Ireland's Participation in the 50th International Mathematical Olympiad

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The 50^{th} International Mathematical Olympiad (IMO) took place in Bremen (Germany) from 12^{th} July until 22^{nd} July 2009. With 565 participants (59 of whom were girls) from 104 countries, this was the IMO with the largest participation so far. It was the first time that a three-digit number of countries participated.

The Irish delegation consisted of six students (see Table 1), the Team Leader, Bernd Kreussler (MIC Limerick), the Deputy Leader, Gordon Lessells (UL) and the Official Observer, Donal Hurley (UCC).

1. Team Selection and Preparation

The IMO is the most prestigious mathematical contest for second level students in the world. Participation in this event is already considered to be a great honour. In order to be able to gain any marks at the IMO exams, it is not sufficient, even for the brightest of students, to rely solely on the Leaving Certificate Mathematics Syllabus (or the equivalent in other countries). Leading countries such as China and Russia are able to identify the most talented students at a very early age and then organise very intense training programmes for them.

Name	School	Year
Jack McKenna	Newbridge College, Newbridge, Co. Kildare	6^{th}
David McCarthy	Midleton CBS, Midleton, Co. Cork	6^{th}
Colman Humphrey	St. Andrew's College, Booterstown, Co. Dublin	6^{th}
Cillian Power	Christian Brothers College, Cork	5^{th}
Colin Egan	Clonkeen College, Blackrock, Co. Dublin	5^{th}
Vicki McAvinue	St. Angela's School, Ursuline Convent, Waterford	4^{th}

Table 1. The Irish contestants at the 50^{th} IMO

In Ireland, we are currently able to identify students with exceptional mathematical talent in three different ways: top performance in the Junior Certificate Examination in Mathematics, or excellent results in the PRISM competition or recommendation by a person who has identified this talent (maths teachers, parents, school principals). At five different locations all over Ireland (UCC, UCD, NUIG, UL and NUIM), mathematical enrichment programmes are offered to the students who came to our attention through one of these sources. These classes run each year from December/January until April and are offered by volunteer academic mathematicians from these universities or nearby third-level institutions.

The selection contest for the Irish IMO team is the Irish Mathematical Olympiad (IrMO), which was held for the 22^{nd} time on Saturday, 9^{th} May, 2009. The IrMO contest consists of two 3-hour papers on one day with five problems on each paper. The participants of the IrMO normally attended the enrichment classes in one of the five centres; however, particpation is open to all interested secondary school students. This year, a total of 69 students took part in the IrMO, which was held at the five centres listed above. The top performer is awarded the Fergus Gaines cup; this year this was Jack McKenna. The best six students (listed in order in Table 1) were invited to represent Ireland at the IMO in Bremen.

During the week before the students travelled to Bremen, they gathered for an intensive training camp at the University of Limerick. In preparation for this camp, they were sent a number of problems which they were expected to have solved by the time they had arrived in Limerick. The camp was organised as usual in a very efficient way by Gordon Lessells. The sessions with the students were directed this year by Jim Cruickshank, Donal Hurley, Kevin Hutchinson, Bernd Kreussler, Tom Laffey, Jim Leahy, Gordon Lessells and Rachel Quinlan. They focussed on problem solving techniques in the core topics that are covered by IMO problems: algebra, combinatorics, elementary geometry and number theory.

In addition to the six members of the Irish IMO-team, six of the best students, who were known to the organisers and who will be eligible to participate in future IMOs, were also invited to attend the camp for three days. This measure aims at building a base for future success.

2. Jury Meetings — The Problem Selection

Donal Hurley and I travelled on the 12^{th} of June to Bremen because the Jury was scheduled to convene on Saturday, 13^{th} June. The location where the students will reside and where the contest is to be held is always well known in advance. The Jury of the IMO however, which is the prime decision making body for all IMO matters, resides at a location which is kept as secret as possible. This is one of the measures which helps ensure that the contest problems will not become publicly known before the exams take place. This year, the hideout of the Jury was in Bremerhaven, the seaport of the free city-state of Bremen. This town was founded in 1827 and is now one of the most important German trade ports. It is located on the North Sea, about 60 kilometres north of Bremen, at the estuary of the river Weser.

The Jury is composed of the Team Leaders of the participating countries and a Chairperson. The Observers at the Jury's site participate, without voting rights, in all Jury meetings. The Chairman for the 50^{th} IMO was Prof. Hans-Dietrich Gronau, who has been the leader of the German IMO-team for many years. He chaired the Jury meetings in a very efficient way, so that the Jury completed all its tasks well within the very tight preassigned timeframe.

In recent years, the seating plan for the Jury meetings was determined by the alphabetical order of a three letter country code (IRL for Ireland). The organisers decided to change the seating plan for the Jury meetings this year in such a way that the countries were seated in the order of their initial participation in the IMO. The first two, according to this order, were Romania and Hungary; Ireland, participating since 1988, was on position 45. Because of the changes in the political landscape during the past 20 years, some nations were seated behind Ireland, even though students from the same geographic location had participated much earlier than students from Ireland. For example, the former Soviet Union belonged to the group of seven countries which participated in the first IMO in Braşov (Romania) in 1959, but the representative of the Russian Federation was seated at position 59, next to the representative from Trinidad and Tobago.

The host country's problem selection committee selected a shortlist of 30 problems from the problems submitted to them in advance. There was general agreement among the Jury members that the shortlist was very well prepared: for the vast majority of the problems two solutions were given, one problem even came with five distinct solutions. Most of Saturday was available for the leaders and observers to get familiar with the problems. Later, at 4 p.m., the solutions were handed out and the evening and night were spent studying them.

The main task of the Jury meetings on Sunday, 12$^{\text{th}}$ July, was the selection of the six contest problems from the shortlist. Under the skilful direction of the Chairman, this task was completed before 4 p.m. on Sunday. During the evening meeting, the final formulations of the six problems were discussed so that the representatives of the language groups of the five official languages (English, French, German, Russian and Spanish) were able to produce their versions over night. After approving these five versions at a Jury meeting on Monday morning, the rest of the day was available for the translation of the contest problems into 50 languages, in addition to the five mentioned before. Most of these translated versions were produced with the aid of LaTeX or MS-Word, but three of them were handwritten: the Armenian, the Mongolian and the Singhalese translations.

3. The Problems

First Day.

Problem 1. Let n be a positive integer and let $a_1, \ldots, a_k$, $k \geq 2$ be distinct integers in the set $\{1, \ldots, n\}$ such that n divides $a_i(a_{i+1} - 1)$ for $i = 1, \ldots, k-1$. Prove that n does not divide $a_k(a_1 - 1)$.
(Australia)

Problem 2. Let ABC be a triangle with circumcentre O. The points P and Q are interior points of the sides CA and AB, respectively. Let K, L and M be the midpoints of the segments BP, CQ and PQ, respectively, and let Γ be the circle passing through K, L and M. Suppose that the line PQ is tangent to the circle Γ. Prove that $OP = OQ$. *(Russian Federation)*

Problem 3. Suppose that $s_1, s_2, s_3, \ldots$ is a strictly increasing sequence of positive integers such that the subsequences

$$s_{s_1}, s_{s_2}, s_{s_3}, \ldots \quad \text{and} \quad s_{s_1+1}, s_{s_2+1}, s_{s_3+1}, \ldots$$

are both arithmetic progressions. Prove that the sequence $s_1, s_2, s_3, \ldots$ is itself an arithmetic progression. *(United States of America)*

Second Day.

Problem 4. Let ABC be a triangle with $AB = AC$. The angle bisectors of $\angle CAB$ and $\angle ABC$ meet the sides BC and CA at D and E, respectively. Let K be the incentre of triangle ADC. Suppose that $\angle BEK = 45°$. Find all possible values of $\angle CAB$. *(Belgium)*

Problem 5. Determine all functions f from the set of positive integers to the set of positive integers such that, for all positive integers a and b, there exists a non-degenerate triangle with sides of lengths

$$a,\ f(b) \text{ and } f(b + f(a) - 1).$$

(A triangle is *non-degenerate* if its vertices are not collinear.) *(France)*

Problem 6. Let $a_1, a_2, \ldots, a_n$ be distinct positive integers and let M be a set of $n-1$ positive integers not containing $s = a_1 + a_2 + \cdots + a_n$. A grasshopper is to jump along the real axis, starting at the point 0 and making n jumps to the right with lengths $a_1, a_2, \ldots, a_n$ in some order. Prove that the order can be chosen in such a way that the grasshopper never lands on any point in M.

(Russian Federation)

4. The Contest

The six Irish contestants, accompanied by the Deputy Leader, Gordon Lessells, arrived in Bremen in the late evening of Monday, 13^{th} July. The Opening Ceremony took place on Tuesday morning at the event centre "Pier 2" in Bremen. The start and the end of the ceremony were marked by a Breakdance performance. In addition to the usual speeches and a short mathematical entertainment entitled "Calculating without a calculator", we saw a parade of all 104 participating teams in the order in which their countries initially participated in the IMO. Some contestants were missing because they hadn't yet arrived, but all made it in time for the first exam on Wednesday.

On Tuesday afternoon the students enjoyed free time so that they could relax prior to the exams on the following two days. After a reception at the 600 years old city hall of Bremen, the Jury, however, were obliged to go back to work. On the agenda for the meetings in the afternoon and evening were the discussion and finalisation of the detailed marking schemes for all six contest problems. The problem captains had done a marvellous job: the only major demand of the Jury was a more explicit marking scheme for Problem 3. Spontaneous applause was awarded to the marking scheme for Problem 2 which included a flow chart type graph giving a very detailed overview of possible partial marks for a few of the anticipated solutions.

The two exams took place on the 14$^{\text{th}}$ and 15$^{\text{th}}$ of July, starting at 9 o'clock each morning. The venue for the exams was a single large hall, so that all students had equal conditions. The seats were arranged in six blocks, each of which did not accommodate more than one student from the same country.

On each day, $4\frac{1}{2}$ hours were available to solve three problems. During the first 30 minutes, the students were allowed to ask questions if they had difficulties in understanding the formulation of a contest problem. These questions were scanned in Bremen and sent electronically to Bremerhaven, where the printout was given to the relevant Team Leader. Each question was discussed and answered in front of the Jury, so that equal standards were applied to all contestants. The approved answer was then sent back electronically to Bremen. Most of the 90 questions on day one were seeking clarification what is an interior point of a triangle (Problem 2). On the second day, there were only 42 questions asked by the contestants.

5. Marking and Coordination

After this last questions-and-answers session on Thursday morning, the Team Leaders and Observers left the beautiful sea-side location in Bremerhaven to join the Deputy Leaders and the contestants on the campus of Jacobs University Bremen. This is a private university which was founded in 1999 and currently has about 1200 students from over 90 countries. The campus of this university is located on the former grounds of the Roland Barracks, which was in use by the German Army until 1998.

Because the scripts of the first day had arrived in Bremerhaven on the Wednesday evening, we already did a first reading before we moved to Bremen. After our arrival, we immediately began to read them in detail, with the help of Gordon. The most remarkable fact about these scripts was that David had produced the record number of 37 pages, neither numbered nor in correct order, for his attempted solution of Problem 1. All three of us, naturally, studied every single page and we tried to put these pages in some kind of order hoping to find the material in it which would be the basis for the points he deserved.

After their return from the second exam, we briefly met the students. They didn't seem optimistic about their performance. During the following two days the Leader, Deputy and Observer were fully occupied with our preparation for the coordination.

At the same time, the students started to relax after their two days of exams. On Friday and Saturday they enjoyed excursions to several destinations, including a Shipyard, a Transrapid Track and the newly opened Klimahaus Bremerhaven. They had a choice and so could spend these days according to their taste. In the evenings, a Football competition was organised. Two of the Irish students participated in a team with students from Georgia, Zimbabwe and South Africa and lost narrowly in the Final.

The marking of the scripts at the IMO is undertaken by two independent groups. One group consists of the Team Leader, the Deputy Leader and the Official Observer. The second group consists of the coordinators, who were appointed by the local organisers. The two groups met according to a tight schedule which was distributed before our departure from Bremerhaven. For each problem there were five coordination tables with two coordinators at each. This means that each pair of coordinators had to study the solutions for one of the problems of about 120 contestants from 20 countries. All coordinators were former IMO participants or involved in the training of contestants for many years, or both.

The coordination is a very important part of the IMO, because it is the well established method to mark fairly the scripts of the students from so many different nations. It is important for the representatives of the teams to be well prepared for each of the half-hour meetings with the coordinators. For example, if a student came up with an incomplete solution which is not exactly covered by the agreed marking scheme, a fully worked out solution which starts with the work done by the contestant could be the argument required to obtain partial points. Sometimes, when the relevant part of the student's work was difficult to find in the forest of unfinished ideas, the many hours we had spend reading and discussing the scripts did pay off. We found that the coordinators were very knowledgable, were always well prepared and ready to listen to our presentation.

Name	P1	P2	P3	P4	P5	P6	total	ranking
Jack McKenna	2	2	0	2	0	0	6	416
David McCarthy	3	1	0	0	0	0	4	444
Colman Humphrey	0	1	0	1	0	0	2	482
Cillian Power	1	1	0	0	0	0	2	482
Colin Egan	1	1	0	2	0	0	4	444
Vicki McAvinue	0	0	0	2	0	0	2	482

TABLE 2. The results of the Irish contestants

In general they were generous in agreeing with our proposals. Table 2 shows the results of the Irish contestants.

The Jury tries to choose the problems in such a way that Problems 1 and 4 are easier than Problems 2 and 5. Problems 3 and 6 are usually designed to be the hardest problems. Table 3 shows that Problems 2 and 4 need to be swapped in order to achieve this aim, but otherwise the results fit very well into this pattern.

	P1	P2	P3	P4	P5	P6
0	83	101	357	188	270	540
1	56	106	127	79	42	2
2	28	43	16	37	50	1
3	20	51	5	23	33	10
4	17	16	2	17	6	6
5	16	15	5	69	4	2
6	21	19	2	52	7	1
7	324	214	51	100	153	3
average	4.804	3.710	1.019	2.915	2.474	0.168

TABLE 3. For each problem, how many contestants have got how many points

Problem 6, the grasshopper problem, turned out to be one of the hardest problems ever: only three students achieved the maximum of 7 marks for this problem, whereas 540 of the 565 contestants could not gain any mark for it. Nevertheless, if the medal cut-offs are taken as an indicator, this IMO does not seem to have been harder than previous ones. Gold medals were awarded to 49 students who scored at least 32 points, 98 students with points in the range 24–31 got silver medals and 135 students who scored at least 14 points but not more than 23 were awarded a bronze medal.

Although the IMO is a competition for individuals only, it is interesting to compare the total scores of the participating countries. This year's top teams were from China (221 points), Japan (212 points) and Russia (203 points). Ireland, with 20 points in total, finished in 89$^{\text{th}}$ palce. Two students, Dongyi Wei from China and Makoto Soejima from Japan, achieved the perfect score of 42 points. The detailed results and statistics can be found on the official IMO website `http://www.imo-official.org`.

6. Celebration of the 50$^{\text{th}}$ Anniversary

The International Mathematical Olympiad is the oldest and largest among all international academic competitions for secondary school students. Exactly 50 years ago, in 1959, the first IMO, with 7 participating countries, was held in Romania, where at least since 1897, national mathematical competitions were organised. At the end of the 1970s, about 20 countries were regularly participating in the IMO. Since then, the number of participating countries monotonically increased to over 100 in Bremen 2009. Because there was no IMO held in 1980, the fiftieth anniversary of the first IMO coincided with the 50$^{\text{th}}$ IMO.

The German organisers organised a marvellous anniversary celebration, which took place on Sunday, 19$^{\text{th}}$ July, at the Bremen Musical Hall. As the highlight of this event, they invited six of the world's leading mathematicians, all of them also successful former IMO participants: Belá Bollobás, Timothy Gowers, Lásló Lovász, Stanislav Smirnov, Terence Tao and Jean-Christophe Yoccoz. Three of them (Gowers, Tao and Yoccoz) have been awarded a Fields Medal, which is considered to be the equivalent of the Nobel Prize for mathematicians.

Each of these six guests gave a short talk centrered around the relationship between IMO problems and mathematical research problems. During his speech at the closing ceremony, József Pelikán, the chairman of the IMO Advisory Board, gave a succinct characterisation of the two types of problems: IMO problems are like wild animals you meet in a zoo whereas dealing with mathematical research problems is more like meeting such animals in the jungle. Considering an IMO problem, you can be sure that there exists a nice solution, whereas this is not always the case for a mathematical research problem.

During two extensive breaks at the anniversary celebration, the students (and, indeed, others) surrounded the six speakers like celebrities. They queued up for autographs and took group photos with them, just as others would do in the presence of famous actors or sports heroes. On the following day, most of the six celebrities also participated in the excursion to Wangerooge, which is the most eastern of the East Frisian Islands.

The closing ceremony took place on Tuesday, 20$^{\text{th}}$ July, in the concert hall "Die Glocke" in Bremen. Two Fields Medalists were among those who handed over medals to the succesful contestants. At the end, the IMO banner was passed on to the delegation from Kazakhstan, where the 51$^{\text{st}}$ IMO will be held from 2$^{\text{nd}}$ July until 14$^{\text{th}}$ July 2010.

7. Outlook

The next countries to host the IMO will be

2010	Kazakhstan	`http://www.imo2010org.kz/`
2011	Netherlands	
2012	Argentina	

While no formal decision has been made about the location of the IMO 2013, it was announced that Coratia has expressed its interest in organising the IMO that year. Two countries have expressed their interest in holding the IMO in 2014 and 2015, details of which are to be given at the next IMO in Kazakhstan.

8. Conclusions

Comparing the Irish performance with that of other nations, it seems not to be sufficient to involve the majority of the students, with exceptional mathematical talent, in mathematical enrichment activities which do not begin until after the Junior Certificate. The youngest participant of this year's IMO, for example, was eleven years old; he was from Peru and got a bronze medal. The PRISM competition (`http://www.maths.nuigalway.ie/PRISM/`) is a promising and valuable initiative to attract younger students. It seems to be important that younger students, who were successful in this competition or whose interest in mathematics was sparked by it, should have the possibility of getting involved in guided problem solving training.

In addition, it was sufficient in Bremen to get a bronze medal by solving two problems completely. In recent years, two of the IMO problems were always from geometry. Therefore, a strong background in elementary geometry will certainly help improve the scores of our participants. A starting point for improved performance of future Irish contestants at the IMO might be to focus on a solid foundation in basic geometry, in particular for students in their Junior Cycle. Geometry is also particularly suitable in introducing interested students to mathematical problem solving. Those who really engage in this activity will experience the satisfaction of success whenever they solve a problem having spent some hours tackling it. Such satisfaction is a most important motivation in continuing to think independently about mathematical problems. Students who do not develop this internal interest in problem solving will rarely be able to win a medal at an IMO.

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Also, thanks to the Royal Irish Academy, its officers, the Committee for Mathematical Sciences, and especially Gilly Clarke, for continuing support in obtaining funding.

The essential part of the preparation of the contestants is the work with the students done in the enrichment programmes at the five universities. This work is carried out for free by volunteers in their spare time. Thanks go to this year's trainers at the five Irish centres:

At UCC: Tom Carroll, Finbarr Holland, Donal Hurley, Edward Lee, Anca Mustata.

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At NUIG: Jim Cruickshank, Niall Madden, Rachel Quinlan, Jerome Sheahan and Emil Sköldberg.

At UL: Mark Burke, Eugene Gath, Bernd Kreussler, Jim Leahy, Gordon Lessells.

At NUIM: Stefan Bechtluft-Sachs, Caroline Brophy, Stephen Buckley, Peter Clifford, Katarina Domijan, Kurt Falk, David Malone, John Murray, Anthony G. O'Farrell, Lars Pforte, Adam Ralph, David Redmond, Rade Stanojevich and Richard Watson.

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