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PROBLEM POSING AND PRE-SERVICE PRIMARY TEACHERS: AN INITIAL STUDY

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Problem solving is an important component of primary mathematics. Central to the activity of problem solving is the task of problem posing. This study examines the skills and predispositions of pre-service teachers when posing problems for use in primary classrooms. The preliminary study was carried out with 36 pre-service teachers at the end of their first semester of a teacher education programme. Participants were asked to construct a problem for use in a primary classroom. Analysis of the problems revealed that problems were very similar across participants and tended to be single step problems with one possible solution. Problems were predominantly computational and focused on procedures and operations relating to quantities. The mathematical content of problems was situated within the number and measurement strands of the primary curriculum. We identify experiences, models and structures that have been shown to be effective in bringing improvements in problem posing in international contexts. Recommendations are made for practice in teacher education contexts to support pre-service teachers in developing the pedagogical skills required to teach problem solving.

INTRODUCTION

Problem solving is a critical domain in mathematics education. Francisco and Maher (2005), in a longitudinal study investigating the conditions for promoting reasoning in problem solving, state that 'providing students with the opportunity to work on complex tasks as opposed to simple tasks is crucial for stimulating their mathematical reasoning' (p. 731). Central to the activity of problem solving is the task of problem posing. Problem posing is a fundamental component of teaching and learning mathematics. Problem posing involves formulating problems that require children to draw on mathematical knowledge and skills in order to answer those problems; or involves the creation of new problems from old ones. These problems usually take the form of word problems.

Traditional efforts at presenting word problems in school mathematics have been criticized in terms of students' lack of sense making (Schoenfeld, 1991; Silver et al., 1993) and for their lack of effectiveness in making mathematics seem connected to real world contexts and thus seem meaningful and real for learners (Lave, 1992). Ng (2002) found that textbooks provide a high portion of routine, closed-ended problems and problems with exactly sufficient

information. Heuristics suggested by curricula are not all covered by the traditional textbook, and textbooks should include more open-ended problems, non-routine problems, authentic problems and problems with insufficient or extraneous information as well as other traditional problems (Fan and Zhu, 2007). Part of the reason accounting for the shortfall of traditional approaches to teaching problem solving is the ways in which text book writers, particularly in the Irish context, conceptualized the role of problems solving in mathematics instruction. Problem solving was seen as an activity that occurred *after* children studied mathematical skills and concepts (hence we saw the placement of word problems at the ‘end of the chapter’) as opposed to being a *means to acquiring* new mathematical knowledge and understandings.

Preparing prospective teachers to teach problem solving

Recent reform views of mathematics posit problem solving as ‘a vehicle for learning new mathematical ideas and skills’ (NCTM 2000, p. 182). This involves reconceptualising mathematics teaching and learning as teaching through problem solving. Carpenter, Fennema, Peterson, Chiang and Loef (1989) emphasise the importance of posing problems to students and listening to how students describe the way they have solved such problems. However, Fosnot (1989) explains that teachers are unfamiliar with such teaching because they are products of systems that emphasised drill and procedure. Any program designed to equip pre-service teachers with the requisite mathematical and pedagogical understandings related to problem solving must take into consideration the mathematical experiences of these same pre-service teachers prior to entry to teacher education. The following studies reveal insights into the influence of school mathematical experiences on prospective teachers views of problem solving and reveal the degree of disconnect between learners experiences of problem context, problem structure and the strategies employed to solve problems.

A study of prospective secondary mathematics teachers’ knowledge of problems and problem solving (Chapman, 2005) revealed that participants made sense of problems in terms of the traditional, routine problems they had experienced from their school mathematical experiences. Their understanding of the problem solving process also aligned with classroom approaches they had experienced as learners. A study of 139 pre-service teachers carried out by Contreras & Martinez-Cruz (2003) found that many of them provided incorrect solutions to additive word problems due to incorrect interpretation of the solution in relation to the problem context. Similarly, Simon (1990) identified difficulties that pre-service primary teachers demonstrated in connecting the semantic features of the word problems and their understanding of division to the procedures that they employed to divide. There is also evidence of the disconnect between pre-service teachers real-world knowledge and the approaches they used to solve school word problems (Verschaffel and De Corte, 1996). Furthermore the same study found that pre-service teachers did not use their real-world knowledge of the problem contexts when considering the children’s responses to the same problems.

Selection of tasks and models

The selection of a problem solving task will depend on the classroom situation of the teacher. Francisco and Maher (2005) explain that students must be provided with the opportunity to work on complex tasks as opposed to simple tasks as such tasks are crucial for the development of mathematical reasoning. Wilburne (2006) has explained that tasks, the best mathematical problems one can employ in the classroom, are non-routine mathematical problems that encourage rich and meaningful mathematical discussions, those that don't exhibit any obvious solutions, and those that require the student to use various different strategies to solve them.

The selection of a problem-solving model to use as a structure within which to situate classroom-based problem solving activity is also an important consideration. Current empirical research specifically reveals the importance of having students justify solutions and fully exploring extension activities that may emanate from the problem solving exercise with them (Elmore, 1996; Francisco and Maher, 2005; Hoffman and Spatariu, 2007). O'Shea (2009) investigated the use of Polya's problem solving model and found the four stage procedure is helpful to provide structure to the problem solving lesson and ensure students engage with the problem, develop a unique method of solution and reflect on the challenge of the situation. O'Shea (2009) concluded that students are successful in their mathematical problem-solving interactions when they are presented with challenging mathematical problems that reflect their current level of understanding and when they are comfortable with a problem solving model. When students engage in reflection it can surprise the teacher and reveal what children are able to do which can often be far more than what the teacher might have given them credit for.

The provision of structured experiences focusing on problem posing has been shown to bring about improvements in how pre-service teachers think about and use word problems. While pre-service teachers often see word problems as opportunities to practice skills that children have already been taught, Chapman (2004) reported improvements in how pre-service teachers view word problems. They shifted from viewing word problems as consisting of a 'cover story' and practice with operations they already know to viewing word problems as consisting of a cover story and both mathematical and semantic structures. Their interpretation of multi-structure word problems improved as did their ability to represent the structure of word problems using a variety of representations (verbally, symbolically, concretely and pictorially) to shown the meaning of operations. A number of studies also examined instructional practices that help prospective teachers to develop their knowledge of problem solving for teaching. A study carried out by Arbaugh & Brown (2004) focused on helping prospective teachers select worthwhile problems by developing their understanding of the relationship between a task and the kind of thinking that task requires of learners. Prospective teachers engaged in task sorting activities that used level of cognitive demand as a determinant. The researchers found that participants enhanced their skills in analyzing problems in terms of the types of thinking required by students to solve them (as opposed to the tendency of novices to focus on surface features of problems). Other studies have focused

on developing prospective teachers' understanding of problem solving through the provision of experiences engaging in the problem solving process themselves as learners (Ebby, 2000; Szydlik, Szydlik, & Benson, 2003) and the provision of academic courses designed to target the problem-solving process and pedagogical processes related to problem solving (Chapman, 2005; Roddick, Becker & Pence, 2000).

Approaching the teaching of mathematical problem solving

There has been very little research examining Irish pre-service teachers understandings of and dispositions towards problem solving. Studies of in-service teachers approaches to teaching mathematical problem solving highlight the emphasis placed on exploring lower level processes with less emphasis placed on teaching higher level mathematical processes such as reasoning and problem solving (O'Shea, 2003; O'Shea, 2009). Furthermore, in a review by the NCCA (2008) of the implementation of the Primary School Curriculum it appears teachers struggle with engaging classrooms of students in group collaborations, which are central to successful problem solving situations. Entry level pre-service teachers carry a multitude of attitudes, beliefs and experiences relating to mathematics and problem solving emulating from their experiences as mathematics learners in the Irish secondary school system. While the secondary mathematics curriculum is under extensive review and redesign, it may take many years for the efforts of Project Mathematics to come to fruition. Consequently we draw on some data arising from review of the leaving certificate programme prior to the implementation and roll out of Project Mathematics.

In a discussion paper reviewing mathematics in post-primary education (NCCA, 2005), the National Council for Curriculum and Assessment comment on the ways that mathematics is taught at second level, with particular reference to the ways in which the teaching of mathematics reflects current trends internationally in mathematics education. With reference to the teaching of *problem solving* (or investigations) at the senior cycle, the report states

‘The exploratory, open-ended style associated with investigations does not seem to fit Irish teachers’ and students’ views of mathematics. Possible reasons for this may lie in the culture of mathematics teaching in this country .. , in the demands that this approach would make on teacher knowledge, skills and attitudes, and in the fact that such work is not currently subject to assessment in the examination.’ (p. 5)

Further insights into the problem solving capacities of Irish secondary school students, a proportion of whom continue on to become teachers of mathematics, can be gleaned from performance on international mathematics assessments. The Programme for International Student Assessment (PISA), coordinated by the Organisation for Economic Cooperation and Development (OECD), tests and compares school children's performance on ‘real-life’ tasks considered relevant for effective participation in adult society and for life-long learning. Student achievement in PISA is categorised along a 6-level continuum. Irish students are proficient at tasks associated with levels 1-3, yet do not compare as well when dealing with mathematical reasoning and developing approaches to analysing, evaluating and working with complex mathematical problems (levels 5-6). According to the PISA 2006 results (Eivers, Shiels & Cunningham, 2007) at levels 5 and 6 Ireland fares slightly less well than the OECD

average and considerably poorer than countries such as Korea and Hong Kong where over 27 per cent of students reached level 5 or higher (as compared to 10 percent of Irish children).

Several studies have been carried out of the mathematical understanding of entry-level pre-service teachers. The outcomes of this research provide insights into the problem-solving abilities of pre-service teachers. Corcoran (2005) carried out a study of 71 pre-service teachers to whom she administered eleven items released from the PISA 2000 assessment (OECD, 2002). Performance on tasks was mapped onto the PISA proficiency levels and results compared against Irish 15 year olds. In examining the two tasks categorized at highest proficiency level, level 6, analysis of responses revealed that less than 10% of student teachers got both items at level six fully correct. More than 20% got both items totally incorrect. The pattern of low performance at task difficulty levels five and six was found to be similar to Irish 15-year-olds performance in PISA 2003, where Irish students performed less well on items requiring higher order mathematical skills. Overall, the author identified a ceiling at proficiency level 4 for up to 80% of students. Analysis of the mathematical tasks with the lowest performance highlighted that more than 50% of student teachers show a concerning low level of the process skills identified in the primary curriculum of applying and problem solving skills together with communicating and expressing skills linked with formal reasoning. Other studies concur that post-primary graduate perform best at lower order mathematical skills such as memorisation of formulae and procedures as opposed to thinking creatively, providing reasons for solutions, or engaging in mathematical problem solving (Hourigan and O'Donoghue 2007; Leavy and Sloane, 2010; NCCA 2005).

METHOD

This study is a preliminary investigation of the understandings that pre-service teachers have of problem solving, with particular attention on the activity of problem posing. As such, the study represents an initial point of departure in assessing the understandings and beliefs about problem solving the Irish pre-service teachers have on entry to teacher education programs. The information arising from this study will be used to inform the design of instructional approaches to facilitate their learning of problem solving.

Participants

Participants were 38 pre-service primary teachers enrolled in the postgraduate diploma in primary education. All participants had primary degrees prior to enrolment in the teacher education diploma. Participants had completed their first semester of their three semester program. They had completed a semester-long course in the pedagogy of mathematics (10 hours) and had completed a 2 week teaching placement in a middle grades primary classroom. During the pedagogy of mathematics courses there was no explicit instruction on problem-solving or on problem posing. On the last class session of the semester, participants were asked to reflect on their experience of teaching practice and on their observations of children engaging in doing mathematics. They were then asked to discuss in groups their experiences

of writing problems for children while on teaching practice. They were provided with two discussion questions to consider during the course of their discussion: what makes a good problem? What different types of stimuli can we provide when designing problems? Participants were then asked to write a problem that they might pose to a group of middle grades children.

Following discussion of these problems, participants were informed of the purpose of this study. Participants were invited to provide a copy of their task for the instructor at the end of the session. Participation was voluntary and anonymous. These written problems serve as data upon which the following analysis is based.

ANALYSIS OF DATA

The sources of data used for the study were the problems participants constructed at the end of the semester. They were analysed according to three categories. These categories focused primarily on problem type, problem feature and mathematics content (Crespo, 2003). Within these categories, a number of criteria were identified upon which to classify problems, examples are: single step versus multistep, single solution versus multiple solution, Focus on procedures versus focus on concepts, low cognitive demand versus high cognitive demand, computational versus exploratory/puzzle-type, arithmetic versus beyond arithmetic. Problems were also categorized in terms of which of the strands of the primary curriculum the problem was situated in: number, shape and space, algebra, measures or data. The criteria above are not necessarily orthogonal nor discrete and we expected some overlap between criteria. For example, a problem that was procedural in nature might also fall into the criteria of being computational. However there is also the potential that a problem may not follow a general pattern and be categorized on opposite ends of similar dimensions. Hence, qualitative notes were also taken for each of the problems to provide context and narrative for the coding and categorization.

RESULTS

Before examining the results we need to be cognizant that study participants are early stage pre-service teachers with only one semester of coursework completed. Also, these pre-service teachers have had no instruction in problem solving or problem posing. Gathering data at this stage of their careers is, however, important as it provides information regarding the point of departure for instruction and provides signposts for the direction that teaching trajectories should take in order to address the needs of pre-service teachers for teaching problem solving.

Analysis of the problems posed by pre-service teachers reinforces the findings of similar international studies. Two thirds of the problems posed by participants were *single step* problems all of which had *one possible solution*. For the most part, these problems focused on *procedures related to operations* on numbers and correspondingly fell into the number or

measurement strands of the primary curriculum. A typical problem was similar to the following (response 16):

If each Justin Bieber ticket costs €25 and 678 people want to attend. How much money would the concert organizers raise?

While one-third of problems were not single step problems, these problems almost always involved two steps, both of which involved the use of an operation, for example addition followed by subtraction. Therefore, while they may be viewed as multi-step problems, both steps were procedural in nature. The following is an example of a problem (response 4) that was coded as multistep and fell within the strand of measures.

John was swimming in his pool last week. Paul and Stephen jumped in and dispersed $\frac{1}{4}$ of the pools contents. The amount of water that remained in the pool was 3,000 litres. How much was in the pool to begin with?

In terms of cognitive demand, the majority of problems posed (89%) were considered to be of low cognitive demand. When considering cognitive demand, we tried to consider the depth of mathematical knowledge required to complete the posed problem. Problems were categorized as consisting of low cognitive demand if they primarily involved the recall of a fact, term or procedure. While most of the one step problems were low in cognitive demand, so too were the multistep step problems which consisted of the implementation of procedures. Problems were categorized as high cognitive demand if they involved mathematical reasoning, higher order mathematical thinking, making logical arguments or developing conclusions. In the following task (response 8), which is an example of one of the four high cognitive demand tasks constructed, the problem is multistep, may have more than one solution (depending on how different children 'fix' the variables), and involves thinking extending beyond the implementation of mathematical procedures.

How many points do Wolves need to be certain of staying in the premiership?
[Present supplementary data such as the league tables, remaining fixtures etc.]

The majority of the problems were computational and focused on operational procedures. The selection of problems of this nature may have reflected participants' own school experiences of mathematics or have reflected the nature of the mathematics they taught while on teaching practice. Only four of the 38 problems were not computational; two of these problems were problems focusing on volume. Both problems (responses 29 and response 33- Figure 1) are presented below:

Sara and John are making a cake for their mother. They need 300ml of milk. However, they only have a 500ml container and a 200ml container. These containers do not have measures on them. How would you solve this problem?

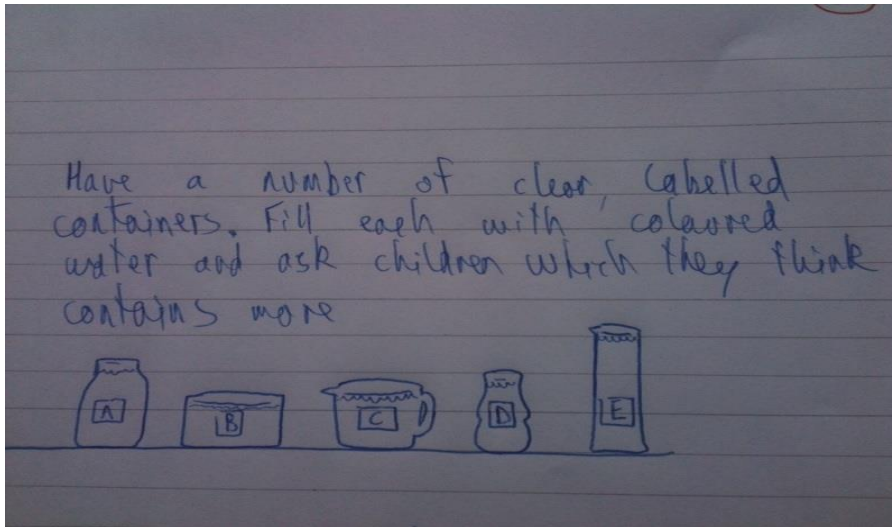


Figure 1: Response 33

As can be seen, both problems involve mathematical reasoning and some element of problem solving. Neither problem involves operations on quantities.

In terms of the mathematical content of the problems, problems were located in a number of the strands of the primary school mathematics curriculum. The table below provides an indication of how the problems were dispersed across the curriculum.

Strand	Number of problems (%)
Number	12 (33%)
Shape and Space	1 (3%)
Measurement	18 (47%)
Algebra	0 (0%)
Data	5 (14%)

Table 1: Dispersion of problems across strand units

Examination of the table reveals the dominance of measurement and number problems. The presence of number is somewhat masked in that many of the measurement problems were in fact problems that involved operations on quantities but used measurement as a context within which to situate the problem. In the problem that follows (response 29), we can see that it is a multi step problem involving the addition of three quantities and subsequent subtraction of the

quantity from a starting quantity. It is primarily an operations problem firmly located within the realm of number work but using length of string as the context. Consequently many of the measurement problems were, in fact, number problems.

If a piece of string measuring 1m 30cm was cut into sections of 8cm, 42cm and 68cm, and used to tie some balloons. What length of string would remain from the original piece of string?

There are several limitations posed by this study. Very little guidance was provided for study participants in terms of the target group for the problem. No specific grade level was specified and there was no guidance provided in terms of the ability of the children involved. This lack of clarity may account for some of the generality in terms of the problems constructed.

DISCUSSION

The goal of teaching mathematics through problem solving is that students will develop a deep understanding of mathematical concepts, skills and procedures. This relationship between understanding and problem solving is symbiotic. In order to solve problems, the learner must have conceptual understanding of the mathematics. Understanding, then, enhances problem solving. At the same time, a problem causes disequilibrium and requires that the learner make connections between the mathematics they already know and apply it to the problem situation as presented. Consequently, engaging in problem solving develops understanding. Not all teachers are adept at asking the appropriate questions in a way that enhances learning because they do not have the sufficient classroom experience or practice in associated discourse (Knott, 2010) and therefore in creating active classrooms, posing and solving cognitively challenging problems, promoting reflection, metacognition and facilitating broad ranging discussions. If we are to move towards teaching mathematics through problem solving, there are many considerations that need to be addressed in pedagogy courses to prepare prospective teachers to teach problem solving. Firstly, designing and/or choosing problematic tasks is not a trivial endeavour. The mathematics that we wish children to learn must be embedded within the tasks. The tasks must also be accessible to learners in that they build on knowledge that learners already have, while at the same time be engaging and draw on contexts and situations that are both interesting and relevant to learners. Pre-service teachers need also to be aware of the landscape of problem solving and of the ways in which children engage in problem solving. This involves attention to common approaches children use, pre-conceptions and misconceptions, in addition to possible learning trajectories that we might expect children to follow when engaging in problem solving activities. It is also useful to highlight attention to the role that textbooks play in guiding the nature of problem solving activities that Irish primary children are exposed to; in addition to engaging pre-service teachers in critiquing the quality and nature of textbook representations of problems.

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