweller, J., Clark, R., & Kirschner, P. (2010) 'Teaching general problem-solving skills is not a substitute for, or a viable addition to, teaching mathematics', *Notices of the American Mathematical Society*, 57(10), 1303–1304.

Szydlik, J.E., Szydlik, S.D. and Benson, S.R. (2003) 'Exploring changes in pre-service elementary teachers' mathematical beliefs', *Journal of Mathematics Teacher Education*, 6(3), 253-279.

Thompson, A. G. (1984) 'The relationship of teachers' conceptions of mathematics and mathematics teaching to instructional practice', *Educational Studies in Mathematics*, 15(2), 105–127.

Vacc, N. (1993) 'Questioning in the Mathematics Classroom', *Arithmetic Teacher*, 41(2) 88-91.

Van den Brink, J.F. (1987) 'Children as arithmetic book authors', *For the Learning of Mathematics*, 7(2), 44-48.

Van de Walle, J. *Elementary and Middle School Mathematics: Teaching Developmentally.*

5<sup>th</sup> Edition Pearson New International Edition.

Van de Walle, J., Karp, K.S., and Bay-Williams J. M. (2014) *Elementary and Middle School Mathematics Teaching Developmentally.* Eighth Edition. Pearson New International Edition.

Verschaffel, L., De Corte, E. and Borghart I. (1997) 'Pre-service teachers' conceptions and beliefs about the role of real-world knowledge in mathematics modelling of school word problems', *Learning and Instruction*, 7(4) 339-359

von Glasersfeld, Ernst. (ed.). (1991). *Radical constructivism in mathematics education.* Dordrecht. Holland: Kluwer Academic Publishers.

Wilburne, J.M. (2006) 'Preparing Preservice Elementary Teachers to Teach Problem Solving', *Teaching Children Mathematics*, 12(9), 454-463.

Wilson, S. and Berne, J. (1999) 'Teacher Learning and Acquisition of Professional Knowledge: An examination of professional development', in Iran-Nejad A. and Pearson, D., eds., *Review of Research in Education*, Washington DC. American Educational Research Association, 173-209.

Winograd, K. (1991) *Writing, solving and sharing original math story problems. Case studies in the cognitive behaviour of Fifth Grade children*. Doctoral dissertation. University of North Colorado.

Xenofontos, C. (2014) 'Defining Mathematical Problems and Problem Solving: Prospective Primary Teachers' Beliefs in Cyprus and England', *Journal of Research in Mathematics Education*, 26(2), 279-299.

## Appendix A

The questionnaire used in the study-  
pre and post-intervention.

1. What is a problem?
2. Choose a class from 1<sup>st</sup> to 4<sup>th</sup> and make a maths problem that would be a problem for those children.
3. What did you think of to make the problem?
4. Why is it a problem?
5. Is it a 'good' problem? Why?

## **Appendix B**

### MIREC Approval and Volunteer Consent Form





## Appendix C.

### Problems used in the Intervention Study.

#### Value-for-money Problem

### Categorise the Problem

In Tesco, a 6 pack of crisps costs €3.30. A single packet of crisps costs 70c. Which is better value?

It is your birthday party and you have €10 to spend on crisps. Tesco is low in stock, they have only salt and vinegar crisps in individual packs and cheese and onion in 6 packs. How many of each could you buy? Remember, not everyone likes the same flavour.

**TESCO**



#### Checklist

- Type of Problem?
- Curriculum strand(s)?
- Single or multiple entry points?
- Number of steps?
- Single or multiple correct solutions?
- Focus on procedures or concepts?
- Level of cognitive demand?

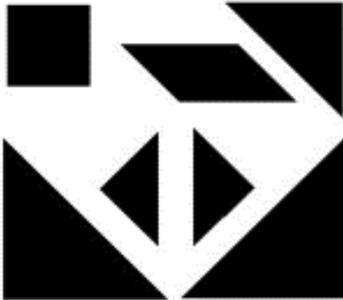
#### School Bus Problem

### Sample Test Items

- 126 pupils in a school are going on a trip to the museum. A coach holds 48 children. A minibus holds 16. Which of these should the school hire so that there are as few empty seats as possible?
  - A 2 coaches
  - B 2 coaches and 1 minibus
  - C 2 coaches and 2 minibuses
  - D 3 coaches

## Tangram Problem

### Problem posing using Tangrams



In your pairs **explore** the tangram pieces.

Write some **4-5 problems** relating to the tangrams.

Explore the **solvability** of your posed problems.

## Open-ended Problem.



**Noah**  
**Stage: 1**

**Noah watched the animals going into the ark. He was counting the legs of the animals and by noon he got to 12. How many creatures did he see? See if you can find other answers. Try to tell someone how you found out these answers.**

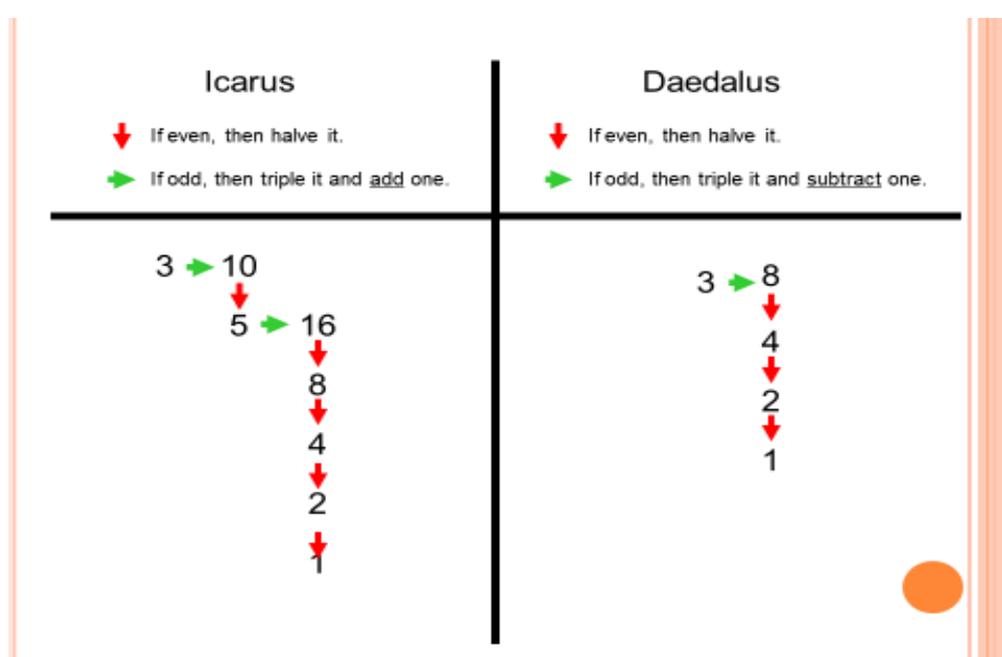
## The Daedalus and Icarus Problem

This problem is presented through the Greek myth of Daedalus and his son Icarus who were attempting to escape from their prison island by flying on wings fashioned from feathers and wax.

The night before the escape attempt, Icarus had a dream in which he had a pile of rocks on which were painted numbers. If the number was even he halved it, if it was odd he tripled it and added one. He was dismayed to find that every rock he picked up and threw ended up in the sea. Was there any number that did not have this fate?

Daedalus also had this same dream. However, if the number was odd he tripled it and subtracted one. Did he find any number that did not end up in the sea?

Can you try out different numbers and see can you save BOTH Icarus and Daedalus? Did you find any patterns emerging?



## Categorise the following problem

In 2011 8 children started playing hurling for their local club. Their ages ranged from 7 to 10 years old. The mode of all the ages was 10. The average of all the ages was 9. There was an equal amount of hurlers who were 9 years old and 7 years old. How many hurlers are there of each age?



**Checklist:**

- Type of Problem?
- Curriculum strand(s)?
- Number of steps?
- Number of correct solutions?
- Focus on procedures/concepts?
- Level of cognitive demand?

## Categorise the following problems

If each Justin Bieber ticket costs €25 and 678 people want to attend. How much money would the concert organizers raise?

John was swimming in a pool last week. Paul and Stephen jumped in and dispersed  $\frac{1}{4}$  of the pools contents. The amount of water that remained was 3000 litres. How much was in the pool to begin with?

Checklist	
Type of Problem?	Curriculum strand(s)?
Number of steps?	Number of correct solutions?
Focus on procedures/concepts?	Level of cognitive demand?

## The Countdown Numbers Game

### COUNTDOWN NUMBERS GAME

Try to get as close as you can to the target using the six numbers:

- You don't have to use all the numbers
- You can't use the same number more than once
- Only +, -, x and ÷ allowed

There are **four** big numbers: 25, 50, 75 and 100

There are **twenty** small numbers: Two each of 1, 2, ... 9, 10



The above task is based on the BBC Countdown programme a classroom version of which appears on [www.woodlandsjuniorschool.com](http://www.woodlandsjuniorschool.com)

## Sample Test Items

- **The time in Hong Kong is 8 hours ahead of Dublin. For example, mid-day in Dublin is 20.00 in Hong Kong. Sheila left Dublin for Hong Kong at 08:00. Her trip time was 13 hours. What was the local time in Hong Kong when she arrived?**

## Categorise the Problem

Mike is mixing cement to fill a hole in his footpath. The mixture he needs to fill the hole is 2 buckets of cement, 6 buckets of sand and exactly 4 litres of water. His bucket holds exactly 5 litres of water and he has a container that holds exactly 3 litres. How will Mike get exactly 4 litres of water to mix the cement?



### Checklist

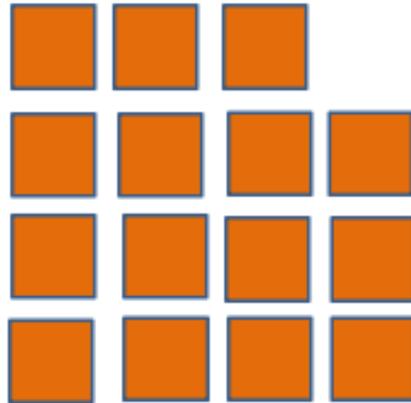
- Type of Problem?
- Curriculum strand(s)?
- Single or multiple entry points?
- Number of steps?
- Single or multiple correct solutions?
- Focus on procedures or concepts?
- Level of cognitive demand?

## LOGIC PROBLEMS

- Team of 4
- Every team player takes one clue card – if less than 4, some students take 2 clue cards
- Each person reads the clues aloud but should not show the clue to another person
- The team is to identify the problem and agree on a method for solving it
- The team members are to solve the problem cooperatively by assisting each other
- The problem is solved when all team members understand the solution and agree that the problem has been solved

## This is step 3

- Using pattern blocks, create 3 different patterns for which the given figure is step 3



## Patterns

- Step 3



- Step 4



- What will step 1, 2 look like? Why?
- What can you say about even/odd steps?

## The Pencil Problem

- In the morning, Ms. Wilkins put some pencils for her students in a pencil box. After a while, Ms. Wilkins found that  $\frac{1}{2}$  of the pencils were gone. A little later, she found that  $\frac{1}{3}$  of the pencils that were left from when she checked before were gone. Still later, Ms. Wilkins found that  $\frac{1}{4}$  of the pencils that were left from the last time she checked were gone. At this point there were 15 pencils left. No pencils were ever added to the box. How many pencils did Ms. Wilkins put in the pencil box in the morning?



## Money, money, money!!

- A business woman went to the bank and sent half of her money to a stockbroker. Other than a €2 parking fee before she entered the bank and a €1 postal fee after she left the bank, this was all the money she spent. On the second day she returned to the bank and sent half of her remaining money to the stockbroker. Once again, her only expenses were a €2 parking fee and a €1 postal fee. If she had €182 left, how much money did she have before her trip to the bank on the first day.



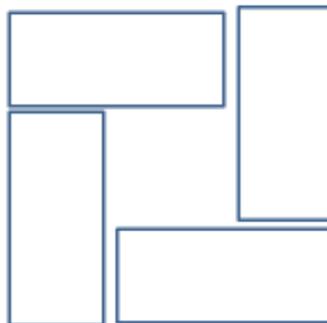
## Wages!

- Sue and Ann earned the same amount of money, although one worked 6 days more than the other. If Sue earned €36 per day and Ann earned €60 per day, how many days did each work?



## Domino Doughnut

- Select the dominoes which when placed appropriately work so that each side adds to 12



## Mystery Numbers

- When two numbers are multiplied, their product is 759, but when one is subtracted from the other, their difference is 10. What are the two numbers?



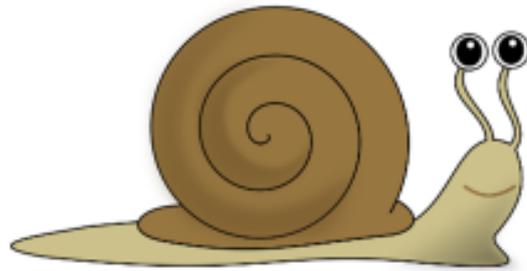
## Worth Going Green?

- Suppose the average annual heating a home with solar energy is €100, with an initial investment of €8000, and the average annual heating cost for a home with oil is €700 with an initial investment of €2000. Find the number of years before the cost of heating with solar energy will equal the cost of heating with oil.



## Slow as a snail 😊

- A well is 20 metres deep. A snail at the bottom climbs 4 metres each day and slips back down 2 metres each night. How many days will it take the snail to reach the top of the well?



## Candy Bar Problem

- Five jars numbered 1 through 5 contain a total of 92 candy bars. If each jar contains 2 more candy bars than the previous jar, how much candy in each jar?



# Handshake Problem

- There are 15 people in the room and each person shakes hands exactly once with everyone else. How many handshakes take place?



## Interior Design!!!

- The two new computers have finally arrived! You can help design the layout for the new computer station in your room. There are 5 tables available. At least 8 children must be able to sit at the station at a time. Only one person can sit at a table with a computer. All the tables must be touching at least one other table. If one person can sit on each side of the other tables, how would you design the space?



## Too good to be true!

- Max bought some plates at a car boot sale for a bargain! When he got home he found that  $\frac{2}{3}$  of the plates were chipped,  $\frac{1}{2}$  were cracked and  $\frac{1}{4}$  were both chipped and cracked. Only 2 plates were without chips or cracks. How many plates did he buy?



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## Cats vs. Dogs!

- The animal shelter has only dogs and cats. There are 25% more dogs than cats. What percentage of the animal shelter are cats. Explain your answer



## What's Bugging You?

- In a terrarium, there are 10% more female bugs than male bugs. If there are 8 more female bugs than male bugs, how many bugs are there in the terrarium?



- Use pictures, words or models to support your explanation.
- What would be a 'typical' error you would foresee?

### **The Census Problem**

A census taker approaches a house and asks the woman who answers the door:

"How many children do you have and what are their ages?"

"I have three children, the product of their ages is 36, the sum of their ages is equal to the address of the house next door." replies the woman.

The census taker walks next door, comes back and says to the woman:

"I need more information"

"I have to go, my oldest child is sleeping upstairs."

Thank you, I now have all the information I need."

**What are the ages of the children?**

## Warm-up

- How many addition signs should be put between digits of the number **987654321** and where should we put them to get a total of **99**?

### *Halving the Squares*

- Set out 15 lollipop sticks in the pattern shown below.
- How many squares are there?
- Now remove three of the lollipop sticks, without moving any of the others, and turn the six squares into three.



- Set out the lollipop sticks in the original pattern.
- Remove two lollipop sticks and turn the six squares into four.

## Dublin Zoo

Dublin Zoo has just received two new sheep for the Family Farm part of the zoo.

The zoo keeper wants to build an enclosure for the sheep.

She decides that the enclosure must be square or rectangular with an area of 100 square metres.

## Dublin Zoo

1. Which different configurations could she build?
2. How many metres of fencing will she need for each possible design?
3. Use your copy or some graph paper to draw all the possible rectangular or square designs.
4. Include a key to tell how much each unit on the grid paper equals.
5. Which fence would you recommend that the zoo keeper builds? Why?



# Noah



Noah saw 12  
legs walk by  
into the Ark.

How many  
creatures did  
he see?



Thousands more problems can be found  
on the NRICH Maths website:

<http://nrich.maths.org>