Insights into Informal Inferential Reasoning in the Primary Classroom

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Introduction

Traditionally, statistics curricula at the primary level focus on descriptive statistics. Instruction and classroom activity centers on describing characteristics of samples or known populations of data. These descriptions take the form of graphs and summary measures using measures of central tendency and variability. This approach to data analysis is indicative of Exploratory Data Analysis (Tukey, 1977) which emphasizes generating hypotheses from analysis of observed phenomenon in contrast to confirmatory data analysis which is hypothesis driven and reliant on probability models. Inferences arising from exploratory data activities pertain only to the specific data under investigation and do not generalize beyond the data.

There is increasing recognition of the importance of developing young students' informal inference skills. Informal inference involves making generalizations that extend beyond the data (Makar & Rubin, 2009; Pfannkuch, 2006; Rossman, 2008). This shift in focus from descriptive statistics to informal inference is motivated by a number of factors. Firstly, the evidenced difficulties with formal inferential reasoning have lead to suggestions that inferential reasoning be tackled at an earlier stage in the educational process, motivated by the belief that 'developing students' informal knowledge related to statistical inference may ease their transition to understanding formal ideas of inference' (Zieffler, Garfield, DelMas & Reading, 2008, p.43). Another argument supporting the early introduction of inference is that the emphasis on exploratory data techniques at the primary level may communicate a deterministic sense of statistics and limit opportunities to look beyond the data (Ben-Zvi, 2006). The current approaches to data analysis at the primary level emphasizing the description of given distributions of data, through summary measures of centre and variability, may communicate that statistics is a field of study where all outcomes can be predicted exactly. The earlier introduction of informal inference may provide experiences in *looking beyond the data* (Curcio, 1987; Friel, Curcio & Bright, 2001) which are central to inferential reasoning thus positioning inferential reasoning as the 'bridge' between primary level exploratory data analysis and formal statistical inference (Ben-Zvi, Gil & Apel, 2007).

Theoretical Background

Informal Inferential Reasoning

There have been a number of efforts to define and posit a conceptual framework for *Informal Inferential Reasoning* (IIR). At a fundamental level IIR involves drawing conclusions and making *generalizations* that extend beyond the data at hand (Makar & Rubin, 2009; Pfannkuch, 2006; Rossman, 2008). This ability to *read beyond the data* (Curcio, 1987; Friel, Bright & Curio, 2001) requires the ability to extrapolate from the data and make inferences about unknown populations. The construction of generalizations about data is concomitant with the need for *data-based evidence* to support these conclusions (Makar & Rubin, 2009; Ben-Zvi, 2006). What we consider as constituting evidence will differ depending on the age and experiences of students.

Another important element of IIR is having an *aggregate* view of data (Rubin, Hammerman & Konold, 2006). Reasoning about aggregates requires the ability to move beyond looking at individual data values and viewing the distribution as a collective. These skills are critical to making inferences about populations based on a given sample and also critical when making group comparisons. Reasoning about aggregates is closely related to *reasoning about signal and noise* and *reasoning about variability* both of which are classified as properties of aggregates (Rubin et al. 2006) and as cognitive aspects of IIR (Ben-Zvi, Gil & Apel, 2007). Two properties of aggregates, which play key roles in inferential reasoning, are *reasoning about signal and* *noise*. Data analysis is characterized by Konold & Pollatsek (2002) as the search for signals in noisy processes–signals are underlying patterns and structures in the data, such as measures of center, whereas noise constitutes randomness and unknown sources of deviation in the data. *Reasoning about variability* involves attributing the sources to noise found in a set of data and establishing whether the variability is due to measurement error, causal factors, or sample-to-sample variability (Rubin et al. 2006). Rubin et al. also characterize informal inferential reasoning as involving consideration of *sample size* (i.e. the potential of a sample to estimate a particular population) and the need to *control for bias* in the selection of samples.

Finally, *language* plays an important role in IIR and provides indicators of a shift from deterministic views of data to more probabilistic ways of reasoning. The expression of uncertainty through the use of probabilistic terms and references suggesting levels of confidence are indicators of a shift to probabilistic language. This articulation of uncertainty has been identified as a key principle of IIR (Makar & Rubin, 2009). The critical role played by argumentation posits language as a central player in IIR. The conclusions derived from analysis of sample data or group comparisons and the concomitant persuasive arguments based on data analysis highlight the central role of language (Ben-Zvi, 2006; Ben-Zvi et al. 2007).

Engaging in Informal Inferential Reasoning in the Primary Classroom

The emphasis placed on introducing students to inferential reasoning earlier in the school curricula leads to the question of how can inference-driven approaches to learning statistics be embodied in the primary classroom? In order to look *beyond the data* learners need to engage in *looking at the data* first and identify trends that may be expected to reflect a population or be due to some causal factor. The initial points of departure for IIR, then, may differ little from the current launching of data lessons where the emphasis is on descriptive statistics. What needs to change, however, are the contexts within which we engage learners in reasoning about data. There seems to be agreement arising from studies that engaging young learners in IIR requires the provision of *statistical investigative activities*. Makar and Rubin (2009) argue for the almost symbiotic interplay between inference and investigation and contend that 'inference and statistical investigation cannot be separated' (p. 84). Statistical investigations, they stress, should be motivated by a compelling question, be situated within an engaging context, and produce data that are sufficiently complex so as to support reasoning and discussion. Pfannkuch (2006) also stresses the importance of situating informal inferential reasoning within the context of an empirical inquiry cycle.

Situating classroom pedagogical activities within the context of statistical investigations supports the design of activities that incorporate informal inferential reasoning. Two activities emerge from the literature: (I) Using sample data to reason about characteristics of a population. Activities falling within this categorization require learners to make generalizations beyond a sample to the population (Ben-Zvi, 2006; Pratt, Johnson-Wilder, Ainley, Mason, 2008; Zieffler et al., 2008). (II) Comparing samples of data to reason about possible differences between populations. These activities involve the comparison of two samples to ascertain whether differences exist followed by the generation or testing of hypotheses to account for observed differences (Makar & Rubin, 2009; Pfannkuch, 2006; Watson & Moritz, 1999). The selection of tasks can be further informed by the degree to which tasks require students to: Utilize prior knowledge to the extent that the knowledge is available (Zieffler et al., 2008); Provide evidence-based justifications for generalizations (Makar & Rubin, 2009); and use probabilistic language in describing the generalizations, including references to levels of certainty about conclusions drawn (Makar & Rubin, 2009).

Preparing Teachers to Focus on Informal Inferential Reasoning

The introduction of IIR poses obstacles for the preparation of teachers. In many countries preservice teachers enter teacher education courses with little more than procedural understandings of statistical concepts and with limited experiences of engaging in statistical investigations. For them, statistics involves the application of a number of formulas for computing measures of center and variability and a focus on the techniques of graph construction. These limited understandings present a number of challenges for the preparation of future teachers to teach informal inferential reasoning.

One challenge is the development of adequate *content knowledge of statistics*. Preservice teachers need to be able to encapsulate properties of distributions and support young learners in differentiating signal from noise when comparing distributions or making generalizations from samples. This requires rich and interconnected understandings of statistics extending beyond the application of skills in computing means and constructing graphs. Unfortunately there is ample evidence to indicate that preservice teachers may not possess these understandings. We know from the work of Skemp (1979) that many students possess instrumental understanding of quantitative concepts consisting of having a collection of isolated rules at their disposal rather than an appropriate conceptual schema. Undergraduate students and preservice teachers have well documented conceptual difficulties relating to concepts that underpin informal inferential reasoning: the mean (Gfeller, Niess & Lederman, 1999; Leavy & O'Loughlin, 2006), median (Friel & Bright, 1998; Groth & Bergner, 2006), variability (Canada, 2004), and reasoning about distributions (Leavy, 2004; Leavy, 2006). Another challenge is the development of *pedagogical content knowledge* (Shulman, 1986). These understandings demand that teachers possess deep understandings of concepts central to IIR. Maker and Rubin's (2009) presentation of classroom episodes of IIR highlight some challenges in terms of the pedagogical content knowledge necessary for the development of IIR in the primary grades, with particular reference to issues selecting sufficiently complex data, choosing engaging contexts, and supporting children in connecting conclusions to evidence and in making predictions. Another challenge is the context within which data handling is taught – moving to a *focus on inquiry based statistical investigations*. Several studies have investigated preservice teachers engaging in statistical investigations (Heaton & Michelson, 2002; Leavy, 2006) but the study of the transfer of this knowledge from college contexts to classrooms needs greater attention.

Research Questions

This study examines the process involved when preservice primary teachers design and teach data lessons addressing informal inference. The study examines the obstacles faced by participants while designing and teaching data lessons and investigates the development of participants own content and pedagogical knowledge relating to teaching (informal inferential) statistics throughout the process.

Context

Lesson Study (Fernandez & Yoshida, 2004; Lewis, 2002; Lewis & Tsuchida, 1998) is used to identify the pedagogical content knowledge needed for teaching informal inference and to investigate how this knowledge is used by teachers when teaching. Lesson Study is an approach for studying teaching that utilizes detailed analyses of classroom lessons. Lesson study is used here study to facilitate the examination of both the planning of lessons and the implementation of those lessons in classrooms and thus provides an avenue to explore problems of practice in primary level statistics education.

The central activity in lesson study iss for participants to work collaboratively on the design and implementation of a study lesson. Participants were organized into groups of 5-6 to engage in the phases of Lesson Study. The first phase involved the *research and preparation* of a study lesson involving researching topics pertinent to the design of a lesson and the construction of a detailed lesson plan. The *implementation* stage involved one preservice teacher teaching the lesson in a primary classroom while the other group members observed and evaluated classroom activity and student learning. Group members then *reflected on and improved* the original lesson design through discussing their classroom observations and modifying the lesson design in line with their observations. The *second implementation* stage involved re teaching the lesson with a second class of primary students and *reflecting* upon observations. The cycle concluded with in-class presentations of the outcomes of each of the five lesson study groups.

Method

Participants

The study was carried out with 26 female final year preservice teachers during the concluding semester of their teacher education program. Participants had completed their mathematics education courses (three semesters) and all teaching practice requirements (at junior, middle and senior grades) and self selected into mathematics education as a cognate area of study. The mean age was 20.61 and all had studied higher level mathematics in secondary school or received an A or upper B grade in college entrance examinations in general level mathematics. 40% studied mathematics in the first year of the degree program and 20% were studying mathematics to degree level. All had studied statistics and probability in secondary school and had covered pedagogical approaches to teaching data handling in their college-level mathematics methods courses. Those studying mathematics to degree level had completed a course on probability and statistics in their second year of study.

Method

The research was conducted over a 12-week semester. The researcher was one of the instructors and had primary responsibility for instruction and supervision relating to lesson study. Participants were divided into five lesson study groups. The research design consisted of three structural components: (I) Introducing lesson study and inferential reasoning; (II) Conducting lesson study; and (III) Reflecting and reporting on lesson study.

During the *Introducing lesson study and inferential reasoning* stage, participants were introduced to lesson study and inferential reasoning over a three-week period. The introduction to lesson study involved the presentation of an overview of the process, the study and critique of seminal readings relating to lesson study (cf. Fernandez & Yoshida, 2004; Lewis & Tsuchida, 1998; Stigler & Hiebert, 1999) and structured discussions arising from these components. Focused time was dedicated to exploring inferential reasoning both from a statistical standpoint and extending into pedagogical perspectives on inferential reasoning. Participants studied a number of readings specifically dedicated to informal inferential reasoning (cf. Ben-Zvi, 2007; Makar & Rubin, 2009) and discussed these in light of the Irish mathematics curriculum. The researcher then modeled a lesson on informal inferential reasoning. The second stage was devoted to *conducting the lesson study*. This stage took place over a period of seven weeks during which time each group met regularly to research, design and undertake the teaching of a study lesson focusing on inferential reasoning. The first lesson was taught in a local primary school to a class of either 5^{th} or 6^{th} class children; the researcher was present for this lesson. Following the teaching of the lesson each lesson study group reflected on and refined the lesson which was then taught in a second school to a group of children in the same class. This lesson was further refined based on reflection. The final stage of the research involved each group *reflecting and reporting on the lesson study.* This stage was centered on three primary activities: a group interview, presentation of an individual reflective paper, and a group presentation to their peers.

Data Collection and Analysis

Data collection methods were synchronized closely with the stages of lesson study. Groups were observed during all phases of lesson study, and the primary methods of data collection constituted: observation, videotaped interviews, and written reports and responses produced by groups. The content of group discussions was recorded and groups kept a log of all planning session held within and outside of the university context. Prior to the teaching of the initial lesson, each group met with the researcher on two occasions to outline their lesson and discuss aspects of the plan. The researcher observed the first classroom teaching session in a local primary school. The researcher also attended the debriefing session following the initial lesson and insights from the reflection stage were used to shed light on what participants attended to when teaching the lesson. At the conclusion of the lesson study cycle each group was interviewed about the lesson study and groups provided an in-class presentation at the end of semester, both of which were videotaped. Each participant compiled a written report outlining a detailed narrative account of their understandings of the content taught and the development of that understanding over the lesson study cycle, all lesson plans developed, a list of sources used to draft the lesson plans, and a reflective critique of what they learned about the teaching IIR in addition to the development of their own understandings. Insights gleaned from the analysis of the data contributed to the development of refined understandings of the process of statistical learning and provided a focus on the development of statistical and pedagogical understandings.

Results

Ability to reason inferentially is necessary but not sufficient to teach IIR at the primary level: Analysis of data arising from the statistical investigation, carried out at the beginning of the study, revealed that pre-service teachers demonstrated many of the skills and understandings fundamental to IIR. They demonstrated relatively sophisticated reasoning about samples and the limitations of making predictions from small samples to a population. They spoke about the homogeneity of the small sample and the implications for constructing a value that would represent the population data. Their descriptions of the distributions indicated the ability to make appropriate generalizations, use evidence to support assertions, and they demonstrated robust understandings of variability, samples and bias. Despite these abilities to reason inferentially, participants demonstrated difficulties 'unpacking' (Ma, 1999) these understandings and making them accessible to primary level students.

The challenge of designing pedagogical contexts conductive to the development of IIR: Participants needed extensive support in the development of pedagogical contexts that would facilitate the development of children's skills in informal inference. Difficulties arose relating to (a) the types of data most suitable to support inference, and (b) choosing investigative contexts. Initial suggestions for investigative activities were the comparison of left/right handedness across groups, the occurrence of birth months, and the incidence of eye colour in different populations etc. These investigations all generated discrete data. Discrete data, as compared to continuous data, severely limit the types of analyses that could be carried out on the data, and the degree to which inferences could be made. Participants did not seem cognizant of the critical role played by data type in supporting the development of statistical reasoning, a finding also emerging from the work of Ben-Zvi (2006) and Makar & Rubin (2009). Secondly, two of the groups initially designed investigations that involved the examination of association between variables rather than the comparison of variables. For example, one group wanted children to compare their shoe size and arm span in an effort to identify if larger show size was related to wider arm span. These investigations were focusing on identifying measures of association and were not, as envisioned, conducive to inferential reasoning.

Problems encountered when teaching IIR in primary classrooms: Despite the design of study lessons focusing on IIR, during initial implementations of the lessons participants reverted to a focus on *teaching statistical procedures*. This arose within four of the lesson study groups and was evidenced in inordinate amounts of time devoted to calculating means and engaging in the techniques of graph construction. As IIR activities were positioned at the end of the lessons, these unplanned activities meant that many groups did not have the *time* to engage children in the IIR activities. Another issue, which dominated initial lessons, was the *lack of justification and evidence-based reasoning* in the lessons. While many of the groups engaged children in conversations about the investigation and the context, at times these conversations did are not succeed in getting children to talk *statistically*. There were two factors which contributed to the lack of justification and evidence-based reasoning. Firstly, participants had difficulty using questions effectively to develop children's informal inference. The questions posed to children were too broad and did not direct children toward statistical aspects of the data. Secondly, preservice teachers struggled to deal appropriately with children's responses to activities. What emerged, then, within classrooms was the tendency for assertions made by children to be 'taken as true' without the provision of data-based evidence to support assertions. There was a failure to focus children on analysis of the data, the identification of patterns, and the generation of assertions arising from those patterns. This challenge of referring to the data at hand as evidence when drawing conclusions about data has been documented with elementary children involved in statistical investigations in small groups (Hancock, Kaput & Goldstein, 1992) and in classroom situations (Makar & Rubin, 2009).

Development of understanding relating to IIR: Participants identified the IIR lesson modeled by the instructor at the beginning of the semester as an activity critical to the development of their understanding. However, data analysis revealed that participants own understandings of IIR developed primarily as a result of engaging in lesson study. The act of teaching IIR to primary level students and being accountable for the development of their understanding provided an impetus for the examination and development of participants' own understandings. The consideration and prediction of children's responses when planning the IIR lesson, and the requirement to respond to and take children's comments into consideration when refining the lesson, provided a valuable learning opportunity for preservice teachers. This, taken in conjunction with the considerable emphasis on observation of and reflection on the development of understandings within study lessons, provided contexts ripe for the development of content and pedagogical content knowledge relating to informal inference.

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