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Indexing Distributions of Data:

Preservice Teachers' Notions of Representativeness

Theoretical Perspective

Distribution is the statistical term for the arrangement of observations along a scale of measurement (Hardyck & Petrinovich, 1969). If the data from an assortment of observations are plotted using a common scale of measurement, the result is a representation of the group data. Repeated observations will result in a set of differently shaped plots, each of which varies in a number of 'dimensions' – most notably deviations occur in terms of center and in variation. A mathematical treatment of distribution entails the use of summary characteristics of the distribution. Four summary characteristics are generally used to index distributions of data: measures of central tendency, variability, skew and kurtosis. If the entire set of characteristics is taken into consideration, the distribution can be described exactly.

In generating descriptions of distributions, the notion of representativeness arises. Representativeness is an important notion in statistics because the ability to use a summary statistic to describe a data set removes the necessity of naming each individual data value. The choice of representative value varies depending on the shape of a distribution. The mean and other measures of central tendency are representative when data sets are normal and non-skewed; however the mean becomes less representative as the distribution deviates from normal. In such situations, the sole use of measures of central tendency is inadequate to index a distribution, and supplemental indices such as variability may be more appropriate.

In recent years the focus of statistics education has shifted from examining individual summary statistics (Friel, 1998; Hardiman, Well & Pollatsek, 1984; Mevarech, 1983; Pollatsek, Lima & Well, 1981; Zawojewski, 1988) and moved to more holistic examinations of distributions of data and the measures that index these distributions (Author & Middleton, 2001; McClain, Cobb & Gravemeijer, 1999, 2000; Mokros & Russell, 1995). Researchers have examined elementary and middle school children's construction of representativeness (Hancock, Kaput & Goldsmith, 1992; Author & Middleton, 2001; Mokros & Russell, 1995). In an investigation of fourth through eighth graders' notions of representativeness (Mokros & Russell, 1995), participants' approaches were grouped into two main categories, those that embody the notion of representativeness and those that do not embody the notion of representativeness. Two approaches did not recognize the notion of representativeness; these were average as mode and average as an algorithmic procedure. Three approaches embodied the idea of representativeness, these were average as 'what is reasonable', average as midpoint, and average as a mathematical point of balance.

A study of elementary and middle grade students understanding of distribution (Author & Middleton, 2001) found that students utilized a number of approaches when constructing representative values. The repertoire and sophistication of strategies developed as a result of exposure to a wide variety of distributional shapes. The initial strategy used by participants to construct representative values was the use of the mode. Coordination of variation and center through the construction of intervals of data was an approach developed by children over the course of the teaching experiment. The ultimate strategy employed was use of the mean in combination with other measures of central tendency or variability.

Difficulties recognizing instances in which the mean can be used as a representative measure were found by Hancock, Kaput & Goldsmith (1992) in a study of fifth through eighth graders. This was indicated by the inability of any students to use the mean to compare two groups of unequal size. A similar lack of understanding of the mean as a representative measure was found in a study of 263 preservice teachers (Author & O'Loughlin, 2002) wherein, when asked to compare two sets of data, only 57% of respondents made an appropriate comparison of means. Hancock et al. (1992) found that despite instruction focusing on looking at trends in the data and the introduction of appropriate vocabulary, students often gave more attention to individual cases than was merited. The majority of students had difficulty looking at the generalized group and frequently referred to the particulars of a specific case.

The increasing use of computer tools to aid in the construction and analysis of data is visible throughout the literature. The use of computer tools provides several advantages: they provide immediate feedback when students make changes to their representations, they facilitate the construction of different representations of data, they help students in the organization of their data, and they function as a calculator carrying out complex operations, hence freeing students to reason about the data.

Studies investigating the use of technology in promoting children's understanding of distribution have found that technological tools can be effective in supporting the development of statistical reasoning relating to distributions of data. The *Reasoning Under Uncertain* (RUU) curriculum (Rosebery & Rubin, 1989) utilizes interactive graphing software, ELASTIC, to teach statistical concepts through the display of histograms, bar charts, box plots, and scatter plots. ELASTIC includes three tools designed to support and develop students' statistical reasoning. One of these tools, the stretchy histogram, allows students to change the distribution of a histogram and watch as the mean, median and quartiles are updated. Another tool, the sampler, is a simulated laboratory where students take repeated samples and compare the shapes of distributions; and the third tool, shifty lines, is an interactive scatter plot that fits lines through varying data points. Through the use of these flexible tools, the curriculum encourages children to investigate statistical questions, and the software supports exploration of these statistical concepts.

In an investigation of the role of discourse in the analysis of data, McClain, McGatha & Hodge (2000) used computer mini tools to support eighth-grade students in developing their data-based arguments. The tools provided a number of approaches to structure and analyze data and representations arising from tool-use were used as evidence to support and substantiate student arguments. Similar tools were used by McClain, Cobb & Gravemeijer (2000) in a twelve-week teaching experiment in a seventh-grade classroom. The tools were considered 'an integral aspect of statistical reasoning rather than as technological add-ons' (p. 174). Both tools used in the study supported the conceptualization of data sets as entities (Hancock, Kaput & Goldsmith, 1992), one tool facilitated students in ordering, partitioning and organizing distribution of data, while the second tool supported the structuring of data into groups and intervals. The researchers found that as students were using the tools to make data based arguments related to distributions of data, deep understandings of statistical ideas were being constructed.

Other researchers have examined the prevalence of computers in college level statistics courses, stating that they assist and enhance instruction (Denby & Pregibon, 1987), hold student attention and act as a motivational tool (Black, 1988), facilitate students in becoming active participants in courses rather than mere spectators (Black, 1988), and can be utilized for data description, multivariate statistics (Bajgier & Atkinson, 1989), regression, and sampling distributions (Gordon & Gordon, 1989). Many of these topics, such as multivariate statistics, are next to impossible to introduce with real data sets without technological assistance.

What is absent from the research base is a coherent model of how preservice teachers conceptualize distributions of data and the measures that represent them. Little is known about how preservice teachers' knowledge of distributions impacts upon instruction. Studies of preservice teacher content knowledge suggests that good mathematical content knowledge results in the incorporation of worthwhile and meaningful classroom mathematical experiences through the increased capacity to pose questions, select tasks and emphasize the richness and meaningfulness of mathematics. Several studies have, however, identified gaps and misconceptions in preservice teacher statistical content knowledge. Studies have shown that preservice teachers' understanding of the mean is limited to computational knowledge with little associated conceptual understanding (Author & O'Loughlin, 2002) and also that computational algorithms are most frequently used in representations when solving mean-related problems (Gfeller, Niess, & Lederman, 1999). An investigation of preservice elementary teachers' classroom practices when teaching statistical investigation (Heaton & Mickelson, 2002) highlighted several difficulties integrating statistical activities. In particular, they identified problems in developing substantive research questions, an overemphasis on technical components of graphing, and a lack of awareness of the importance of relating the data back to the investigation questions thus leading to a superficial exploration of the data. These difficulties related to understanding and teaching relatively simple statistical concepts support Shulman's (1986) argument for the need to provide experiences that allow teachers to acquire statistical content and pedagogical knowledge.

This exploratory study focused on uncovering the strategies employed by preservice teachers when constructing representative measures for a variety of distributions of data. Two specific research questions were addressed:

1. What types of representative measures do preservice teachers construct to index distributions of data?
2. Does the representational form of the data influence the nature and type of representative values constructed to index data?

Method

Participants

Study participants were 283 preservice elementary teachers in their second year of a three-year Bachelor of Education Degree program. The mean age of the group was 19.8 years old, with 14% male and 86% female. The preservice teachers had studied a component of descriptive statistics at high school, which was focused on computing measures of central tendency and variability and constructing graphical representations. Study participants had also completed an undergraduate mathematics methods course, part of which focused on approaches to teaching data modeling to elementary children. In addition, they had completed two teaching practice placements at early and middle elementary grade levels.

Instrumentation

The research instrument was designed to investigate the types of representative measures that preservice teachers considered appropriate to index distributions of data. The instrument consisted of five tasks designed to reveal participants' strategies when constructing representative values for a variety of distributional shapes and representational forms. The distributional shapes of the data were varied to reflect bimodal, skewed, flat, and normal distributions. Each of the tasks presented a set of data and a problem situation wherein participants were asked to construct a representative value(s) that indexed the entire data set. Two of the tasks (tasks 1, 2) presented 'raw' data sets in which data were presented in an unordered and unorganized manner while three of the tasks (tasks 3, 4, 5) presented the data graphically on a line plot. The raw data sets, in tasks 1 and 2, consisted of identical data values as presented graphically in tasks 3 and 4, however individual data values were listed rather than presented graphically. The problem context was changed so as to prevent participants from recognizing the identical problem situations. This would allow examination of the effect of representational form on construction of representational index, while keeping distributional shape and size constant. Figure 1 shows a graphically presented data set (task 3) and figure 2 shows the same data presented in a non-graphical context (task 1).

The participants were administered the tasks in a whole group setting and given one hour to complete them. All students had completed the tasks within 50 minutes. Participants were provided with calculators and asked to engage in a 'written-think-aloud' protocol wherein they recorded the main justifications for engaging in specific mathematical practices, documented mathematical dilemmas they faced when engaging with tasks, and mentioned other aspects of their thinking that would facilitate the researcher in understanding the reasons for construction of specific representative values.

<u>Figure 1</u>: A positively skewed distribution of data presented on a line plot.

The following line plot describes the speeds of 64 drivers, over one week, ticketed by the police in the suburbs of a large city. These drivers were traveling through a 30 miles per hour speed zone. Generally, what was the speed of a driver who received a ticket?

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                     X
                     X
                X    X            X   X
              XXX X  X  X   XXX    X         X
          X   XXX X  XXX   XXX   XX   X    X     X
          XXXXX XXXXX XXXXX XXX X X XX   XXXXX  X X  XX
________________________________________________________
 30     35        40        45        50       55       60        65    70
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<u>Figure 2</u>: A positively skewed distribution of data presented in 'raw' data form

Are great discoveries generally made by people who are young and vigorous or by persons who are old and wise? Below is a tabulation of the ages at which scientists (Copernicus, Galileo, Newton, Franklin, Lavoisier, Lyell, Darwin, Maxwell, Planck, Einstein, Schröedinger etc.) made great discoveries.

39, 73, 39, 41, 70, 42, 43, 43, 52, 43, 44, 41, 44, 45, 36, 45, 45, 41, 47, 48, 49, 50, 48, 48, 49, 36, 37, 38, 46, 47, 38, 38, 39, 49, 49, 51, 51, 52, 52, 53, 55, 58, 47, 59, 59, 61, 62, 63, 40, 41, 63, 40, 39, 40, 41, 47, 41, 55, 56, 72

What approach would you use to answer the question: ***At what ages are great discoveries made?*** Please show your work.

Analysis of Data

Responses to tasks were examined and codes constructed to describe the representative values constructed by participants. Task responses were analyzed from two perspectives: nature of representative value and adequacy of representative measure. The nature of representative values was ascertained for individual responses on each task, representative values were coded as characterizing measures of central tendency, variability or some combination of measures. The adequacy of the constructed representative value was then recorded for individual tasks by determining the extent to which the measure reflected the variability and density of scores. Hence, once identified, the representative measure was coded as poorly representative or highly representative. An index was considered highly representative when the respondent constructed a value that took into consideration the landmarks and trends in the data. Factors influencing participants' selection of representative score were noted from written comments, inscriptions and other referents visible on the transcript. Final coding categories and respective frequencies are outlined in tables 1-5.

Results

The researcher and a member of the mathematics department examined the data sets prior to administration. A list of possible representative measures was constructed, codes assigned, and the degree of representativeness of the specific measures ascertained.

Question 1: What types of representative measures do preservice teachers construct to index distributions of data?

Preservice teachers were asked to construct representative values for normal and skewed distributions of data. Individual measures of central tendency were not considered representative of the positively skewed distributions of data. The mode was not necessarily representative of the skewed distribution due to its position at the lower end of the distribution and hence its poor ability to represent data values at the upper end of the distribution. Similarly the mean was not considered very representative due to the degree of influence of the outliers present at the upper end of the data set. Many of the preservice teachers seemed unaware of these issues, with 60% choosing an individual measure of central tendency to index the distribution, thus resulting in a value that did not provide an accurate picture of the data set (see table 1). 17% of the students attempted to coordinate variability and central tendency through the use of a variety of different strategies, hence taking into consideration the patterns or landmarks in the distribution. These responses included those strategies that constructed intervals of data (code A_2) and combinations of central tendency and variability (code C). The remainder of the responses utilized the range (11%), or other strategies (12%).

Table 1:

Distribution (in percent) of types of representative values constructed for a positively skewed distribution of data presented on a line plot

A.	Measures of variability			26%
	A_1	Range	11%	
	A_2	Interval	15%	
B.	Measures of central tendency			60%
	B_1	Mean	27%	
	B_2	Median	30%	
	B_3	Mode	3%	
C.	Combinations of variability and central tendency			2%
D.	Other			12%

Responses were categorized into measures that were representative of the data set and measures that were poorly representative of the data set. An index was considered representative when the respondent examined the data set and attempted to choose a value that took into consideration the landmarks and trends in the data. Hence, the provision of appropriately placed intervals of data, the construction of adjusted mean or mode values, and median values were considered representative. Categorizations of responses in terms of degree of representativeness are presented in table 2. As can be seen for the positively skewed data sets, 24% of constructed measures were considered representative of the data set, 65% were considered poorly representative of the data set and 11% were indeterminable.

Table 2:

Degree of representativeness of indices used to describe a variety of distributional shapes.

	Highly representative indices	Poorly representative indices	Indeterminable	Non representative indices
Positively Skewed	24	65	11	0
Negatively Skewed	34	64	2	0
Bimodal Distributions	67	24	4	5

Negatively skewed data sets were also indexed using a multiplicity of approaches. Similar to positively skewed data sets, individual measures of central tendency were considered poorly representative of negatively skewed distributions. The majority of the student-constructed representative values were measures of central tendency (64%) consisting predominantly of the mode, which represented 50% of the overall values (see table 3). In this particular distribution, the mode was situated at the upper extreme of the data set and was poorly representative. 27% of the students constructed an interval of data to index the distribution. These intervals represented an attempt to coordinate variability and center and were highly influenced by the mode (codes A_1 and E). 7% of the responses attempted a visual balancing of the data set wherein the mode, range and clusters of data were all taken into account. Hence we can see awareness on the part of 34% of the students of the specific features of the distribution that influence representativeness and hence the corresponding construction of representative values (see table 2).

Table 3:

Distribution (in percent) of types of representative values constructed for a negatively skewed distribution of data presented on a line plot

A.	Measures of variability			25%
	A_1	Interval	25%	
B.	Measures of central tendency			64%
	B_1	Mean	12%	
	B_2	Median	1%	
	B_3	Mode	51%	
C.	Combinations of variability and central tendency			2%
D.	Visual Construction			7%
E.	Other			2%

Measures of center were the predominant index utilized to represent the bimodal distribution and accounted for 66% of the responses, three quarters of which consisted of the mean (see table 4). Measures of variability accounted for 16% of the responses, the majority of which consisted of the construction of an interval of data within which a representative value would lie. One third of the intervals encompassed the first clump of data and the other two thirds encompassed the two major clumps of data. All intervals ignored the upper and lower data values, and hence represent attempts to summarize the data. 17% of the respondents presented an individual data value to represent the data set; analysis of the constructed values indicated that the values were not the result of an algorithm such as the mean or median, rather they represented an attempt at ‘eyeballing’ or visually balancing the data set. As the bimodal data set was symmetrical, the mean was considered a representative value, as was the median, and a large proportion of the intervals. Participants were most successful in constructing representative values for bimodal distributions, with 67% succeeding in constructing a representative value. This large proportion is mainly due to consideration of the mean as a representative value. More difficult to determine however is whether those participants who constructed mean values did so as a result of reflection on appropriateness of the mean as a representative value or whether they would have calculated the mean irrespective of the distributional shape. Analysis of approaches used by participants across tasks revealed that 59% of those who used the mean in the bimodal distribution had not constructed a mean in the skewed distributions and thus implies that they did so based on analysis of the distributional shape. Interestingly, 5% of the respondents constructed an unrepresentative value, deemed unrepresentative as it fell outside the range of the data values.

Table 4:

Distribution (in percent) of types of representative values constructed for a bimodal symmetrical distribution of data presented on a line plot

A.	Measures of variability			16%
	A_1	Range	12%	
	A_2	Interval	4%	
B.	Measures of central tendency			66%
	B_1	Mean	48%	
	B_2	Median	3%	
	B_3	Mode	11%	
	B_4	Median of median	4%	
C.	Combinations of variability and central tendency			0%
D.	Visual Construction			17%
E.	Other			1%

A chi-square analysis indicated that ability to construct representative values and distributional shape are not independent ($\chi^2_{(6)} = 889.771$, $p < .000$). Examination of residuals indicates that the ability to construct representative values for bimodal distributions exceeded expected values, and correspondingly fewer than expected participants constructed poorly representative values for symmetrical distributions.

Question 2: Did representational form influence the type of student-constructed representative values?

Measures of central tendency were the predominant distributional indices used to construct representative values, and accounted for over 50% of the responses in each of the tasks. However, the proportion of strategies using a measure of central tendency was higher for raw data sets (76% for task 1; 85% for task 2) than distributions presented using a graphical representation (60% for task 3; 64% for task 4; 66% of task 5). Correspondingly, measures of variability were less prevalent with raw data sets (2% and 11%) than for graphically represented data sets (26%, 27% and 16%). Using a chi-square analysis, the selected of representational index (i.e. measures of central tendency, variability) and representational form (graph, raw data) were found not to be independent ($\chi^2_{(6)} = 810.38$, $p < .000$). In particular, the use of measures of center to index graphical distributions was larger than predicted.

The higher use of measures of variability as descriptors for graphically represented data sets may be due to the fact that the graphical representation permits easier viewing of the variability of the data set, whereas in the raw form the data need be reorganized in order to facilitate examination of the variability. Examinations of the responses showed that less than 1% of the students reorganized or graphed the raw data sets.

Analysis and discussion

Mokros & Russell (1995) argue that summary statistics can have no meaning for students unless the data set can be thought of as more than a series of numbers - an understanding of the data set as a unit must exist. Results of this study indicate that this notion of data set as an individual entity may not exist for a proportion of preservice teachers, as over half of the preservice teachers utilize inadequate measures when attempting to index distributions of data.

The mean was the predominant measure used to represent distributions regardless of the distributional shape, indicating a lack of awareness of the repertoire of indices available. The propensity to calculate means regardless of distributional shape suggests that the mean represents a significant measure for preservice teachers. Other studies have suggested that use of the mean reflects a computational act more than a conceptual one, and that while many students may be able to compute the mean, few possess strong conceptual understanding of the nature of the mean (Skemp, 1979). Author & O'Loughlin (2002) asked preservice teachers to explain what the mean represents in the context of a distribution of scores and found that conceptualizations of the mean were limited to a formulaic notion of average. The fact that 41% of participants in this study constructed a mean to represent two or more distributions, regardless of distributional shape, may suggest that poor conceptual understanding exists of the function, role and limitations of the mean. This is concerning as preservice teachers may pass on their predispositions for the mean as a representative index to students in their future classrooms.

The study also highlights a lack of attention paid to measures of variability, particularly when the data were not presented in graphical form. A study of elementary and middle grade students constructions of typicality carried out by Author & Middleton (2001) found that younger students were more likely to employ measures of variability in their descriptions of distribution, and also that as students gained more experience working with measures of center they dispensed with reference to measures of variability in their descriptions of distributions. This may account for the lack of reference to variability in this study, an unfortunate pattern as measures of variability are necessary to supplement measures of location, which do not describe the dispersion among the data points. This suggestion of the mean computational formula gaining primacy to the detriment of the other forms of understanding was also proposed by Pollatsek, Lima & Well (1981) in a study of undergraduate psychology students' understanding of the weighted mean.

In addition, a lack of awareness of the functionality of graphical representations was evident in the absence of responses that reorganized or graphed the raw data sets. This finding has serious implications for classroom instruction as graphing data represents a significant stage in the data modeling process and influences the subsequent interpretation and analysis of data. If preservice teachers do not use graphical representations as tools to analyze data, then they are unlikely to advocate their use in classroom contexts. This hypothesis is supported by a study carried out by Heaton & Mickelson (2002) which found that preservice elementary teachers, when engaging children in statistical investigation, focus predominantly on the technical aspects of graph construction and do not use the representations as tools to analyze data or to make connections back to research question.

The findings of this study have several implications for preservice education. The ability of preservice teachers to construct measures of central tendency and variability, when presented with graphical distributions of data and raw data sets, indicates that they possess statistical content knowledge. This is an important and encouraging finding. Less encouraging, however, was the fact that many of these measures chosen to index distributions of data were not adequately representative. This may be due to the lack of experiences that preservice teachers have had with data. What this study highlights is the need for teacher educators to build on preservice teachers' content knowledge and intensify their efforts to help future teachers reason about distributions of data. Preservice teachers need to be provided with opportunities to engage in statistical investigations as learners first and then as teachers (Heaton & Mickelson, 2002). This will support explorations of how to best scrutinize, describe, index and represent data. Preservice teachers need to be afforded opportunities to see data sets as entities and examine the relationships between distributions of data and the measures that index them. Studies have shown that technological tools can support such investigations; hence the incorporation of such tools at preservice levels may be necessary. A small proportion of preservice teachers coordinated measures of variability and center in an effort to represent distributions of data, these students' reasoning needs to be explored and tasks designed to facilitate the development of similar approaches within the preservice elementary teacher population.

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