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Mathematical under-preparedness: the influence of the pre-tertiary mathematics experience on students' ability to make a successful transition to tertiary level mathematics courses in Ireland

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Internationally, the consequences of the 'Mathematics problem' are a source of concern for the education sector and governments alike. Growing consensus exists that the inability of students to successfully make the transition to tertiary level mathematics education lies in the substantial mismatch between the nature of entrants' pre-tertiary mathematical experiences and subsequent tertiary level mathematics-intensive courses. This paper reports on an Irish study that focuses on the pre-tertiary mathematics experience of entering students and examined its influence on students' ability to make a successful transition to tertiary level mathematics. Brousseau's 'didactical contract' is used as an intellectual tool to uncover and describe the contract that exists in two case mathematics classrooms in Irish upper secondary schools (Senior Cycle). Although the authors are professional mathematics educators and well informed about classroom practice in Ireland, they were genuinely surprised by the very restrictive nature of this contract and the damaging consequences for students' future mathematical education.

1. Introduction

Mathematical proficiency of graduates is increasingly cited as a major factor in a country's economic success and competitiveness, especially graduates emanating from the numerate disciplines such as Science, Engineering, Technology and Business. Consequently, the findings of international reports [1–3] stating that many graduates of the numerate disciplines demonstrate deficiencies in mathematics are a source of serious concern. This phenomenon is clearly related to longstanding international disquiet (e.g. UK, USA) that students are entering mathematics-intensive courses with fewer of the basic mathematical skills essential for course success [2–6]. This situation, especially in the UK, has come to be known as 'the mathematics problem'. This phenomenon has far-reaching implications for the individual student, institution and economy alike. While wastage and failure rates in mathematics and dependent areas are a direct consequence of this problem, a lack of facility in mathematics impinges on a graduate's ability to fulfil aspects of his/her job description, e.g. ability to be innovative [2, 5, 7].

This paper reports on an Irish study that specifically sought to identify factors in the pre-tertiary mathematics education of students entering tertiary level mathematics courses that explained their under-preparedness for these courses. The so-called 'Mathematics problem' is interpreted in the Irish context leading to some unexpected insights. The study is framed by a discussion of the context and motivation for the study, and criticisms of secondary mathematics education in Ireland. Details of the study are given followed by a discussion of findings and the paper concludes with some ideas for local improvement.

2. The 'mathematics problem'

Although the body of Irish research on this phenomenon is limited, Mathematics Departments within Irish Third Level Institutions (e.g. Cork Institute of Technology, University of Cork, and University of Limerick (UL)) have expressed dissatisfaction with the mathematical ability of entrants since the mid-1980 s [5, 6, 8, 9]. Students who enter tertiary level courses lacking basic mathematical knowledge and skills are categorized as being mathematically 'under-prepared' or 'at-risk' [5]. On entry to mathematics-intensive courses, much of the mathematics that students find difficult shocks lecturers. Under-achievement within the Irish context seems to defy all logic, as first year mathematics courses at tertiary level generally overlap and extend the school Leaving Certificate Syllabus [4, 5]. Many

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students demonstrate knowledge that is fragmented, variable, and insecure [6]. Internationally (e.g. USA, Australia, UK, Ireland), ‘under-prepared’ students demonstrate some combination of the following characteristics:

- Large gaps in their knowledge (e.g. trigonometry and complex numbers) [6].
- A lack of numerate skills necessary to cope with everyday life not to mention studying mathematics in college [5, 8].
- Little competence in algebraic manipulation/simplification [10].
- An inability to use or apply mathematics except in the simplest or most practised way, i.e. to solve problems that move beyond 1-step or practised 2-step solutions [6, 11].
- Inability to make valid judgments and interpretations or to reason mathematically e.g. 1/xþ1/y¼1/xy [11–14].

A cause for further unease is the fact that under-preparedness does not correspond directly to terminal school examination results either in the UK or Ireland. In the UK for example, pupils achieving ‘A’ grades in mathematics are demonstrating characteristics traditionally associated with poor pupils, e.g. panic when presented with mathematics not seen before [15]. In the University of Limerick similar trends are evident, where 31% of students who achieved good grades, e.g. Ordinary Level ‘A1’ and ‘A2’ grades at Leaving Certificate, were diagnosed as being ‘at-risk’ [5]. Such data creates serious doubts regarding the presence of any definite correlation between examination achievement at school level and true student understanding. While all students entering tertiary level in Ireland have achieved above the minimum cut-off point in mathematics, i.e. D at Ordinary Level Leaving Certificate, this safeguard seems to have failed in one aspect of its prescribed role (i.e. to prevent the take-up of courses by students with sub-standard mathematics). Currently, there is consensus among university mathematics lecturers in Ireland that mathematics grades achieved in state examinations today are not comparable to the same grades 10 years ago, i.e. grade depreciation is evident [5, 9]. The Irish experience in this regard is very similar to the UK experience as reported by Lawson, who found that students with an N-grade at A-Level in 1991 out-performed D-grade students of 1995 in a diagnostic test [3].

All in all this is an unenviable position for tertiary level institutions. They are left in the difficult position of trying to educate students, despite what they see as their ‘substandard’ preparation in mathematics, to the standards of excellence previously attained [7]. Consequently a growing number of institutions internationally have had no alternative but to accommodate these ‘at-risk’ students by setting up short-term strategies to counteract the effects of their under-achievement [12] and to maintain exit standards upon course completion. While many universities in the UK have either adapted their first year courses (introductory year-core zero) or lengthened their courses’ duration to cater for under-prepared students, the most common practice internationally is the testing of entrants followed by the implementation of an appropriate intervention [15, 16]. Aftercare services used in mathematics departments range from self-help to one-to-one tutoring, including a range of ICT supports.

In the Irish context, the University of Limerick developed a package to detect and help ‘at-risk’ students entering the university. A 40 item open-ended diagnostic test was administered to entering students who were directed to a variety of support services based on their results. Some were asked to attend workshops based on a front-end basic skills package developed in-house followed by a series of support tutorials. Students also had access to additional support, e.g. drop-in service through the university’s Mathematics Learning Centre from 2001 [17].

3. Context and motivation for the study

The international move from elite to mass education has meant that the number and range of mathematical abilities of students entering tertiary level institutions have grown dramatically over the years [18]. Consequently, those who previously were not eligible for many courses, now have access [2]. The UK system, for example, accepts students on their mathematics-intensive courses from various routes including academic (General Certificate of Secondary Education (GCSE) and Vocational (Business and Technical Education Council (BTEC) and General National Vocational Qualifications (GNVQ)), as well as mature students. Consequently, it is impossible to identify topics and prerequisite knowledge that all students are guaranteed to be familiar with [19]. The UK identifies this issue as the single most detrimental factor leading to under-preparedness within mathematics-intensive courses at tertiary level.

The Irish system, on the other hand, does not exhibit the same degree of diversity in the mathematical preparation of its tertiary level entrants. While the intake to Irish universities and other higher education institutions is now more heterogeneous than ever before (i.e. entrance routes include Post Leaving Certificate (PLC)/FETAC awards and courses and direct entry mature students), this system is still considered homogeneous in nature as the vast majority of potential tertiary level students enters via the Leaving Certificate [5]. These facts prompted the authors to question why Irish students are exhibiting the same problems on entering university mathematics courses as their UK counterparts given the different contexts. On reflection the sameness of the descriptions of the problem in the UK and Ireland was surprising given two contextual factors that represented distinct advantages for Irish students over their UK counterparts:

- Irish students are obliged to study mathematics for the entire duration of their secondary school experience (five or six years).

- The vast majority of Irish students enter tertiary education with a mathematics qualification obtained on the State School Leaving Certificate Examination.

The authors concluded that the nature of mathematics teaching at pre-tertiary level, i.e. in secondary schools, directly influences the ability of students to successfully make the transition to tertiary level mathematics-intensive courses. Consequently, the authors sought to explore the nature of the ‘typical’ pre-tertiary mathematics experience (secondary school mathematics) of Irish pupils in an effort to better understand the nature of the transition from secondary to tertiary level mathematics. While there is consensus that the problem of under-preparedness always existed within the Irish system to some extent, what has altered considerably is the general acceptance in educational circles of the large number of students affected by it. While many internal factors within tertiary level institutions have been identified as potential causes of poor mathematical performance (e.g. large class groups, lecture format), the proposal that the root of the ‘mathematics problem’ lies in the failure of many students to make the transition to tertiary mathematics has gained momentum [5, 20, 21]. Prior to this, the authors, like so many others, would have associated failure in mathematics at tertiary level exclusively with a combination of a lack of effort, attendance and independent study on the part of students. This insight into the causes/sources of mathematical under-preparedness prompted the initiation of this investigation into the problem in the Irish context.

4. Criticisms of secondary school mathematics education in Ireland

A review of the relevant literature reveals that concern and discontent have been expressed for decades regarding the predominant practices in and standards of pupil learning and understanding of secondary school mathematics. Agreement exists that the ‘points system’ (selection mechanism for tertiary level entrance) causes the Irish Second Level Education

System to be extremely 'exam-oriented', a scenario that results in a very pragmatic approach, where teachers and pupils focus on the results required to guarantee third level entrance rather than the knowledge and skills essential for the future [6, 22]. National and international studies suggest that Irish mathematics classrooms are largely traditional, where exam-focused teaching relies on recall and routine procedures. In response to this phenomenon, there is evidence that pupils rely on rote memory and special-purpose algorithms as alternatives to understanding [4, 5].

As the Leaving Certificate course is assessed by terminal examination only, course dilution is a common strategy. This method involves omitting sections of the syllabus for the purposes of examinations, e.g. objectives which do not lend themselves to being assessed by pen and paper [4, 6]. This approach undoubtedly contributes to the existence of gaps in pupils' mathematics understanding on entry to third level courses. While it has been repeatedly recommended that the Irish mathematics education community must move away from the prevailing inflexible and unresponsive learning environment, which promotes mediocre learning and poor study habits, the reality is that there is no incentive to change because students can achieve high standards in the predominantly abstract, context-free Leaving Certificate examinations containing few challenging questions [23, 24].

The effects of the implemented curriculum pupil learning (i.e. the attained curriculum) can be gauged through an examination of the performance of Irish pupils in both national and international studies. The Chief Examiner's Reports have examined national Leaving Certificate mathematics standards and trends since 1995. The 2000 report was especially critical of the serious decline in ability, effort and understanding among Leaving Certificate students at all three levels (Higher (H), Ordinary (O) and Foundation (F)). The report suggested that a major gap existed between what was expected of these students and what in reality many of this cohort could competently do. A major concern was the lack of facility among many pupils in carrying out the most basic calculations and techniques. It also documented an apparent lack of relational understanding [25].

The mathematical performance of Irish Second Level pupils in the various international studies of achievement, e.g. Third International Mathematics and Science Study (TIMSS) and the Programme for International Student Assessment (PISA) has been categorized as ‘satisfactory’, that is, while Irish pupils achieved in the average range, their performance was significantly lower than countries such as Singapore, Korea and Japan [26–30]. While Irish pupils tended to perform better on tasks involving basic mathematical operations, limited proficiency in performing higher-level mathematical processes was consistently evident [26, 27, 29, 31]. In general Irish pupils performed relatively poorly on questions that required abstractions or the presentation of mathematical tasks in non-routine formats [6, 31].

The most recent comparative study, PISA, which endeavours to gauge the level of ‘mathematical literacy’ among pupils at/near the end of their compulsory education, i.e. 15 years, provides further insight into the mathematical ability of Irish pupils. Detailed investigation into the compatibility of PISA items with the Junior Certificate examination (content, context and processes tested) found that Irish pupils in lower secondary education did not excel in this test because its focus on conceptual understanding in realistic situations was at odds with the predominant ‘exam-oriented’ approach [32, 33].

The relevant literature highlights that the pressure on schools to deliver good examination results seriously weakens the greater aim of providing a high quality mathematics education [14, 34]. The literature examined provides strong evidence to support the conviction that, for many pupils, the predominant approaches are not producing the desirable mathematical skills, knowledge and attitudes required of pupils in modern society [35]. Thus the authors concluded that the nature of the mathematics education available to many Irish Second Level pupils, characterized by such a limited experience of mathematics as these reports suggest, would potentially diminish one's ability to make a successful transition to tertiary level mathematics-intensive courses.

5. The study

The authors sought to investigate the extent to which the 'typical' Irish Senior Cycle (Upper Secondary) mathematics experience contributes to under-preparedness among students entering tertiary level mathematics intensive courses. It was not intended to point the finger of blame in any direction, but rather to highlight unknown factors that determine the extent to which one can successfully make the transition to subsequent mathematics learning or not, with a view to suggesting alternative practices to improve the situation. To achieve this aim, it was essential that the data collection methods used provided a detailed and realistic insight into the precise nature of behaviours and attitudes of teachers and pupils alike within this context.

5.1. The case schools

A case study approach was adopted for the study. The study focused on a fifth year class group (i.e. group studying the first of a two year Senior Cycle Mathematics syllabus) selected from a community school (co-educational) and a convent school (all girls).

For the purposes of this study the former class group is referred to as the 'Mixed group' and the latter the 'Single sex group'. Selection of these sample groups was based on the voluntary co-operation of the class teachers and Principals of each school. Interestingly these schools had different methods of pupil grouping. The prevalent method in the convent school was streaming into levels for mathematics (Higher, Ordinary, Foundation) and further division into fast/slow moving Ordinary Level classes determined by Junior Certificate results. The community school undertook similar division of levels but all classes at Ordinary Level were mixed ability. The Single sex group was studying the Higher level course, while the mixed group was an Ordinary level class. The author had intentionally requested such variation (ability). The rationale behind this process was to have access to data relating to possible variations in teacher/student attitudes or behaviours that could possibly be associated with these factors.

Both participating teachers were assured of total confidentiality regarding the location and identity of any person involved. They were informed that all participants would be assigned pseudo-names in order to ensure total anonymity. T1 and T2 were used as a code for the teachers of the ‘Mixed’ and ‘Single sex’ groups respectively. In return the teachers were asked to treat the lesson preparation as for any other mathematics lesson they gave.

Being a case-study, a primary limitation was its narrow focus. While it is not proposed that this study is representative of all Senior Cycle mathematics classrooms, it can potentially offer practitioners and other interested parties a starting point/guidelines for future progress and development.

5.2. Theoretical framework

Brousseau’s concept of ‘didactical contract’ was exploited as an intellectual tool to further the objects of this research. This theory argues that an unspoken didactical contract exists between the participants of every mathematics classroom determining the roles, behaviours and attitudes of all ‘actors’ (teacher and students) within the classroom situation [36]. This research concluded that a child’s acquisition in a mathematical learning situation is not simply regulated by her level of intelligence, but is equally affected by many other relationships. The authors’ draw on Lim’s (2000) interpretation of Brousseau’s ‘didactical contract’ and the approach and findings are relevant to this study. Lim concluded that seven common elements were apparent almost exclusively in the four classrooms observed in that study. The authors could strongly identify with the conditions described in Lim’s ‘contract’. The teacher-led, variety-starved environment of Lim’s study reflected many of the characteristics of the typical Irish exam-orientated Senior Cycle mathematics classroom. This research provided the author with an appropriate conceptual framework to guide data collection and analysis of classroom practices [37].

5.3. Methodology

Qualitative non-participant observation was chosen as an optimum strategy for data collection [38] because Brousseau [36, p. 226] reported 'the hypothetical process of finding a contract is the contract', claiming that it is only possible to uncover the didactical contract present in any particular classroom by undertaking an in-depth study of the routine happenings (i.e. behaviours, interactions, and attitudes) of the mathematics classroom. This method provided a unique access to the otherwise somewhat 'closed world' of the mathematics classroom, allowing the author (MH) to capture all participants in their natural settings in everyday conditions [22].

5.3.1. Data collection. Although the author could never pass unnoticed, the aim was to be as unobtrusive as possible, in order for behaviours to be as normal as possible. To facilitate this, observations took place in both schools over a period of 10 weeks. The author felt that this lengthy contact period would enhance the development of a thorough insight into the true situation, in which participants' behaviour should progressively become more natural and valid, once initial restraint and caution were overcome [38].

To facilitate increased objectivity and validity, a structured manner of collection was selected [39]. The author prepared an observer checklist, which acted as a vital mechanism in the pursuit of this unspoken agreement in each classroom setting. While the observation criteria were loosely defined during the preliminary pilot observations, a refining process took place during this phase i.e. the checklist was cut and confined to factors considered vital in gaining a holistic picture of the school environment. The general areas of focus for formal observations were:

A – The individual mathematics classroom (lesson content, planning and methodologies).

B – Quality of interactions.

C – Teacher behaviour and classroom management.

D – Pupil learning and attitude.

E – Miscellaneous.

In addition, each area was broken into various sub-headings, which in turn were subdivided into relevant key words for the purposes of focusing the researcher's attention on specific detail. Throughout the observation period, the observation checklist remained interactive, thus receptive to adjustments caused by any practical or theoretical insights gained.

The author was positioned at the rear of the classroom throughout the classroom observations, making no attempt to participate or interfere in activities or classroom discourse.

The author also kept a reflective journal throughout. The reflection process took place after every session. The aim was to deliberately set aside time to look beyond the event itself, thus identifying significant events, overall patterns and conclusions [40]. This activity was vital, as much of the recording during observations was notetaking/ ticking the occurrence of certain criteria. A reflection guide was developed to facilitate thorough post-observation reflection.

Semi-formal interviews were subsequently utilized as a means of information backup, providing clarification of the behaviours and intentions of participants detected during observations. These interviews offered a means of confirmation of observed behaviours [22]. This process offered the study a source of triangulation, thus adding to reliability and validity of results and negating any claims of bias or subjectivity [41]. The interviews were carried out after the final observation in both schools, thus facilitating ample time and scope for reflection prior to question formulation. Questions were designed to further expand the data available relating to the thoughts, beliefs and attitudes of all the participants. These factors were almost impossible to detect through observation alone.

5.3.2 Data analysis. Constant comparative analysis ensured that all findings were both grounded and relevant. As it was not possible or necessary to highlight and examine every statement and action, the focus was narrowed to factors deemed influential to the nature of the 'didactical contract' present in each classroom [42]. The wealth of the material was pulled into an elementary structure through the establishment of general units/chunks of related data, which were labelled under codes e.g. 'Exam-reference', 'Routine' and 'The use of hints and tricks'. Through a succession of examinations of the potential compatibility of existing units, the author found that some codes were subsets of other and therefore could be merged. This regrouping process highlighted the richness of the data, as substantial relationships existed between units. The above sequence of structured steps altered pages of random information into a group of focused themes and issues central to mathematics experiences of the Senior Cycle classrooms investigated. The final headings were: 'Exam-oriented mathematics', 'Daily mathematics class' and 'Quality of interaction'.

6. Findings

The actual experiences of two relatively 'typical' Senior Cycle Mathematics classes highlighted relevant shortcomings, which may contribute to the poor transition made by a sizeable number of pupils to tertiary level mathematics-intensive courses in Ireland [22].

6.1. *Exam-oriented mathematics*

The evidence accumulated and analysed suggests that in the Senior Cycle classes observed, exam-focus whether mentioned or implied by action, was the central concern, even though these pupils were in the first year (i.e. 5th year) of the two-year exam course. Accordingly, the behaviours of teachers and pupils alike were dictated by the implicit conviction that the Leaving Certificate Examination is the principal reason for studying Mathematics at Senior Cycle.

Observations revealed that the Leaving Certificate exam was used as a motivational ploy to gain and maintain interest and concentration levels among pupils. Comments like '*This always comes up in the Leaving Certificate*' (Observation (O) 5, T2) and '*You'll get good marks for the table and the graph and once you understand the question you'll have no problem*' (O3, T1) were especially popular and a central component of each lesson observed. Further evidence of the central importance placed on the Leaving Certificate exam lies in the fact that the only voluntary pupil participations within the majority of the lessons observed were queries relating to exam questions such as '*In the exam, is there lee-way to which a question is right?*' (O3, T1).

6.2. *Pacing*

Pacing was also affected by the need to cover topics for the Leaving Certificate exam. While a challenging pace was evident in both settings, it was more pronounced in the Honours class. This phenomenon had implications for pupil behaviour as reflected by one student in the interview session:

> *I feel I can't ask the questions I want or need to even though she says 'Well any problems with the homework'. I might have had a problem with all of it especially now that we are doing the papers . . . I know at the end of the day it's me who's going to lose out but I just feel she's always giving the impression that she's under pressure to get the course done. It's always as if we have to move on . . . I don't want to hold the class back* (Interview (I), T2: Anna).

A preoccupation with topic progress diminished the willingness of pupils to participate fully in the mathematics class. Such pupil passivity undoubtedly disadvantaged many of the pupils.

6.3. *Routine*

The exam-focused educational environments also had knock-on effects on many aspects of the day-to-day functioning of the mathematics lesson, i.e. the predominant methodology, participation and relationships. Gerard's comment '*It's pretty much the same everyday*' (I, T2) captured the reality that set routine formed a central component of both settings. The predominant resources were the blackboard and the primary text. Methodologies were traditional in nature and teacher-centred. The repertoire of approaches observed ranged from exposition to consolidation and practice. Mathematics tended to be taught in isolation from all relevant subjects and every-day links. Instead the focus was on the mastery of algorithmic procedures as isolated skills.

6.4. *Quick-fix mathematics*

Ample evidence of quick-fix approaches and drill arose during the investigation period. On a regular basis the teacher would advise '*You need to know that off* ' (O1, T1).

Both teachers tended to do the pupil's thinking for them, providing 'ready-made' mechanisms that would facilitate memorization. For example, in the honours class, the teacher gave the pupils 'the four steps to curve-sketching' prior to making any reference to the actual process itself. One would imagine that genuine understanding and know-how could have been promoted through students actually sketching the curve and then co-operatively formulating their own sequence of steps. Consequently, comments such as '*Do you always start with the smaller number?*' (O2-T2) demonstrated that the pupils expected a 'learn-off ' approach. The teachers in question adopted a 'reductionist orientation', where her role was defined as 'effective teller' [43]. Mathematics for the pupils observed was about manipulating numbers and letters and filling in the right formula. 'Problem-solving' in these settings is best described as text-based story problems, where the teacher would lead the pupils through the steps necessary to achieve the 'right answer'. While the pupils were extremely familiar with the procedures to solve practised problems in a known context, the limitations of their knowledge often became apparent. Problems that strayed slightly from the textbook format were met with responses such as '*I didn't know what formula to use*' (O2-

T2-Gerard). In reality the pupils' learning was 'situated' [44]. Confirmation of this came from one of the teachers, who stated '*If it is not like the examples in class, many won't even attempt it*' (I: T2).

Another shocking reality that faced the author was the fact that none of the pupils interviewed had ever completed practical or investigation work in their post primary education to date. A frank and honest response from a teacher again highlighted the effects of the exam-oriented approach on classroom practices: '*Experiments are fine if you have resources, time and your own classroom, which I don't have any of* . . .' (I: T2).

6.5. *Interactions*

During the analysis of the nature of interactions it soon became clear that, for the most part, dialogue was teacher-initiated. As individual questioning was rare during exposition, the opportunity to assess the level or even presence of understanding was lost. Questions generally lacked direction e.g. '*Anyone not get it*' (O1-T1), thus facilitating a chorus answer which naturally camouflaged the confused. The statement '*Listen and learn; that's what you're there for*' (I, T2: Marian) reflects pupils' beliefs that their role was primarily one of listener and copier. The passive nature of pupils' involvement is captured by one pupil's comment '*Sometimes you feel like you're taking it down like a robot. You're really not involved in it* . . .' (I, T2: Olivia). There was a widespread reluctance among pupils in both groups to openly announce their views for fear of failure and public humiliation. A discomfort in relation to individualized questioning was also apparent. The practice sessions proved to be the only stage of the observed mathematics lessons where two-way interaction and pupil-initiated dialogue was evident. In this component of the lesson one-to-one support was provided to all pupils requiring assistance. In their effort to promote positive attitudes, teachers repeatedly coaxed the groups, offering positive reinforcement for all efforts regardless of relevance or accuracy. Catch phrases such as '*Well spotted* ' (O5, T2) and '*Go* *again*' (O1, T1) saturated the observed lessons. Pupils expected this caring learning environment, describing a 'good teacher' as '*Someone who's tolerant of you if you don't understand* ' (I, T1: Jim).

In general the respective teachers did not force pupils to work extremely hard, but rather opted to encourage them. The teacher of the Ordinary Level group demonstrated very flexible expectations where retention of previous topics was never assumed. In fact an inability to recall even the most basic elements of previous topics was deemed acceptable, a practice which meant that the teacher repeatedly revised basic principles without any protest or attempt to probe. On the other hand, the teacher of the Higher Level group revealed that she took basic knowledge for granted among her pupils, an approach which the author felt would promote ineffective learning of new material if pupils had partially forgotten the prerequisite information [22].

7. The nature of the 'didactical contract' in the classrooms investigated

Because the research was sourced in data it was possible 'to generate a theory which fits the data, as opposed to finding data to fit a theory' [44, p. 45].

The development of a logical chain of evidence, as presented above, facilitated the author in the formulation of a 'didactical contract', which was representative of the actions and attitudes of all participants in both educational settings. This unspoken contract, which was a negotiated agreement between the participants in the classes observed, included many mutually acceptable demands as well as involuntary sanctions, obligations and restrictions on all concerned. While many of the common characteristics applied to both classrooms, some variations occurred between the Ordinary and Higher level groupings.

Readers should be cognizant of the word 'should' throughout the contract, signifying that these conclusions are objective interpretations of the evidence collected and analysed. The common elements of this 'didactical contract' identified by the author are as follows:

- The Leaving Certificate terminal exam should be the central aim of the mathematics class. This exam should be the core component of each lesson, present as a sole motivation to learn a new topic. The teacher should present work referring to its inclusion/importance in the Leaving Certificate and provide details of the gain loss of marks at every opportunity. All class tests should be based on previous exam questions. No time should be wasted in class, as the adequate chapters for exam preparation require completion in the shortest possible time.
- The teacher should introduce the lesson by correcting the homework swiftly and orally if possible and move onto a new topic. Blackboard work should be curtailed, unless pupils have major problems and ask for help. During the homework correction, the teacher should interact mainly with the group, asking for the answer to the exercises. The introduction of a new topic should consist of the illustration of a number of worked examples on the blackboard. After pupils have copied these into their copies, they should practise similar exercises from the course text entitled, Text and Tests. During individualized practice work, the teacher should circulate, offering personal feedback and help to all needy pupils. The pupils should use this opportunity to ask questions regarding the homework etc. Before the bell, similar type exercises ought to be set for homework from the primary text. Homework ought to be attempted in order for pupils to gauge how much they actually know.
- The teacher should not depart from the set lesson routine unless the class are preparing for a test, in which case the entire class should be used to review formulas, techniques and standard problems. The teacher should, if possible, get something new

done every day, as pace is vital. The standard lesson should never include practical work or investigation or make logical links to other subjects or everyday life unless it is directly relevant to the exam, e.g. statistics.

- The teacher should not ask pupils complicated individualized questions, as the pupils should not be publicly scrutinized. Such questioning in class should be pitched at the group, to allow more outspoken and well-able pupils to volunteer. The only directed questions that ought to be permitted are those on new topics or on straightforward workings.
- The teacher should 'break things down' and simplify for the pupils. The pupils in turn should listen and try to learn off. The teacher should provide pupils with tricks to remember mathematics methods and a step-by-step breakdown of problem-solving techniques, i.e. the use of trigger phrases. Pupils should not be expected to persevere with difficult questions or to be able to solve questions requiring a combination of procedures.
- Pupils should not be expected to remember previous topics and the teacher should be patient and ready to re-explain any section/method on request. The pupils should listen and learn, participation in class activities is not vital. Once the minimum standard is achieved, i.e. homework attempted, on task in class and able to answer directed questions, the teacher shouldn't request any more. The teacher should always be positive and encouraging even if pupils are not working to their ability.
- Any individual teacher or pupil should not interfere with the contract described above, even if they are unhappy with many elements of it. Pupils should not interrupt the lesson unnecessarily, thus holding back the group, even if they are confused. Pupils should work on passively in class even if they require extra blackboard reference, examples or time on a particular topic. The teacher should not ask directed questions

> that may demand thought and reflection, or set questions which have more complex wording or layout than that of the main text. The class form should not be disrupted, as the present momentum is deemed vital in order to keep on target in the pursuit of exam success. All loose ends and confusion should be dealt with during seatwork or after class [22].

In reality, all of the above beliefs and behaviours led towards entrapment within a vicious cycle, where the teacher believed that pupils would/could not think independently and as a result the pupils did not think independently, thus confirming his/her views. Therefore ‘learned helplessness’ was repeatedly reinforced [45].

8. Discussion

These findings are generally consistent with the relevant research reviewed. It is the authors’ belief that the ‘typical’ Irish Senior Cycle experience, as described above, has direct implications for students’ long-term mathematical competence. The outcomes of this study are most enlightening and undoubtedly have direct and significant relevance in alleviating the detrimental effects of under-preparedness at tertiary level in Ireland. A major issue, which has emerged from this research, is the wide gap apparent between the ideal and real practices in Senior Cycle mathematics classrooms (i.e. the planned and implemented curricula). This is the result of the powerful backwash effect of the Leaving Certificate exam on both what is taught and how it is taught.

While mathematics teachers, like all other subject teachers, are endeavouring to achieve whatever ‘is in the best interest’ of his/her pupils, the national obsession with the state examinations means that many teachers see their primary function as preparing their pupils for this vital exam. Unfortunately though, in doing so, teachers are narrowing such pupils’ future potential, as exam-driven practice generally results in regimental thinking, limited problem solving ability and a lack of self-confidence and perseverance in the face an unseen/different challenging problem. The limitations of a quick-fix, teacher-led didactic approach, which has the terminal exam as a primary focus, means that the 'typical' pre-tertiary mathematics experience fails to provide pupils with the necessary foundations for everyday life, not to mention tertiary level mathematics courses.

While mathematics-intensive courses at tertiary level need independent learners possessing conceptual and transferable skills required to solve unfamiliar problems, the development of these essential skills is not fostered within the classrooms studied. It is hard to change such behaviours in a system where teachers and pupils are judged by their performance in the Leaving Certificate examination, which focuses predominantly on memorization, the practice of technique and ability to spot cues [22].

9. Concluding remarks

It has been suggested that the problem of the mathematical under-preparedness of students entering tertiary level education in Ireland and elsewhere is not a transient one, but rather a permanent feature of many educational systems in developed countries [5].

While it has already been acknowledged that the present study is not comprehensive, it does represent a step forward in terms of finding solutions to the plight of mathematically under-prepared students entering tertiary level mathematics courses in Ireland. There is scope for further research into this phenomenon in the Irish context and further afield. Undoubtedly much work remains to be done before lecturers of mathematics at tertiary level cease to be concerned about incoming students' mathematical preparation and ability.

The authors conclude this paper by engaging in some reflexive thinking, applying insights gained from this study to their own situation. If one of the aims of Second Level education is to educate pupils who are adequately prepared to participate in mathematics courses at tertiary level, it is essential that pupils leave Second Level education in a ‘high energy state’, i.e. demonstrate a predisposition to

. . . confront any problem given to them/invented by themselves with their previous learning in Mathematics, in an active and accessible state and with the assumption that they have the ability to progress [46, p. 105].

If mathematics education in Ireland is to move successfully towards this ideal mathematical state, significant changes are inevitable in the Second Level mathematics provision. First, it is essential that all the interested parties (e.g. practitioners, Department of Education and Science) become fully informed of the present scenario. Teachers need to recognize that pupils’ achievement in the Leaving Certificate mathematics examination is only one aspect of their mathematics education. A keen awareness of the mathematical skills and abilities required by pupils beyond this examination, e.g. pupils who hope to enter tertiary level mathematics-intensive courses, is also necessary if teachers are to serve their pupils well.

Second Level mathematics practitioners must move away from over-reliance on traditional approaches and instead focus on the provision of a more active, varied and positive mathematics experience for their pupils. Mathematical concepts should be taught with a heightened awareness of what learners already know, i.e. there should be an emphasis on ‘connection-making’. Such a process, unlike reliance on memorization, will develop re-usable mathematics understanding.

The teacher should endeavour to portray the belief that mathematics is useful and vital for all our people. The development of learning that ‘transcends boundaries’ can be achieved through integration and extra-curricular reference and activities [45]. Further, the teacher’s notion of caring and helping pupils needs to be transformed into a construct that attends to their intellectual development and not just the protection of pupils’ comfort levels. Research suggests that both can be achieved side by side, rather than one being subservient to the other

[47]. Pupils need to realize that answers do not always drop out smoothly, but require struggle and that out of difficulties encountered, real know-how emerges. The creation of an environment where risk-taking, discussion and solving relevant problems with many possible solutions are central elements of each lesson encourages pupils to extend their values beyond the desire to attain the 'right answer'. Consequently errors and misunderstanding are seen as an opportunity for learning, rather than a source of embarrassment or discomfort that should be swiftly avoided or dismissed. Effort must be perceived as central to success. Otherwise pupils will have no motivation to persevere if success is equated directly with ability [48].

The authors are cognisant that many of the factors leading to the predominant examination-focused practices lie outside teachers' control and must be addressed at national policy level if there is to be change. Consequently, they are heartened by the government's recent move to seek a 'root and branch' review of Second Level mathematics education with special reference to Senior Cycle mathematics. As the present examination system seems to dictate the mode of teaching and learning employed by most teachers and pupils, one suspects that the alteration of the mode of assessment to include project work, for example, would foster the development of problem-solving and higher-order thinking skills [22]. The National Council for Curriculum and Assessment has raised such issues in a publication entitled *Review of Mathematics in Post Primary Education as a prelude to reform of the system in Ireland* [6].

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