

Wittgenstein Looking at Wittgenstein

*A critical analysis of the self reflexive logical
evolution of Wittgenstein's work*

*From the Tractatus to his later work on Aspect
Seeing considered as exemplars of closed and open
logical models, and critiqued through the influence
of the set theoretic concept of the infinite, Russell's
Paradox, and self reference*

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Declaration of Originality

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Declaration: I hereby declare that this thesis is the result of my own original research and that it does not contain the work of any other individual. All sources that have been consulted have been identified and acknowledged in the appropriate way.

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Abstract

This thesis traces the evolution of Wittgenstein's work through the set theoretic concept of the infinite and the generated problems of Russell's paradox and self reference, which I argue develops through three distinct phases as presented in his early, middle and later work. I contend that considering Wittgenstein's work through the lens of these specific concepts is pivotal in identifying and understanding the logical evolution from his early to later work, which I argue represents a logical evolution from a closed to an open logical model wherein the influence of Russell proves critical. I identify these concepts as the key to understanding the crucial self reflexive dynamic operative between Wittgenstein's early and later work, which I claim further extends to understanding Wittgenstein's diametrically opposed early and later positions on the status of the activity of logical analysis, the referential and non referential use of language and his position on the logical distinction between a primary and secondary language. His position on the latter proves methodologically essential, illustrating how these core concepts are the logical means by which this distinction evolves, ultimately framing the move from a closed to an open logical model. Through the inversion of the primary secondary language distinction, I consider Wittgenstein's later work on aspect seeing as structurally and conceptually reflective of Russell's open logical model of type theory, arguing that Wittgenstein comes to accept in his later writings aspects of Russell's position that he had rejected in his early writings. In Wittgenstein's later open logical model of aspect seeing the primary secondary language distinction functions not only as the mechanism by means of which logical analysis operates, but also as a meta-analysis of Wittgenstein earlier work. This allows us to retrospectively engage with Wittgenstein's own particular form of linguistic aspect seeing, where we encounter the inherent self reflexive nature of this logical evolution allowing us to linguistically observe Wittgenstein looking at Wittgenstein.

Introduction

This thesis contends that the transition from Wittgenstein's *Tractatus* to his later work on aspect seeing represents the evolution from a closed to an open model of logic, within which the work of Russell occupies a critical and influential role. I argue that this logical evolution is specifically informed by Wittgenstein's changing position on the logical status of the connected issues of the problem of infinity and Russell's paradox which progress through three distinct phases during his early, middle and later work. While the problem of paradox has been expressed in many fascinating forms, it is the issue of linguistic and logical self reference and self reflexivity, which remains Wittgenstein's central concern throughout these three phases of evolution. In the work of the early Wittgenstein the concept of the infinite as presented by Cantor in its theoretical form and as presented through infinite space filling curves – which today we know as fractal curves - is rejected as logically untenable. I claim it is this specific rejection which in turn serves as the mechanism by means of which Wittgenstein claims to have resolved Russell's paradox.

After the failure of the closed model of the *Tractatus*, Wittgenstein returns in his middle period to reconsider his earlier position on the problem of the infinite, Russell's paradox, and self reference where the catalyst for this dramatic reconsideration is the fractal connection. In contrast to his earlier rejection, both the infinite and Russell's paradox now become central in laying the foundations for his later open logical model of aspect seeing. Consequently, in his later work both the infinite and Russell's paradox occupy an implicit role in the *Investigations*, albeit the logical analysis of the infinite, paradox and self reference, which brought Wittgenstein to this new logical position is no longer present in its explicit form as presented in his middle period.

In using this conceptual framework I show how, through tracing the influence of the infinite, Russell's paradox, and self reference, Wittgenstein ultimately moves from a position which regards logical theories such as type theory as being logically impure and therefore nonsense, to a position which regards such theories as logically essential and valid. I further contend that this logical evolution is underpinned within the parallel evolution of Wittgenstein's primary secondary language distinction, which features implicitly in the picture theory of meaning presented in the *Tractatus* and explicitly in his later work on aspect seeing. The critical difference in the logical structure of the primary secondary language distinction in his later work is the fact that it is presented in an inverted form. I contend that this inversion not only reflects the changed logical status of the infinite, paradox and self reference, but also completes the move from a closed to an open model of logic. Furthermore I maintain that the logical evolution of these core concepts lead to two diametrically opposed views of the process of logical analysis itself, and the role therein of non referential language use. In this regard I suggest the non referential use of language is represented by the ladder metaphor in the *Tractatus* which Wittgenstein rejects. With his later rejection of the *Tractatus* and the implicit Augustinian picture of language therein, the ladder of non referential language use is now accepted as a critical component in the process of logical analysis. These two opposing positions regarding the status of logical analysis itself thus prove foundational and are reflected in both Wittgenstein's early and later logical distinction between primary and secondary uses of language, and within his early and later closed and open logical models respectively.

Methodologically my analysis utilises a retrospective focus which applies Wittgenstein's later criticisms retrospectively to the *Tractatus*, making explicit the role of the primary secondary language distinction in his early work. As such I present Wittgenstein's later work on aspect seeing as a conceptual device which when retrospectively applied to both the *Tractatus* and his criticisms of Russell's type theory, allows each respective logical model to be considered as exemplars of aspect seeing. Through interweaving remarks from his middle and later work the

sense of his self reflexive logical evolution is evidenced, allowing the reader to see the conceptual evolution of the infinite, paradox and self reference which leads to the open logical model of aspect seeing. Furthermore I consider Wittgenstein's later work on aspect seeing as a meta-analysis of the process of logical analysis itself as operative in the non referential secondary use and meaning of language, thereby functioning as a retrospective device which allows language to look at itself in a self referencing and self reflexive manner, thus illustrating the very mechanism of self reference between a primary and secondary language which Wittgenstein argues is the means by which non referential language use generates all logical models. Not only therefore, is Wittgenstein concerned with the concepts of self reference and the infinite, but his later work retrospectively and actively demonstrates the activity of self reference as operative in both the language game of aspect seeing, and within his own secondary use of language and meaning, as expressed in his earlier closed logical model. I contend that central to this new position is his later reconsideration of both the infinite in its visual or fractal expression and the problem of paradox, where they now serve as illustrations of the aspect switch which characterises the experience of aspect seeing. In considering the later Wittgenstein's work on aspect seeing as an open logical model similar to Russell's type theory, and as an infinite language game which logic and language cannot quantify over, this retrospective analysis allows the Tractatus to be self reflexively contained or accommodated within his later open model of aspect seeing. Retrospectively I conclude that the different ways of considering the problem of the infinite, the problem of paradox and self reference, as they are presented in the three different periods of Wittgenstein's work, serve to explicitly illustrate the concept of seeing as or aspect seeing. In addition my application of the concept of aspect seeing, in a methodological manner reflects the interconnected relationship between the pedagogical process and the process of logical analysis, wherein looking back at language, and recognising the self referencing and self reflexive ability of language to look at itself, and reconsider itself in new ways, proves essential to the language game of linguistic aspect

seeing. It is in this sense that Wittgenstein's own retrospective meta-analysis of logical analysis itself via the concept of aspect seeing - in a general sense and within his own work specifically – creates a significant applicative potential.

Chapter one addresses the evolution of the primary secondary language distinction in the work of Wittgenstein, and in the work of Russell and Frege as presented in their logicist project. Firstly I present the implicit primary secondary distinction which informs the closed logical model of the Tractatus, before turning to an exposition of the non referential use of language by both Frege and Russell in the process of logical analysis epitomised in the logicist project.

Chapter two introduces the background of set theory which proved a fundamental influence in the work of both Russell and Wittgenstein. As Russell's paradox arises from the concept of the infinite as originally presented by set theory it is essential to present this conceptual framework in order to illustrate the centrality which the concept of the infinite occupies. Briefly the fundamental problem which the concept of the infinite generates, in relation to the logicist project of Frege and Russell, is that of a totality which cannot be quantified over and cannot therefore be accommodated within a closed logical mode. I then turn to Russell's efforts to address the problem of the infinite in the context of its role in generating Russell's paradox and the problem of self reference. I contend Russell's consideration of the infinite ultimately leads to the creation of the open logical model of type theory, which rather than rejecting the infinite attempts to accommodate it. Implicit in the open logical model of type theory is I argue a dynamic understanding of the role of non referential language use in the process of logical analysis, which Russell regards as both logically unproblematic and logically unlimited. While Wittgenstein rejects the work of Russell both on the basis of his accommodation of the infinite, and on the basis that such non referential language use is nonsense and involved in a vicious circle of self reference which gives rise to the paradox, he would later adopt a structurally similar open logical model in his work on aspect

seeing, thereby casting his earlier criticisms of Russell in an entirely new light, while also illustrating the critical influence of Russell from his early to later work.

In Chapter three, I introduce Wittgenstein's closed logical model of the Tractatus through the conceptual framework of set theory and begin to analyse the first stage in the evolution of his analysis of the concept of the infinite. Unlike Russell, Wittgenstein's critical appraisal of the concept of infinity and the generated problems of paradox and self reference, involve not only its logical or theoretical expression but critically its visual expression, which is presented by way of what today we know as fractals – examples of which include the Cantor Set, and the Peano and Hilbert space filling curves. I also highlight the central role which the Augustinian picture of language occupies in Wittgenstein's and Russell's work, connecting it with Wittgenstein's rejection of the ladder of non referential language use in the Tractatus. In 3.2 I introduce the fractal connection providing a clear historical overview of its significance. While fractal geometry is a highly complex mathematical field, my use of the term is in a conceptual and explanatory context. Considered in this context, in 3.3, I highlight the implicit role of fractals considered as the visual expression of the infinite in Wittgenstein's construction of the closed logical model of the Tractatus, as revealed in the Notebooks through his analysis of complex spatial objects. My analysis on this particular issue offers a new way of considering the picture theory of meaning which the Tractatus presents. I contend this is a significant and original contribution, as it ultimately forms part of the logical link which allows us to understand the evolution from his closed logical model of the Tractatus to his open logical model of aspect seeing, where we encounter his parallel changing position on the nature of pictorial representation and logical analysis. Unlike Russell Wittgenstein rejects in its entirety the concept of the infinite in both the visual field and in language, therefore there is no logical need or incentive to accommodate it from Wittgenstein's perspective. In the Notebooks and subsequently in the Tractatus the infinite in both its theoretical and visual expression is thus rejected.

In 3.4, I connect Wittgenstein's rejection of the infinite to Russell's paradox, where the rejection of the infinite functions as the mechanism by means of which he maintains he can banish Russell's paradox and with it the problem of linguistic self reference. In rejecting the infinite Wittgenstein in effect rejects the use of non referential language by means of which logic talks about such logical objects as the infinite. As Russell's paradox arises from a consideration of the infinite – which has been rejected - and from the non referential use of language to talk about logical objects – which has also been rejected - the paradox cannot emerge within the closed logical confines of the Tractatus. Parallel to making Russell's paradox vanish, I highlight Wittgenstein's use of the Necker cube as a means of briefly dealing with the issue of self reference in the visual field, whereby the perception of ambiguous perception which he would later term aspect seeing is 'accounted' for by stating that what we see are two different facts. Underpinning this position Wittgenstein establishes in contrast to Russell's infinite type theory, the idea of the atomic proposition as signalling the internal limit of the closed logical model. Here Wittgenstein's closed logical model can be considered as a linear construction, in contrast to Russell's hierarchical approach in his theory of types. From logical form through to the logical terminus of atomic propositions the Tractatus presents a closed model of logic which is self reflexive in so far as logic is mirrored into all its subparts. However its termination with atomic propositions denies language the potential to reflect upon itself and thereby to re create itself within the process of logical analysis. As such the closed model of logic utilises the principles of self reflexivity, but imposes a logical limit on this process unlike Russell's type theory. This ensures that we cannot talk about the system of logic or language from within that same system, as any efforts to do so will be self referential. This is criticised in terms of an absence of methodology and examples, regarding both how this would be achieved given that language in its non referential use is unacceptable, and also in terms of the ambiguity regarding what exactly an atomic or elementary proposition is. In addition the Tractarian conception of logical form is criticised as representing grammatical acrobatics rather than sound logical premises, given that

Wittgenstein has of necessity to use the ladder of non referential language to establish its very reality. While Wittgenstein perhaps felt that the concluding remarks which self reflexively condemns the propositions therein as nonsense resolves this particular dilemma, I argue that this is not the case. I suggest that his rejection of the infinite in both its theoretical and visual context as a means of resolving paradox and self reference, leads to an untenable limited and closed logical model of both the visual field and language itself.

The most significant and negative effect of this is that in so doing Wittgenstein banishes the entire activity and process of logical analysis and the non referential use of language therein to an eternal silence. Critically I highlight that if we accept the closed logical model of the Tractatus we accept a model wherein the possibility of aspect seeing is excluded. In this context I indicate that Russell's paradox serves as an illustration of the aspect switch inherent in the experience of aspect seeing in a manner similar to the duck-rabbit and the Necker cube. I conclude on the suggestion that logical form or the logical foundations of language which assumes the paradoxical position that while it can be shown it cannot be spoken of, gravitates on the very duality of an aspect switch which the Tractatus expressly rejects. This point becomes critical from the retrospective position of the open logical model of aspect seeing.

Chapter four, 4.1, moves to consider the failure and rejection of the Tractatus, highlighting the central role which the infinite assumed in that process. In this context I suggest that the colour exclusion problem assumes a critical significance in so far as the logical problem it reveals is that the atomic structure of propositions is not independent, as Wittgenstein had maintained but rather self referential and self reflexive. As the atomic structure Wittgenstein proposed is now untenable, it becomes essential that the concept of the infinite which had led to this initial conclusion is re-examined. In recognising this I focus on how Wittgenstein's return to the problem of the infinite – which he had rejected in the Tractatus as a means of addressing Russell's paradox – signals the beginning of the self reflexive nature of

the relation between his early and later models of logic, and the self reflexive nature of the process of aspect seeing which both constitutes his later open logical model and, is retrospectively illustrative of the process of logical analysis and the non referential or secondary use of language. Extending this analysis in section 4.2, I return to the primary secondary language distinction of the early Wittgenstein, highlighting his rejection of that particular configuration and present its inverted form as indicative of the role which his later consideration of the infinite, paradox and self reference assume therein. I conclude by highlighting Russell's criticism of the Tractatus indicating that his proposal for a potential solution to the problems he identified in the Tractatus ultimately anticipating Wittgenstein later open logical model of aspect seeing.

Chapter five, 5.1 represents the second stage in the evolution of the infinite in Wittgenstein's work via the fractal connection. Here I provide exegetical analysis of Wittgenstein's middle period which demonstrates how the infinite complex spatial objects of the Notebooks – fractals - are reconsidered, ultimately operating as the logical catalyst for his later concept of aspect seeing. In 5.2 Wittgenstein's acceptance of the infinite within the visual field is extended to an acceptance of the infinite in language itself, where it is now understood and logically presented as indicative of an infinite possibility of language rather than an infinite logical reality which had been the case in the Tractatus. Parallel and central to this new logical positioning of the infinite is Wittgenstein's focus on the pedagogical process as a means of revealing the nature of the process of logical analysis.

In 5.3 this is extended to how Wittgenstein re considers the role of non referential language use in the process of logical analysis. Here we see the middle Wittgenstein focusing on the significance of rule following and the extended significance of the distinction between rules of reference and rules of invention as a means of accommodating the infinite. Ultimately Wittgenstein's conception of the grammar of infinite possibilities now operates as the new logical bolt of his later work, replacing the fixed and immovable logical bolt of elementary

propositions. From a retrospective position this validates my exposition in chapter three, clearly highlighting the centrality of the infinite in Wittgenstein's move from a closed to an open logical model.

In Chapter Six, The second stage in the evolution of Wittgenstein's analysis of Russell's paradox and the problem of self reference, 6.1 I present Russell's paradox as illustrative of the experience of aspect seeing. Here Wittgenstein's middle period position on Russell's paradox and the problem of self reference as generated by the problem of the infinite is examined through exegetical analysis. Using Russell's work as a retrospective contrast to Wittgenstein, I illustrate how Wittgenstein's new open logical model of aspect seeing demands the principle of self reference and reflexivity, which he had earlier rejected, in order for his new position on the logical possibilities of secondary language use and meaning to function. Retrospectively considered this illustrates how both the problem of the infinite and the problem of paradox and self reference are inextricably linked from Wittgenstein early to later work. In 6.2, I introduce Wittgenstein's distinction between bounded and unbounded language games from the Blue and Brown Books, which serve to illustrate how Wittgenstein's new position on the status of the infinite, paradox and self reference find an applicative use in the language game of secondary language use and meaning. Here Wittgenstein's use of the pedagogical as a methodological device is again highlighted. Methodologically using excerpts from his middle and later periods side by side highlights the self referential and self reflexive logic inherent in both his own logical evolution and within his later open logical model of aspect seeing. In considering logical analysis as such an unbounded language game, his distinction between bounded and unbounded language games can be seen as the means by which the later Wittgenstein accommodates the logical principle of the infinite in an unlimited, and in principle infinite language game.

Chapter Seven, focuses on the third and final evolution of Wittgenstein's analysis of the problem of the infinite, paradox and self reference as presented in

Wittgenstein's later work specifically the *Philosophical Investigations*. Here I illustrate how the principle of linguistic aspect seeing - the self reference between primary and secondary language as the generative force in creating new language use and meaning – functions as a retrospective device of meta-analysis allowing us to see Wittgenstein's earlier work as self reflexively related to his later work. This allows a final definitive retrospective view on how the ladder of non referential language use which was rejected in the *Tractatus*, now occupies the most central position in his open logical model. This presentation of the secondary use and meaning of language in the linguistic language game of aspect seeing, allows us to see Wittgenstein looking back at his earlier work in a retrospective manner, both as an exemplar of aspect seeing/blindness, and as a meta analysis of the non referential use of language in the activity of logical analysis. Here we see the self reflexive nature of Wittgenstein's own language looking at itself, and in so doing generating a new and secondary open logical model of aspect seeing within which the early closed logical model is both dependent on and ultimately consumed by.

In Chapter seven, *Inside the Self Reflexive Process of Aspect Seeing - Wittgenstein Reflecting Russell*, I illustrate how the open logical model of aspect seeing can retrospectively be considered as being implicit in both Russell's work – specifically the theory of types - and in his conception of the open ended nature of logical analysis itself. This presents a new understanding of the extent and depth of influence which Russell's work exerts upon Wittgenstein's logical journey from the closed model of the *Tractatus* to the open logical model of aspect seeing. While traditionally overshadowed by his student, my analysis reveals new aspects of the interconnectedness of the work of Russell and Wittgenstein, which ultimately reveals more similarity than difference. Retrospectively considered Wittgenstein's logical journey demonstrates that ultimately he ends up adopting a logical model similar to that expressed by Russell's open logical model of type theory, which Wittgenstein so passionately rejected in the *Tractatus*. In coming full circle, Wittgenstein, like Russell ultimately had to accommodate the problem of the infinite, paradox and self reference, as rather than logical problems to be removed

or banished they are now understood to represent logical indicators of the open ended language game of aspect seeing. In accommodating the infinite paradox and self reference Wittgenstein revealed not the end of logical analysis as he had believed in the Tractatus, but the beginning, and thus now can freely speak about the process of logical analysis which he had once condemned to an eternal silence.

Literature Review

Resolute and Non Resolute Readings of the Tractatus

The traditional resolute and non resolute readings of the Tractatus have dominated Wittgenstinian scholarship for many years. Their relevance centres on the fact that while both make large claims regarding both the purpose of the Tractatus itself and most significantly how it relates to his later work, the mathematical issues of the infinite and the problem of paradox – which I contend are essential to both claims – remain entirely unaddressed. The reading of nonsense which this thesis advances is a view which demands taking the nonsense of the Tractatus as representing a serious logical position wherein an understanding the complex role of both the infinite and the generated problem of paradox, self reference and contradiction are essential. In line with the resolute position I contend that the logical model of the Tractatus is a deeply held logical position and was certainly a position to which Wittgenstein adhered.

Traditionally academic positions on the Tractatus are divided between adherents and non adherents of the New Wittgenstein movement. Here the most famous and well known academic readings of Wittgenstein's work divide into two camps – the New Wittgenstein movement which supports a resolute reading of Wittgenstein's work, led by Cora Diamond and her followers, and the more traditional anti resolute reading led by P.M.S Hacker and adherents to his view. The intellectual division between the New Wittgenstein movement and the traditional or standard anti resolute reading of Wittgenstein's work centres on two issues. Firstly whether or not there is a decisive break between the early and later Wittgenstein or if there is a clearly identifiable consistency spanning both periods; the second issue centres on the disagreement between both groups on the status which ought to be attributed to the concept of nonsense as it appears in the Tractatus.

In relation to the first issue the resolute reading of nonsense which it extends to the work of the later Wittgenstein wherein they argue no decisive change has occurred

leads to an untenable position. In response to this my work seeks to show how and why – via an analysis of the concept of the infinite and Russell’s paradox - there is consistency between the early and later Wittgenstein while also showing how and why there is a decisive break between the two periods which rather than weakening consistency in fact strengthens it. The second issue regarding the status of nonsense in the Tractatus is understood by the resolute reading as indicating all is nonsense including the propositions of the Tractatus and crucially extends to a further position which contends there is no logical argument being claimed by the Tractatus. On the other hand according to the non resolute standard interpretation there are ineffable truths which the Tractatus somehow gestures at or shows, further arguing that the nonsense of the Tractatus has a constructive role. The reading I present accepts that both viewpoints regarding nonsense have merit and are not mutually exclusive.

The logic of the Tractatus operates like a tightrope- at one end is logical form and at the other are elementary or atomic propositions. My contention is that the picture of logic it sets forward is that of a closed logical model, and stands in complete opposition to Russell’s open logical model of type theory, the recursive self referential method of which delivers a picture of logic operating in a manner akin to a Russian doll sequence continuing ad infinitum. I suggest that the validity of the resolute position regarding the nonsense of the Tractatus extends only to the extent that the closed logical model of showing had unavoidably to employ language to present its arguments, albeit in a novel manner. I maintain that in order to appreciate the nonsense of the Tractatus the reader must acquiesce with the Tractatus as representing two different views of logical analysis – the process of non referential language use which he rejects as represented in open logical models such as type theory, and his new logic of showing which is presented as a superior model which avoids the problem of linguistic self reference. The structure of the Tractatus is I suggest intended to suggest two logical models but there is only one to which Wittgenstein adheres the closed logical model of showing. Of course paradoxically Wittgenstein has to employ the non referential use of language in

order to both reject its use, and in order to present his alternative closed logical model, thereby making the Tractatus an impossible object by virtue of having to use the rules of non referential language which he rejects to deliver the show say distinction. The paradox of interpretation is therefore an intentional and unavoidable one from Wittgenstein's perspective. Retrospectively considered from the perspective of his open logical model of aspect seeing Wittgenstein recognises this as a flawed position but also as reflecting of the language game of linguistic seeing as or the secondary use and meaning of language.

The resolute position recognises the essential purpose of Wittgenstein's work – both early and late - as therapeutic. While this position is unproblematic their position that the Tractatus has nothing at all to show is I argue untenable. This view is underpinned by the resolute interpretation of nonsense in the Tractatus whereby the propositions of the Tractatus itself are regarded as nonsense thereby removing the possibility– as Hacker and others argue – that the Tractatus implicitly suggests something ineffable or metaphysical which cannot be logically articulated but is somehow shown by the work itself. Alice Crary describes Diamonds and Conant's position as one which entails that the propositions of the Tractatus:

are not logically distinct from gibberish – i.e., to use one of Diamonds examples, not logically distinct from “piggly wiggle tiddle” – and that we should give up the idea that they are trying to say anything. (Crary, Read, 2000, p.12)

Firstly it must be noted that in the Ogden version of the Tractatus which Wittgenstein approved of the word nonsense features only once. Here the nonsense of the Pears McGuinness edition is presented by the word senseless apart from the Necker cube reference – which in itself is significant as it is this particular visual illustration occupies a key role regarding Wittgenstein's view on the independence of atomic propositions. While the resolute position is valid in taking Wittgenstein

at his word whereby he states that the propositions of the Tractatus itself are nonsense and not “logically distinct from piggly wiggle tiggles” the reason *why* he is in a position which demands this assertion remains problematic. As I will show the central problem with the resolute position is its failure to address the quintessential *purpose* of the Tractatus – which I contend is to resolve the problems generated by the concept of the infinite presented in set theory and its subsequent involvement in the generation of Russell’s infamous paradox, self reference and contradiction. If we are to fully appreciate the resolute position which sees no difference between the propositions of the Tractatus and “piggly wiggle tiggles” we must first fully understand the complexity of the logic informing this work. It is I maintain the ladder of the non referential use of language and its efforts to resolve the problem of the infinite and paradox – the equivalent of piggly wiggle tiggles– which Wittgenstein rejects, and not the closed logical model of showing which he is offering in its place.

The resolute interpretation of nonsense is absolute. What the Tractatus shows is “real nonsense, plain nonsense, which we are not in the end to think of as corresponding to an ineffable truth” (Diamond 1988, p.8) While it is the case that the Tractatus does not “correspond to anything”, the sense of this point is rooted in Wittgenstein’s belief that the logic of showing can operate without the necessity of referring to or corresponding to logical form or logical objects – as is the case in the non referential use of language within traditional logical analysis which he criticises specifically in the work of Russell. Wittgenstein’s logical atomism thus stands in opposition to the non referential use of language – the equivalent of “tiggly wiggly piggles” - which claims to do precisely this - i.e. refer to a logical realm. While it is therefore strictly true that the logic of showing cannot be stated within linguistically without the problem of self reference emerging, there is no mechanism other than language available to Wittgenstein to reject the non referential use of language as operative in traditional logical analysis and epitomised for Wittgenstein by Russell’s type theory. Despite this, Wittgenstein did adhere to the view that the closed logical model of the Tractatus could reveal –

through showing as opposed to the non referential use of language – the atoms of language. While Wittgenstein fails to provide an alternative methodology to non referential language use, and also fails to provide any example of an atomic proposition he believed these problems would be addressed at a later point. This adherence is clear in his later work albeit is disregarded by the resolute reading who claim he adhered to no position. Hacker in opposing the resolute reading suggests the ineffable dimension of the Tractatus demands an understanding that “the nonsense of the pseudo-propositions of philosophy, in particular of the philosophy of the Tractatus, differs from the nonsense of ‘A is frabble’, for it is held to be an attempt to say what cannot be said but only shown.”, and is considered by Hacker as “illuminating nonsense” with the added qualification that Wittgenstein was indeed trying to whistle it. (Hacker 2000).

The central problem with Diamonds analysis – which is the *raison d’être* of the resolute position she adheres to – is her argument that the Tractatus has nothing to *show* us. While she points out that “the idea of a science of logic is, on Wittgenstein’s account, nothing but an illusion. (Diamond 1988, p.27), she fails to highlight that this view is informed by Wittgenstein’s rejection of non referential language use in the process of traditional logical analysis, and therefore the Tractatus is disregarded as having any logical status at all. Hacker highlights this problem in his analysis of ‘Throwing away the Ladder’ where Diamond credits Frege’s work as the catalyst for the show say distinction of the Tractatus. Contrary to this Hacker points out that the specific catalyst for the show say distinction was Russell’s theory of types – albeit the fatal flaw in the contributions of both Hacker and Diamond is their entire neglect and omission of the problem of paradox:

It is evident that the distinction emerged in the final section of the ‘Notes on Logic’ of September 1913. It resulted from reflecting on Russell’s theory of types, and not, as Diamond and Conant assert without textual support, from reflecting upon

Frege's puzzlement about the assertion that the concept horse is (or is not) a concept. (Hacker, 2000)

While it is of course entirely valid for Diamond to highlight the central influence of Frege it is misleading to ignore the textual evidence which clearly demonstrates that Russell's theory of types – specifically as a response to the problem of the infinite and paradox - was the central influence in the show say distinction. Moreover any analysis of Frege ought to logically lead to the problem of paradox as Russell's paradox emerges in response to Frege's Axiom V. However such a transition fails to materialise within both the resolute and non resolute position. The extent to which the resolute position makes contact with the problem of paradox is through the Frege connection. However while Diamond gives a detailed analysis of the role of a function in Frege's work and makes an implicit reference to the problem of paradox she fails to identify either the paradox or its centrality in the failure of Frege and Russell's logicist project - to this extent the analysis is flawed.

Diamond's analysis of nonsense as resolute stems from her criticism of logical form "the one thing which according to the Tractatus shows itself but cannot be expressed in language"(Diamond 1988, p.7) Diamond's account of this central logical component of the Tractatus is dismissive and definitive in its conclusion that the concept is entirely devoid of any significance:

So it looks as if there is this whatever-it-is, the logical form of reality, some essential feature of reality, which reality has alright, but which we cannot say or think that it has. What exactly is supposed to be left of that, after we have thrown away the ladder? (Diamond 1988, p.7)

For Diamond the central problem is how seriously we can take Wittgenstein's throwing away the ladder remark:

The problem is how seriously we can take that remark, and in particular whether it can be applied to the point (in whatever way it is put) that some *features of reality* cannot be put into words. (Diamond 1988, p.7)

Maintaining that the Tractatus has something ineffable to show is according to Diamond the "chickening out" position offering instead what she believes is the correct reading of the Tractatus:

What counts as not chickening out is then this, roughly: to throw away the ladder is, among other things, to throw away in the end the attempt to take seriously the language of "features of reality". (Diamond 1988, p.8)

This same issue is clear in James Conant's work who claims that "the illusion the Tractatus seeks to explode, above all, is that we *can* run up against the limits of language." (Crary, Read 2000, p.197) concluding in his paper 'Elucidations and Nonsense in Frege and Early Wittgenstein' that:

The aim of the work is to show us that beyond "the limits of language" lies – not ineffable truth, but rather – (as the preface of the Tractatus warns) *einfach unsinn*, simply nonsense. (Crary, Read 2000, p.198)

Against this reading I argue that running up against the limits of language is not an illusion which Wittgenstein seeks to explode, but rather is the central logical

argument of the Tractatus which is directed at logicians and the pictures which non referential uses of language as presented specifically in the open logical model of type theory propagate. The illusion under attack is rather the illusion of the logician who believes he can use non referential language in a referential manner to refer to the foundations of logic and language, without the problem of self reference emerging. For Wittgenstein the problem of self reference – specifically within the work of Frege and Russell - did indeed signal running up against not only the limits of language but most crucially the limits of the visual field within which his closed logical model is rooted. However as the resolute position does not consider the problem of paradox to be in any way central to the concept of logical form the possibility of considering the Tractatus in this way is impossible.

Diamond thus maintains that the show dimension of the Tractatus – which I argue was the foundation of Wittgenstein’s logical atomism - is used by Wittgenstein for ironic purposes only. This position is entirely untenable given Wittgenstein’s retrospective remarks on the Tractatus. She argues that if the implicit thesis of the Tractatus is true (while simultaneously denying there is one) it would be impossible to use ordinary language. She asserts thus “if my account of Wittgenstein is correct, his is a lunatic account. (Diamond 1988, p.19) Firstly Wittgenstein is decisive in his view that ordinary language is in perfect logical order as it is, therefore “carrying out a complete analysis” is a concern for logic and not for the ordinary use of language which as Diamond notes “we in practice never do”. (Diamond 1988, p.19) It is not Wittgenstein’s intention that we should!

5.5563 All propositions of our colloquial language are actually, just as they are, logically completely in order. (Wittgenstein and Ogden, 1990, p. 88)

While the Tractatus is of course open to be read by anyone, identifying the target audience - the world of logicians - is crucial in order to understand the salient

purpose of the show say distinction – an observation absent in the resolute position. For Conant the “only insight that a Tractarian elucidation imparts is one about the reader himself” to the extent that “he is prone to such illusions of thought”.(Crary, Read 2000, p.197)The view here is that Wittgenstein is involved in a grand plan where the sole purpose of the Tractatus is to lead the reader – whoever it may be – to a place where they imagine themselves as being able to do “what the Preface warns we will fall into imagining ourselves able to do”.(Crary, Read 2000, p.198) Rather Wittgenstein’s point is that once a limit to what can be thought or said is in place – which Wittgenstein believed would be achieved through the discovery of atomic propositions – then the idea of thinking outside of this limit (which is the illusion of the logician) resolves to become mere nonsense.

The most problematic dimension of both Diamond and Conant’s position is their extension of the resolute reading from the Tractatus to the Philosophical Investigations. For Conant the link revolves around the following illusion. The illusion in question is the position Wittgenstein ‘lures’ the reader to occupy, namely a “certain sort of a perspective” where “we take ourselves to be able to survey the possibilities which undergird how we must represent things. (Crary, Read 2000, p.197) Most problematically Conant maintains that Wittgenstein is concerned in both his early and later work with the *same* illusion and more significantly that there has been no change in his position spanning both periods. On the other hand Diamond links Wittgenstein’s supposed irony in the Tractatus regarding the show dimension of the text to a specific remark in the Investigations which is considered retrospectively as simply a reiteration of what has been said in the Tractatus. The position of both Diamond and Conant that the Investigations is simply a reiteration of the Tractatus as understood by the resolute position is untenable in the light of Wittgenstein’s own remarks, particularly his paper Some Remarks on Logical Form. His adherence to a closed logical model of logical atomism - in the Tractatus is made retrospectively explicit in Remarks on Logical Form; however this critical paper is never mentioned by Diamond or Conant.

In this paper Wittgenstein is crystal clear that he did adhere to the project of logical atomism in the *Tractatus* explicitly stating that:

It is, of course; a deficiency of our notation that it does not prevent the formation of such nonsensical constructions, and a perfect notation will have to exclude such structures by definite rules of syntax.

Such rules, however, cannot be laid down until we have actually reached the ultimate analysis of the phenomena in question. This, as we all know has not yet been achieved. (Wittgenstein, 1929)

To suggest that Wittgenstein's own retrospective analysis of the *Tractatus* is imbued with intentional conspiracy and irony is incredulous and without any textual support. Diamond and Conant simply do not recognise the paper as it really doesn't quite fit with the resolute position, and absolutely doesn't tally with the idea of irony they both suggest. As Hacker points out in his criticism of the resolute position:

Nowhere is there any suggestion that of course, he really did not believe these things, that he knew at the time that all these assertions were 'plain nonsense' written in the spirit of Kierkegaardian irony or in the manner of a Zen master. On the contrary what he explicitly accused himself was dogmatism. (Hacker 2001)

Wittgenstein's own remarks on the *Tractatus* are decisive on this issue:

Formerly I myself spoke of a 'complete analysis'.....At the root of all of this there was a false and idealised picture of the use of language. (Kenny 1994, p.41)

On the non resolute side Hacker also offers a reading of Wittgenstein's Tractatus which seeks to link it to his later work noting:

The author of the Tractatus laboured to reveal that the structure of the world cannot be described but only shown. The author of the Investigations bent his efforts to reveal how what seemed to show itself was an optical illusion. (Hacker 1986, p.168)

Hacker's analysis of the transition between Wittgenstein's early and later work is limited, however unlike Diamond and Conant Hacker at least registers the fact that there is a significant change in the logic of the later Wittgenstein, from which his early work can be considered – a change which I contend consists in the move from a closed to an open logical model. While Hacker is correct that Wittgenstein was under an illusion of sorts in the Tractatus he ultimately fails to locate the correct source of the illusion. The source of the illusion in this case was his belief that logical atomism definitively resolved the problem of paradox. In respect of what I referred to as the gold standard in approaching exegetical analysis of Wittgenstein's work the resolute position is entirely untenable, not only in relation to its analysis of the Tractatus but more significantly in relation to the extension of this analysis to his later work. What Wittgenstein does have to say can leave us in no doubt that from a retrospective consideration Wittgenstein most certainly did adhere to a particular logical position – the position of logical atomism.

That he did adhere to a logical position is also explicit in the Tractatus. Here Wittgenstein is clear on a division between ordinary language and logical language (primary and secondary language), unambiguously indicating in the following remark that the purpose of the Tractatus is to bring logic and the logician to a new logical domain, one where showing takes logical precedence to saying:

3.323 In the language of everyday life it very often happens that the same word signifies in two different ways...Thus there easily arise the most fundamental confusions (of which the whole of philosophy is full). In order to avoid these errors *we must employ a symbolism which excludes them...* (Wittgenstein and Ogden, 1999, p.41)

In his criticism of the resolute position Ian Proops also focuses on this very point noting:

I say in the Tractatus that you can't say anything about the structure of atomic propositions: my idea being the wrong one, that logical analysis would reveal what it would reveal. (Proops 2001, p.392)

Even in more non resolute, resolute adherents such as Juliet Floyd this obvious problem is essentially avoided and the Tractatus rather than a coherent work of logic is considered as somewhat of a mishmash of concepts which don't "fit together" an observation which I maintain is entirely incorrect:

When looking at the text of the Tractatus (as opposed to meditating on the concept of nonsense), it has never been clear to me, prima facie, that all of Wittgenstein's remarks concerning showing or nonsense fit together in one way, or, perhaps, even at all.(Floyd 2007, p.182)

As I will show rather than a mishmash of concepts which do not fit together the closed logical model of the Tractatus is informed specifically by the concept of the infinite and Russell's paradox, which in turn inform his later rejection of the Tractatus and the generation of the new open logical model of aspect seeing. I thus contend that the evidence for Wittgenstein's adherence to a very particular logical position in the Tractatus is clear while the evidence to support no such adherence – as Diamond and Conant argue - is entirely absent.

Hacker cites a letter Wittgenstein wrote to Russell regarding the show say distinction which expressly details that the main contention of the Tractatus “is the theory of what can be expressed by propositions – i.e. by language – (and which comes to the same what can be thought) and what cannot be expressed by propositions but only shown; which I believe is the cardinal problem of philosophy.”(Hacker 2000) As Hacker remarks “It is implausible to suppose that he was pulling Russell's leg and that the *real* point of the book is that there is nothing at all to be shown.” (Hacker 2000) In his rejection of the resolute view Hacker categorically rejects – and correctly in my view- Diamond and James Conant's stance that the show say distinction is merely an illusion which we are meant to throw away once we come to the end of the Tractatus. Central to this argument is acknowledging Wittgenstein's retrospective analysis of the Tractatus which explicitly establishes that Wittgenstein himself was under the very illusion he thought he was ridding logical analysis of.

For Wittgenstein in order to be meaningful a proposition must be able to refer in a referential manner to what it names. However this is precisely what Wittgenstein argues *cannot* be achieved by the language or ladder of non referential language use, as language cannot he claims refer to its own logical form. Rather logical form constitutes the logical foundation of language and as such cannot be referred to in any meaningful way. In failing to identify the role of the infinite and the role of paradox and self reference, the resolute position reflects an essentially aspect

blind insistence that we must see the Tractatus in one way only – the resolute way. As I will show it is the influence of the problem of the infinite and the garneted problem of paradox which leads the early Wittgenstein to assert the independence of atomic propositions in the closed logical model of the Tractatus. Retrospectively considered this central logical bolt of the Tractarian closed logical model, serves as the means by which Wittgenstein’s later work on aspect seeing evolves to become an open logical model, where the logical bolt now becomes the interdependence and self reference between primary and secondary language use and meaning in the experience of seeing something as. As such my work argues that the work of the early and later Wittgenstein correspond to two very different logical models both of which he adhered to during these respective periods.

The problem of paradox

All secondary literature dealing specifically with the problem of paradox from Davant to Urmans Sutrop, Jose Zalabrado, and Graham Stevens identify logical form as foundational in the Tractatus itself and in Wittgenstein's resolution of the problem of paradox – a view which stands in direct opposition to the view presented by the resolute position which maintains the Tractatus has nothing to show.

In his analysis of the problem of paradox Davant highlights the central role of logical form and the role of a function. Unlike the resolute position the role of Frege is directly linked to the problem of paradox - a link which fails to materialise in the resolute debate. Both logical form and the role of logical functions are presented as being intrinsically bound with both the problem of paradox itself and with its resolution. Davant locates Wittgenstein's central problem with Russell's theory of types in his use of logical functions:

How is it possible to determine that
function at type level n means the
same as function at type level $n+3$?
(Davant 1975, p.104)

This problem as I have indicated rests on the lack of any public criteria which could verify or refute such claims. Wittgenstein rather sees such efforts as inventions. Crucially the closed logical model of the Tractatus has no room for the domain of invention or creativity - a position which would be abandoned entirely in his later work on aspect seeing. Davant further links the role of a function to the role of logical form - an essential link – which brings to the fore the fundamental importance of the logic of showing which Wittgenstein adheres to. Disposing of Russell's paradox demanded denying to logical functions the possibility of being their own argument owing to the intractable problem of self reference which this

involved. As Davant points out “if a function could be its own argument” then “the form is different from the original and the signs do not have the same meaning”(Davant 1975, p.106) Moreover Davant notes that Wittgenstein’s logic of showing precluded in a de facto manner the very idea “that a function could even take itself as an argument.”(Davant 1975, p.106) It is as such “not even a possible value of the variable” (Davant 1975, p.106) because it shows us nothing but its own self reference. The possibility of inventing logical terms to substitute what cannot be shown and as such neither verified or refuted leads not to a solution but to an iteration of the original problem – which while it might look different linguistically becomes nothing more than a ladder of nonsense propositions. The set which the theory of types is trying to construct can therefore never be internally consistent as it is of necessity an infinite set embroiled in infinite self reference:

“signs such as ‘function’ must constantly be reintroduced, the completeness condition may not be attained”(Davant 1975, p.105)

In Urmas Sutrop’s paper Wittgenstein’s Tractatus, 3.333 and Russell’s paradox adopts a more non traditional approach to the resolution of the problem of paradox setting out his position as follows:

My position is that Wittgenstein created his own transcendental logic that can show self referential (reflexive) sentences without reflexivity. (Sutrop 2009, p.179)

Crucially Sutrop identifies the centrality of set theory as a methodological device in the logic of showing presenting a reading of Wittgenstein’s solution to the problem of paradox – in its technical expression – as indicating “a class membership language (CML). This approach has a clear parallel with my presentation, however the focus in this paper is exclusively on technical issues

regarding the logical formulation of the paradox which while important in its own right fails to get to the heart of the logic of showing by ignoring the visual dimensions of paradox. Sutrop's conclusion is that:

Russell's paradox (his Theory of Types) contains self reference and for that reason is circular. Wittgenstein's solution in his formula contains no self reference (reflexivity). The symbols show the same sentence that lies behind Russell's paradox, but without saying it. Wittgenstein's logic is not a theory but a reflexion of the world. Nevertheless we have an access to what is shown by Wittgenstein's transcendental logic. Using CML one can say in ordinary language what is only shown by symbols of logic. (Sutrop 2009, p.193)

The idea that Wittgenstein's logic is transcendental, runs counter to everything the closed logical model of the Tractatus is rejecting – specifically the non referential use of language as encountered in relation to the infinite. I maintain contrary to Sutrop that the logic of showing is intended to remove the metaphorical ladder of non referential language use which is under the illusion that it can reach the transcendental realm. It is this which demanded the peculiar and paradoxical role of logical form – that which both is and is not a member of the closed set of the Tractatus in so far as it is shown but cannot be in turn spoken of. Sutrop's view that we can say in ordinary language what the symbols of logic show is also problematic. As already indicated the closed logical model of the Tractatus retrospectively identifies a primary and secondary language – wherein secondary language is ordinary language and has no essential role in logic. The function of a secondary language as conceived by Wittgenstein – no matter how misguided – was to function as a means of showing or revealing and not saying what the innermost and final logical component of the closed set was. This final logical

internal limit was the atomic proposition which Wittgenstein maintained was hidden or disguised by ordinary language. As he observes in *Some Remarks on Logical Form*:

Our analysis if carried far enough must come to the point where it reaches propositional forms which are not themselves composed of simpler propositional forms. We must eventually reach the ultimate connection of the terms, the immediate connection which cannot be broken without destroying the propositional form as such. (Wittgenstein 1929)

As is clear in the above we cannot get outside the propositional form – this must remain intact and as such the logic of showing would not be using language to refer to its foundation but rather would be revealing the innermost “connection which cannot be broken without destroying the propositional form as such.” (Wittgenstein, 1929)

The position of Dale Jacquette that “the theory of types is unnecessary as a solution to the paradox, because there is no paradox to be solved.” (Jacquette 1998, p.142), is also problematic. If interpreted as there is no paradox to be solved once the closed logical model of showing assumes logical precedence to the non referential use of language, then the point is acceptable. However if intended to suggest that the problem of paradox is entirely insignificant, then the position is untenable. The position I believe to be accurate is that Wittgenstein regards it impossible for language to refer to its own foundation thereby making the problem of paradox insoluble within the traditional non referential use of language in the process of logical analysis. Critically the problem of paradox, self reference and contradiction are generated by the problem of the infinite and it is this which I contend has greater significance for Wittgenstein, in so far as its removal is essential in resolving Russell’s paradox. Moreover as I contend, the problem of

paradox is also central to his later work on aspect seeing, so the idea that the problem doesn't exist at all does not tally with what we are presented with by Wittgenstein in either his early or later work.

The work of Graham Stevens presents a unique integration of the problem of paradox, the theory of types and the theory of judgment, an approach I believe is essential. In *Re-examining Russell's Paralysis: Ramified type theory and Wittgenstein's objection to Russell's theory of Judgment* Stevens describes the orientation of Russell's logical atomism as follows:

The ultimate constituents of propositions are atoms – entities which are logically independent of one another and stand on an ontological par at the end point of analysis. (Stevens 2003, p.10)

Such atoms were of course never reached either by Russell or Wittgenstein. While Stevens correctly identifies this motivation in Russell's work he fails to identify it in Wittgenstein's. While this point may seem minor it is significant in light of the argument which follows. Firstly Steven correctly identifies logical form as the central problem for Wittgenstein both in the theory of types and in the theory of judgment, noting that Russell's introduction of "the form of the judgment into the judgment complex" was intended to "ensure the required unity and order."(pg. 19) However as Stevens points out:

There is a problem as to the status of logical forms. Russell seems unsure as to precisely what they are. On the one hand they appear to be complex constituents of judgment, but on the other hand they appear to demand special treatment compared to the other constituents of judgments. (Stevens 2003 p.19)

The problems this generates stand parallel to the problems inherent in the theory of types where “we are faced with the threat of a regressive argument” (Stevens 2003, p.20) However as Stevens explains “invoking the logical form to explain the structure of the judgment will serve little purpose if we are now left wanting an explanation of the structure of logical form.”(Stevens 2003, p.20) Stevens concludes that if the theory of judgment was to be successful “it required the imposition of the type part of the ramified hierarchy onto Russell’s ontology in exactly the same way that the multiple relation theory had been intended to help him avoid.”(Stevens 2003, p.26) Thus the “very situation he strove so hard to avoid in his solution to the paradoxes now becomes unavoidable if the multiple relation theory is to be maintained.”(Stevens 2003, p.26) While I agree with Stevens analysis, I contend that the integration of Wittgenstein’s rejection of the theory of types and the theory of judgment is rooted in the respective implicit theories of visual perception which the Tractatus and Russell’s theory of judgment advance. The theory advanced by Wittgenstein and essential to his construction of a closed and therefore consistent logical model is that there is only one way of seeing the world. This position is adopted to ensure the independence of atomic facts from one another thereby signalling the limits of the closed logical model of showing. At the core of Wittgenstein’s difficulty with Russell’s account of logical form is Wittgenstein’s belief that the non referential use of language in the traditional process of logical analysis cannot avoid the problem of self reference and therefore asserts nothing but invented nonsense. The significance of Wittgenstein’s concern with the problem of self reference is illustrated by the Necker cube ensuring by way of analogy with the ladder of non referential language use, that as we can’t have two contradictory states of affairs co existing in logic – as expressed in the problem of self reference in Russell’s paradox - a closed logical model is the only remedy.

A final interesting and significant approach to the problem of paradox features in the work of Saul Kripke. In 1979 Saul Kripke identifies the paradox of the Philosophical Investigations which is as follows:

This was our paradox; no course of action could be determined by a rule, because every course of action can be made out to accord with the rule. (Wittgenstein 2009, p.87)

While *a* problem of paradox has been identified in the Investigations by Saul Kripke, *the* problem of paradox and self reference has not. The fractal connection offers a new way of considering Kripke's paradox. Kripke's understanding of the paradox revolves around a simple mathematical problem of adding $68 + 57$. The individual in question has never performed this particular sum and upon following the rule for addition achieves the result 125. He next meets a bizarre sceptic "who questions my certainty in a metalinguistic sense." (Kripke 1982, p.8) He suggests that "as I used the term plus in the past the answer should have been 5!". Kripke goes on to suggest that his use was previously informed by what he calls the quus function which would determine the answer should be 5. The paradox is thus "who is to assert that this is not the function I previously meant by '+'". (Kripke 1982, p.9)

By 'plus' he says I always meant quus, now under the influence of some insane frenzy or a bout of LSD I have come to misinterpret my own previous usage. (Kripke 1982, p.9)

In essence the quus sign is an illustration of a 'private' rule. Ultimately Kripke concludes that he can appeal to no fact which can resolve the paradox. What is really presented here is what Wittgenstein would refer to in his writings on

mathematics – which deal directly with the problem of paradox - as “defective surroundings” to which aspect seeing is presented as a response. Critically Kripke only references the opening line of the section where Wittgenstein raises the problem of paradox in the Investigations. The section he omits is however far more significant and proceeds as follows:

The answer was: if every course of action can be brought into accord with the rule, then it can also be brought into conflict with it. And so there would be neither accord nor conflict here. (Wittgenstein 2009, p.87)

The paradox as presented in the above is concerned with rule following presenting two contradictory propositions similar to the original paradox of Russell. Immediately Wittgenstein points out that “there is a misunderstanding here” which is shown:

By the mere fact that in this chain of reasoning we place one interpretation behind another, as if each one contented us for a moment, until we thought of yet another lying behind it. (Wittgenstein 2009, p.87)

In the illustration of the problem of paradox Wittgenstein provides “we show that there is a way of grasping a rule which is not an interpretation, but which from case to case of application is exhibited in what we call “following the rule” and “going against it”. This type of rule following relates to the primary use of language within the domain of language acquisition where cases of following a rule and going against a rule are clear and transparent. They are not as he points out based on an interpretation of a rule, because the rule operates within the public language game where we are being taught the rule for the first time – the domain of consistent foundations. The crucial point is the second case he refers to:

One should speak of interpretation only when one expression of a rule is substituted for another.(Wittgenstein 2009, p.87)

In this second case we are dealing with a different language game than the first. We are dealing with the language game of aspect seeing or the secondary use and meaning of language, where one expression of a rule – or one way of seeing an object or language itself – can be substituted for another rule or another system which corresponds to a new way of seeing. Wittgenstein's paradox of rule following as presented here can be understood as being retrospectively directed at the Tractatus, which had attempted to finalise the rules of the language game of logical analysis itself. It is not a logical paradox which is intended to be solved or resolved – in the manner of the Tractatus - but rather is a paradox which is made understandable once we understand the language game of aspect seeing which the logician is playing. In substituting one rule for another the logician is interpreting a rule or set of rules of a given system in a new or secondary way by linguistic self reference to ones primary understanding of a rule, and thereby creating or inventing a new rule or system of rules. Here the new location of self reference and paradox operates between the primary and secondary employment of language – where both uses point at each other just as in Russell's paradox. What the logician is not doing is interpreting some hidden logical realm which he has privileged or private access to. The entire purpose of his later work is to address this apparent problem of paradox which originates in logicians becoming entangled in their own rules by a comparison of primary and secondary uses of language parallel to primary and secondary ways of seeing:

Our clear and simple language games are not preparatory studies for a future regimentation of language – as it were first approximations ignoring friction and air resistance. The language games are rather set up as objects of comparison which are meant to throw

light on the facts of our language by way not only of similarities but also of dissimilarities.(Wittgenstein 2009, p.130)

In this sense Kripke's illustration of the sceptical problem is no different than that of the logician or mathematician who created the symbol π . As Wittgenstein remarks:

The fact that we cannot write down all the digits of π is not a human shortcoming, as mathematicians sometimes think. Teaching which is not meant to apply to anything but the examples given is different from that which 'points beyond' them. (Wittgenstein 2009, p.89)

In the public language game of language acquisition we are implicitly taught to register and follow rules which are pattern based. Secondary language use within the language game of aspect seeing does not create private rules but rather extends the primary experience of rule following and creates new rules based on a mastery of the public language. It is in this way that every system of rules becomes understandable even the paradoxical rules emanating from the problem of self reference in Russell's paradox.

Aspect seeing and perception

In turning to the final cohort of academics who address the popular topic of aspect seeing both Michel Ter Hark and Michael O'Sullivan stand apart from the others in their respective efforts to unite the Tractatus and Wittgenstein's later work on aspect seeing via an implicit theory of perception. In the most recent publication on the links between the logic of the Tractatus and Wittgenstein's later work on aspect seeing Michel Ter Hark explores the relevance of the Necker cube in the Tractatus to aspect seeing where it also plays a key role. Ter Hark identifies the lack of research to date regarding precisely these links. In the opening of his chapter in the publication *Wittgenstein and Perception* Ter hark makes the following observation:

The aim of this chapter is to investigate in some detail how the later writings on aspect perception relate to Wittgenstein's early solution of the 'Necker cube' in the Tractatus. Detailed study of the passage on the Necker cube is relatively rare. If and how the later concerns with aspect seeing has to be seen as a response to the Tractatus equally still awaits further research. (Campbell and O'Sullivan, 2015, p.165)

Ter Hark highlights the difficulties in providing such a link noting that with the exception of the Necker cube "in the Tractatus itself perception is hardly a topic of discussion." (Campbell and O'Sullivan 2015, p.165) Similar to my presentation Ter Hark identifies the location of the link between perception in the Tractatus and perception as explored through the concept of aspect seeing, as being rooted in Wittgenstein's response to Russell's work. On this point Ter Hark argues that other readings "have missed Wittgenstein's background in Russell's treatment of perception, notably in his Theory of Knowledge" (Campbell and O'Sullivan 2015, p.165). While Ter hark does identify that *a* problem of paradox is in fact inherent

in the later Wittgenstein's work on aspect seeing, he does not link this to the influence which Russell had on both the early and later Wittgenstein. He thus comments on the visual paradox of aspect seeing wherein something can be seen in two opposing ways:

But this is a paradox only if one fails to recognise the intersection of word uses. Now a paradox arises because the new report seems unsupported by a change in the perceptual situation. (Campbell and O'Sullivan 2015, p.179)

Ter Hark argues that resolving the paradox of aspect seeing operates as follows:

The solution seems to be to say that something must have changed after all, be it that we see two facts, as TLP 5.5423 claims, or that there is a change of organisation as Kholer claims. But when the linguistic practice is such that we rely on the form of an account of perceptual objects while at the same time using this form in order to express an experience no paradox is forthcoming. (Campbell and O'Sullivan 2015, p.180)

The above conclusion ultimately fails to provide a credible and coherent account of how the Necker cube relates his later work on aspect seeing to the Tractatus. The last sentence in the above in particular does not seem in any meaningful way to indicate that "no paradox is forthcoming". The fact is that aspect seeing necessarily involves the problem of paradox, self reference and contradiction – wherein a given aspect both is and is not a member of a given content considered as a set. In contrast to Ter Hark I suggest that the problem of paradox is in fact essential to the language game of aspect seeing. As I will show Russell's paradox can be seen

retrospectively as an instance of aspect seeing, an observation which in tandem with his treatment of the infinite would lead Wittgenstein to the open logical model of aspect seeing. However Ter Hark does not identify Russell's problem of paradox as a significant influence, which I contend in opposition is central to the evolution of aspect seeing from Wittgenstein's early to later work. Other areas of critical influence such as set theory, and Wittgenstein's distinction between a primary and secondary language are also untouched. On this point of criticism it must be noted that Ter Hark qualifies his analysis by noting that "the material is controversial and requires a more sustained treatment than can be given here" (Campbell and O'Sullivan 2015, p.166)

In *Wittgenstein and Perception*, Michael O'Sullivan poses the following crucial question "does the Tractatus contain a theory of perception?". If the problems in Ter Hark's account are to be overcome this question must be addressed. O'Sullivan observes that the idea of a theory of perception in the Tractatus is not commonly accepted view in academia noting the work of William Child as a case in point. However O'Sullivan contends – and correctly in my opinion – that such a view is implicit in the Tractatus albeit it is not presented as a theory per se:

As I see it the Tractatus does contain a substantive philosophical view on the nature of perception, though not one that deserves to be called a 'theory'.
(Campbell and O'Sullivan 2015, p.162)

Of all the positions thus examined O'Sullivan comes closest to the root of an implicit theory of perception in the Tractatus observing the following in relation to the Necker cube:

Part of the Tractatus view is that there is no room for a distinct philosophical theory of perception: an adequate philosophy of perception is in a sense already present in a theory of

judgment. (Campbell and O'Sullivan
2015, p.163)

This is of course precisely the point. The rejection of Russell's theory of judgment via the Necker cube illustration which contends that it must be impossible to judge a nonsense - the crucial point on which Wittgenstein claims Russell's theory falls apart – is I suggest in essence a rejection of the problem of paradox and self reference in the visual field. For Wittgenstein as logic must be mirrored in the world and as logic must of necessity exclude these problems, then it was essential that this problem of visual paradox was also addressed. In a more recent paper *The Visual Field in Russell and Wittgenstein* (O'Sullivan 2015) he alludes to this implicit theory of perception and its relation to the theory of judgment but stops short of accounting for its significance. He remarks that Wittgenstein's criticism of Russell's theory of judgment "entails that even at the level of judgment we cannot speak of world views or ways subjects take things to be" however leaves the issue untouched noting that "an examination of this would take us beyond the confines of this paper. (Campbell, O'Sullivan 2015, p.331) O'Sullivan's instinct that uniformity takes precedence to different ways of taking things is precisely on point but remains just a point. While O'Sullivan's paper is very significant, I contend that the role of both the infinite and the problem of paradox and are essential in completing the picture and in establishing both the link and the cause of the change, between Wittgenstein's early and later closed and open logical models respectively.

Secondary literature dealing specifically with the topic of aspect seeing comprises a vast body of work which is rapidly increasing. In *Seeing Wittgenstein Anew* three relevant perspectives on aspect seeing are presented by Avner Baz, Stephen Mulhall and Juliet Floyd.

Avner Baz's contribution *On Learning from Wittgenstein* critiques the work of Stephen Mulhall who he claims misrepresents Wittgenstein's work on aspect

seeing in so far as he argues that “it is only against the background of “continuous aspect perception” that what he calls “the inherent paradoxicality” of the experience of the dawning of an aspect can be “accounted for”.(Day and Krebs 2010, p.229) While Mulhall’s aim is to remove the paradoxical nature of aspect seeing as presented in Wittgenstein’s later work, Baz counters the very sense of this ambition observing that if it was such as simple a matter as Mulhall contends why did Wittgenstein “feel the need to write hundreds of remarks on the seeing of aspects, if what he was really trying to say can be put in twenty pages or so?”(Day and Krebs 2010, p.230) Baz’s central argument centres on Mulhall’s view that “aspect dawning acquires its significance by revealing our basic relation to the world as unproblematic – something we all have already earned for ourselves as a matter of course.(Day and Krebs 2010, p.248) In contrast Baz’s view is that:

I find that aspect dawning reveals our basic relation to the world to be one in which we are continually in danger of losing our world, by, as it were taking it as a matter of course. The continual danger in other words is that in succumbing to habitual and convenient ways of treating or regarding things, we will lose our ability to *see* them. (Day and Krebs 2010, p.248)

Both points articulated by Baz in terms of his criticism of Mulhall and his alternative view have merit and are not as he suggests mutually exclusive. The danger of losing our ability to see as Baz points out in the above is I agree a fundamental dimension of the concept of aspect seeing – specifically as I contend that it is retrospectively applied by Wittgenstein to the closed logical model of the Tractatus. What is entirely absent in all of these considerations is that Wittgenstein’s ideas on aspect seeing - specifically the danger of becoming aspect blind and failing to see in a conceptual and linguistic sense- developed originally as a response to both the problem of the infinite and the problem of paradox as

expressed in the crisis of mathematics, and consequently as a retrospective response to the logical shortcomings of the Tractatus. Wittgenstein's entire focus in his development of the concept of aspect seeing is I suggest to present it as a language game as operative in the language game of logical analysis, wherein the secondary use and meaning of non referential language becomes the most critical component. In this sense Wittgenstein utilises the concept of aspect seeing to offer a meta-analysis on the very process of logical analysis itself. In his efforts to dispel the idea of a hidden idea logical language – which he adhered to in the Tractatus - which once found would provide consistent foundations and undo the damage generated by paradox and self reference, it was essential that an alternative consistent foundation was offered. On this point Mulhall's position that "our basic relation to the world is unproblematic" is a critical point. Mulhall's understanding of the continuous aspect dawning is he points out "made manifest in ones tendency to sort an object with others as one of their kind – picture rabbits, Louis XV chairs with Louis XV paintings rather than Bauhaus chairs and so on"(Day and Krebs 2010, p.255) While the problem of consistency in the crisis of mathematics is not addressed by Mulhall his understanding of continuous aspect dawning is essentially what emerges once one has acquired mastery of a primary language and it is this public language nexus which Wittgenstein uses as the desired consistency foundation. While the term continuous aspect dawning is somewhat problematic, Mulhall's use of the term is correct, to the extent that it is aligned with Wittgenstein's aim is to demystify the concept. In this sense its occurrence in everyday contexts and in the world of children's language games becomes central in Wittgenstein's efforts to show that the language game of logical analysis is no more mysterious than a child seeing a chest as a house. Similar to the presentation here Mulhall presents Wittgenstein's distinction between primary and secondary senses of language as central in understanding the concept of aspect seeing noting that:

He declares that secondary uses of a word do not illustrate but rather presuppose (even while transforming)

its primary use...any experience of aspect dawning –whether linguistic or pictorial – is doubly dependent on the inclination to take over an expression from its standard technique of use and employ it as the immediate expression of an experience. (Day and Krebs 2010, p.258)

As such Mulhall contends “the language of aspect dawning experiences in general is an instance of what Wittgenstein means by primary and secondary senses of words.”(Day and Krebs 2010, p.259) Mulhall’s dissolution of the paradox of aspect seeing consists in dissolving “paradoxical turns of phrase” such as seeing something both as a duck and rabbit, “by relocating that specific kind of experience against the broader background that is constituted by the general role of pictures in our lives”(Day and Krebs 2010, p.264) While Mulhall identifies the issue of paradox within the general concept of aspect seeing, to truly understanding the complexity of this paradox one has to trace its roots back to the original mathematical problem of the infinite and the generated problem of paradox understanding how they influence Wittgenstein’s understanding of both the visual field and language in both his early and later work. Mulhall also highlights some crucial exegetical points which are central, noting that Wittgenstein’s “treatment of what is now Part I suggests that the text of Part II, as we have it, would have been subject to further revision if its author had lived long enough to do so.”(Day and Krebs 2010, p.252) Mulhall also points out that the phrase aspect seeing “is rather Wittgenstein’s – a coinage of his own for experiences of a kind that do not seem to have their own handy, generic labels of which we might be reminded.”(Day and Krebs 2010, p.256)

While Mulhall does not touch on the mathematical domain of Wittgenstein’s work such an offering is provided by Juliet Floyd. Floyd presents a reading of aspect seeing which similar to mine considers how Wittgenstein’s “uses of aspect perception bridges the evolution in his thought from earlier to later.”(Day and

Krebs 2010, p.314) One central problem with Floyds work, is the view that Wittgenstein uses the concept of aspect seeing in an explicit and intentional manner in his early work. On this point Floyd uses the illustration from the Notebooks where the concept of probability is considered by Wittgenstein. In considering probability by analogy of a black urn containing “equally many black and white balls the number of black balls that are drawn will approach the number of white ones if the drawing is continued.” (Wittgenstein 1998, p.28) Contrary to Floyd I suggest Wittgenstein in this instance is using the analogy to support his view that in order to be meaningful a proposition must reveal an isomorphic relation to the world. As such considering different ways of how the pattern formation of the coloured balls will be drawn is insignificant to Wittgenstein in the Tractatus, rather his point is that we can only represent what we can actually see and what we can actually see is limited and fixed to the actual draw that takes place – to one way of seeing. That the arrangement could have been different is insignificant. While retrospectively we can highlight that aspect seeing has an implicit role in the Tractatus, the evidence that Wittgenstein is addressing the concept of aspect seeing in an explicit manner is untenable. Rather what he is doing is rejecting the non referential use of language operative in the process of logical analysis which contends that we can go beyond the limits of the world we can see and represent, and articulate a logic by using language to refer to its own logical foundations. This was the problem the Tractatus tried to overcome through the closed logical model of showing.

Many of Floyd’s observations on the seeing of aspects in mathematical problems are correct and insightful. One such observation parallels my consideration of how set theory proved fundamental to both his early and later work and inadvertently highlights the point that aspect seeing was not explicit in his early work – only in a retrospective sense:

The general form of proposition is a
scheme whose physiognomy is fixed,
not open ended, not subject to

elaboration of new aspects. (Crary and Read 2000, p.333)

While Floyd's exegetical approach is different than my presentation the conclusion reached that the "grammars of different systems.....the evolution of language and of mathematics and logic in particular is both open-ended and unforeseeable in general" (Crary and Read 2000, p.333) is a conclusion with which I agree. Despite agreement here the most important mathematical problem – the problem of the infinite and the problem of paradox remain untouched by Floyd. While Floyd contends that we are dealing with two different approaches to the concept of aspect seeing in Wittgenstein's early and later work, we are I suggest rather dealing with two different approaches to the language game of logical analysis, which is duly reflected in the two opposing closed and open logical models which represent his early and later work respectively. I suggest rather that just as the infinite and the problem of paradox were central in the formation of the closed logic of the *Tractatus*, they also serve as the catalyst which led to the formation of the concept of aspect seeing as both a visual experience and a language game in its own right, therefore indicating that Wittgenstein did adhere to two very different logical positions in his early and later work.

A final unique approach to the problem of paradox and Wittgenstein's involvement in the crisis of mathematics is presented by Francesco Berto in *There's Something About Gödel*. While Berto is primarily concerned with Gödel's work and does not consider Wittgenstein's work on aspect seeing, or his logical efforts to resolve paradox and self reference in his early and later work, Berto does offer a new way of considering Wittgenstein's criticism of Gödel, which is very significant in terms of considering the applicative potential of Wittgenstein's work. Berto's suggestion is that Wittgenstein's rejection of Gödel's incompleteness theorem can be considered as an anticipation of paraconsistent logic. I suggest that Berto's view is supported by and reflected in the development of the open logical model of aspect seeing considered as a language game. In this sense the language game of aspect

seeing can be considered in light of recent advances in logic as an inconsistency tolerant logical model which is diametrically opposed to the closed logical model of the Tractatus.

Chapter One

Referential and Non Referential uses of language - The Evolution of Primary and Secondary Uses of Language in the Work of Wittgenstein and Russell

1.1 The early Wittgenstein's conception of Primary and Secondary Language

From a methodological perspective understanding the links between Wittgenstein's early and later work demands establishing a solid link between these two very different phases in the logic Wittgenstein adhered to. The assumption of the resolute tradition that Wittgenstein's logic is entirely consistent between both phases, parallel to their argument that Wittgenstein does not adhere to any logical position in the Tractatus, is I argue entirely untenable. In order to fully appreciate the nonsense of the Tractatus it is essential to appreciate the complex role which the infinite, Russell's paradox and the subsequently generated issues of self reference and contradiction, play in the construction of his early closed model of logic. That Wittgenstein himself retrospectively identifies inconsistencies in the Tractatus serves as a decisive indicator that the Tractatus did adhere to a very specific view, which was later rejected and indeed criticised by Wittgenstein. The evidence for taking this position is critical and emanates from Wittgenstein's own remarks in the introduction to the Philosophical Investigations where he states:

Four years ago I had occasion to reread my first book (TLP) and to explain its ideas. Then it suddenly seemed to me that I should publish those old ideas and the new ones together: that the latter could be seen in the right light only by contrast with and against the background of my older way of thinking. (Wittgenstein 2009, p.4)

Given Wittgenstein's retrospective position on the relation between his early and later work, it is clear that both logical enterprises, while inextricably linked are fundamentally different. As such any link which is proposed must satisfy this criterion as set out by Wittgenstein. Wittgenstein's remarks on this issue are indisputable and serve as the gold standard by means of which any analysis must be measured. Most significantly Wittgenstein's own remarks provide the only neutral axis of reference and will be used as such in the work which follows. Wittgenstein's retrospective comments thus serve as an indispensable methodological springboard in exploring the complex relation between his early and later models of logic.

The following remark from *Philosophical Remarks* decisively aligns Wittgenstein's Tractarian logic to a logical model which was based on a distinction between a primary and secondary language - a distinction which would later become foundational to the development of the concept aspect seeing. While this distinction features in a negative manner in the following remark - clearly in response to Wittgenstein's rejection of the closed logical model he adhered to in the *Tractatus* and his belief that a logical language lay hidden within ordinary language, which he refers to in the following as a "primary language"- it reappears in a diametrically opposed context in the *Philosophical Investigations* featuring as what I argue is one of the most important and revealing remarks concerning aspect seeing:

We talk for instance of an *optical illusion* and associate this expression with the idea of a mistake, although of course it isn't essential that there should be any mistake; and if appearance were normally more important in our lives than the results of measurement, then language would also show a different attitude to this phenomenon.

There is not - as I used to believe - a *primary language as opposed to*

our ordinary language, the 'secondary' one. But one could speak of a primary language as opposed to ours in so far as the former would not permit any way of expressing a preference for certain phenomena over others; it would have to be, so to speak, absolutely *impartial*. (Wittgenstein 1998, p. 84)

Wittgenstein's early conception of a primary and secondary language expressed above, underpins my consideration of the Tractatus as a closed logical model. In the above remark, primary language is conceptually located as lying behind secondary or ordinary language, indicating that ordinary or secondary language evolves or emerges from a logical or primary language. This assumed primary language in effect mirrors its logical form through to ordinary or secondary language wherein it is presented in a disguised or ambiguous manner. This early conception of primary language thus signals an internal logical limit of the closed logical model as presented in the Tractatus, which Wittgenstein maintained was accessible through his variant of logical atomism. As he remarks in the Tractatus:

4.002 Language disguises the thought; so that from the external form of the clothes one cannot infer the form of the thought they clothe, because the external form of the clothes is constructed with quite another object than to let the form of the body be recognised. The silent adjustments to understand colloquial language are enormously complicated. (Wittgenstein and Ogden 1990, p.44)

Here Wittgenstein's idea is that primary or logical language, which is disguised by secondary or ordinary language, is united in a consistent manner by logical form,

as indeed is thought and visual experience. In this sense finding the correct logical notation or pattern within secondary or ordinary language which would reveal this universal logic of showing, which he contends is hidden within ordinary language, would not breach the isomorphic limits of the Tractatus. Most critically for Wittgenstein the methodology would avoid self reference, as by his rationale the logic of showing was not using language to refer to its own foundations. What is of course problematic is that the proposed methodology is absent as are any examples of atomic or elementary propositions. It seems Wittgenstein believed that in *somehow* revealing the structure of language and logic *within* the closed logical model of the Tractatus, he would avoid the problem of linguistic self reference generated by using language in a non referential manner, from an assumed position *outside* of language. While Wittgenstein's model concluded on a note of absolute silence, this was regarded as logically superior to what he regarded as the nonsense of Russell's open logical model of type theory. Wittgenstein maintained that by using his new logical method of showing, language was not being used to refer to itself - as he maintained it was in the Russell's open logical model of type theory - but rather was revealing its own inherent logical structure, in a non referential manner.

Retrospectively this position would prove deeply flawed and the atomic propositions he believed would be uncovered via the logic of showing at some later point in the future would prove illusory. In effect the essential difference which he maintained separated the closed logical model of the Tractatus from the open logical model of type theory would prove to be entirely misleading. While claiming logical superiority for the logic of showing, Wittgenstein is ultimately speaking from within the same system as Russell – i.e. language - and like Russell is also trying to set in place sound rules which will serve as the basis for a logical language. Ultimately Wittgenstein would come to recognise by rejecting the Augustinian picture of language which informs the Tractatus that the closed logical model of showing is entirely dependent on language, as without linguistic articulation it cannot be expressed as a competing logical model in the first

instance. In this sense the checks and balances of the closing statement of the Tractatus which commits logical analysis to eternal silence can retrospectively be seen as mere linguistic acrobatics rather than sound logical conclusions.

1.2 Primary and Secondary Language in the work of Frege and Russell

We also encounter the logical distinction between a primary and secondary language in the logicist programme of Frege and Russell, which has its historical roots in set theory:

Between the end of the nineteenth century and the beginning of the twentieth, the great philosophers and logicians Gottlob Frege and Bertrand Russell, attempted to provide a definitive, unassailable logical and philosophical foundation for mathematical knowledge precisely by means of set theory. (Berto 2009, p.5)

For Russell language is referential in two distinct ways. Firstly it is referential in an objective manner – e.g. to a physical table or chair within the objective world – and secondly it is referential in a logical manner. Russell's conception of language being referential in a logical manner is essentially based on the view that language is capable of referring to its own foundations to reveal a perspicuous view of the logical form or structure of logic:

For my part I believe that partly by means of the study of syntax we can arrive at considerable knowledge concerning the structure of the world. (Russell 2007, p.129)

Wittgenstein would argue in the *Tractatus*, that Russell's conception of language being referential in a logical manner proves intractably problematic, not because it is based on the assumption that ordinary language hides or disguises a second logical language which remains to be discovered - as Wittgenstein also held this view – but because Russell believed language could be used to talk about or refer

to itself. Both Frege and Russell held the view that the realm of logic and mathematics shared a reality on a par with the physical world of objects, which however could only be accessed via logical analysis:

Frege's own theory of mathematics, so called logicism, involved the idea that all truths of arithmetic are deducible from a few purely logical truths, which themselves are self evident, indubitable and independent of the human mind. Logicism also involved the claim that numbers are genuine objects, although not physical or mental, but abstract ones situated in a "*Third Realm*". (Stroll 2000, p. 32)

This idea of an abstract realm of logical objects was a central pillar of logicism and proved crucial in both Wittgenstein's analysis and rejection of Russell's theory of types and in the creation of his alternative logic of the *Tractatus*.

Both Russell and Frege "thought that natural languages were logically defective" and most significantly maintained that "ordinary grammar was a fallible guide to the real structures which in their view, logic and philosophy must investigate." (Hacker 1996, p.20). The logical concept of a secondary language first appears in the context of Frege's ambition to establish a concept script, which would function as a perfectly logical language devoid of all the ambiguities of ordinary language. For Russell and Frege their secondary language or "concept script" stood "to ordinary language as the microscope stands to the eye." (Hacker 1996, p.20) Quoting from Frege, Hacker recounts the following, which sets out clearly the ambitions of the logicist programme and their particular understanding of ordinary language use:

It cannot be the task of logic to investigate language and determine what is contained in a linguistic expression. Someone who wants to learn logic from language is like an adult who wants to learn how to think from a child. When men created language, they were at a stage of childish pictorial thinking. Languages are not made to match logics ruler. (Hacker 1996, p. 19)

Implicit in the position set out above is the view that the non referential use of language can be successfully used to discover a hidden logical language and in turn to set forth a consistent logical model which would definitively establish that all mathematics are derived from logical foundations. The ultimate criteria by which the resulting concept script or secondary logical language would be measured and assessed centred on whether or not the logical model it produced proved consistent. In opposition to this Wittgenstein opted to use not only logical consistency as a criteria but also and crucially to stipulate that the logical consistency must be rooted in the publicly verifiable set of criteria inherent in the visual field. Ultimately this criterion would become the show say distinction which was itself subject to the public isomorphic check for veracity or falsity. Rather than establishing a decisive public criteria which would clearly demonstrate how language operates in the public world of visual perception as Wittgenstein would do via the isomorphic principle, Frege and Russell considered language to be entirely at their disposal:

In logical investigation 'we need not be concerned with what linguistic usage is. Instead we can lay down our linguistic usage in logic according to our logical needs'. (Hacker 1996, p.19)

The “logical needs” in question were those concerning the discovery of a perfect logical language which was internally consistent and which definitively proved that mathematics is itself derived from logical principles. This would necessitate that all of mathematics could be accounted for within the logical model of logicism. The idea that language could be used to refer to the foundations of logic demanded that language was capable of referentially referring its own foundations, and could thereby talk about itself in an unproblematic manner. Once the correct logical language was developed, it would succeed in expressing the foundations of logic as a definitive logical content, which the logical language could refer to unambiguously and over which the logician would have a perspicuous view.

This assumption rested for Wittgenstein on an entirely incorrect view of how language functions. For Wittgenstein the situation was precisely what Frege and Russell claimed it was not. On the contrary, Wittgenstein would contend that pictorial thinking and pictorial representation were logically significant, and would prove central in his picture theory of meaning, his implicit primary secondary language distinction, and his resulting closed model of logic. Most crucially it was the logical combination of all of these which would prove the means by which Wittgenstein believed the problem of the infinite and self reference could be overcome. The idea that the logician could lay down his “linguistic usage in logic according” to his “logical needs” was to be rejected by Wittgenstein and moreover was to be identified as the source of philosophical nonsense. While there were stark and unbridgeable differences between the logicist project and Wittgenstein, the difference rested more on methodology rather than aim, as Wittgenstein also maintained via his own unique variant of logical atomism that a primary language underpinned our ordinary ambiguous secondary one.

While for Frege and Russell, ordinary or primary language was considered entirely flawed, they recognised that it “may nevertheless provide invaluable clues for the logician”, albeit they would “not overlook the deep gulf that separates the level of language from that of thought, and which imposes certain limits on the mutual

correspondence of the two levels.”(Hacker 1996, p.19) While Wittgenstein rejected the logicist project of Russell and Frege he did adhere to their view that ordinary language conceals a hidden logical matrix and duly acknowledges this in the Tractatus:

4.0031 All philosophy is “Critique of language” Russell’s merit is to have shown that the apparent logical form of the proposition need not be its real form. (Wittgenstein and Ogden 2009, p.45)

Unlike Russell and Frege, Wittgenstein’s methodology positions the logic of showing as a superior logical model, which he claims can comprehensively deal with the concept of the infinite, and the associated issues of paradox, self reference, and contradiction, by denying all of the aforementioned the status of a genuine logical problem. If a problem arose – as indeed it did in the form of Russell’s paradox – the logicist conception of language meant that language itself could be reinvented and remodelled to deal with the issue – an option denied by the early Wittgenstein’s closed logical model.

Unlike Wittgenstein Russell and Frege utilised non referential language use as an essential tool in the process of logical analysis. For Russell specifically the non referential use of language is prioritised as the very means by which the problem of the infinite, and the generated problem of paradox self reference can be overcome. However for Wittgenstein it is precisely the non referential use of language which shows us the limits of language through its inability to move beyond the constraints of self reference.

While Russell like the early Wittgenstein attempted to bar all paradoxes and potentials for self reference from Principia Mathematica, his open logical model did manage to accommodate the infinite. Unlike Russell Wittgenstein deemed the concept of the infinite as logically untenable. The reason why is rooted in the early

Wittgenstein's rejection of the use of non referential language in the process of logical analysis as traditionally employed, and in his adherence to the view that a primary form of logical language - atomic or elementary propositions – lay hidden beneath our ordinary or secondary language. Wittgenstein's genius lay in the claim that such atomic propositions could be shown without having to be spoken of, and would ultimately prove untenable. Central to the efforts of both Russell and Wittgenstein is their respective positions on a primary and secondary language

Chapter Two

Russell's logical Accommodation of the Infinite

Non Referential language use in Russell's Open Logical Model of Type Theory

2.1 Russell's Paradox and the Influence of Set theory

The logical roots of the problem of paradox originate in the work of Georg Cantor and the mathematical field of set theory which he discovered. Within set theory the complex logical issues of infinity, paradox, self reference and the continuum hypothesis emerge, all of which would prove formative in both Wittgenstein's early and later models of logic. The logical roots of the Tractatus which has both bewitched and fascinated readers can be traced to the mathematical field of set theory which Wittgenstein encountered via the work of Russell. My contention is that the very basic conceptual structure of set theory concerning finite and infinite sets informs the logical backdrop to a consideration of the logical models we encounter in the respective works of Russell and the early Wittgenstein as open and closed logical models respectively. Using language to refer to finite sets is unproblematic, however when the logician attempts to use language to refer to and quantify over an infinite set, intractable problems of paradox, contradiction and self reference emerge. This crucial linguistic problem of expression is not only confined to the theoretical dimension of finite and infinite sets, but also to their pictorial geometric counterparts - fractals. Visual illustrations of sets and geometric curves such as the Cantor set, the Hilbert and Peano curve display – in an even more accessible manner than the theoretical dimension - the fascinating and perplexing characteristics of the tension between finite and infinite, open and closed sets and most importantly the centrality of self reference. Most critically for Russell and Wittgenstein, resolving this conceptual tension would demand resolving the grammatical and syntactical problem of self reference which emerges within the seemingly irreconcilable chasm between the grammar of the finite and

the grammar of the infinite as presented in set theory. What both Russell and Wittgenstein required to resolve the problem of self reference was a secondary or logical language which was devoid of the ambiguity inherent in ordinary or primary language. This would ultimately serve as the foundation upon which their respective open and closed models of logic were built. The logical conception of a primary and secondary language is a view deeply held by both Russell and Wittgenstein. There is the language of the common man and there is the language of logic. However while Russell believes language can be used to reference its own foundations once the correct model of saying has been deciphered, Wittgenstein recognises this problem of self reference as a logical limit which cannot be breached without generating nonsense. The central problem for Wittgenstein is that a theory such as the theory of types has no apparent relation to the objective world and is as such a metaphysical fiction generated by the logician's inability to recognise the limits of language. In short it is for Wittgenstein a misapplication or a misuse of non referential language which crucially has no public criteria to either verify or falsify it. For Wittgenstein when it comes to the foundations of logic and language, there is no referential application, only a self referential one signalling the limits of its capabilities.

The premise of set theory is the remarkably simple one of categorising objects into different sets by means of logical whole part relations. Anything can be categorised by means of set theory - chairs, teapots apples etc. However within set theory sets also count as the objects which can be categorised and as such the relationship between different sets can be studied. While the premise is simple the application of set theory to mathematical objects and to sets themselves would prove more complex ultimately giving rise to the concept of the infinite and the infamous paradoxes which precipitated what is known as the 'crisis in mathematics'. The problem of paradox enters the academic world via the work of Georg Cantor's early work on set theory and the concept of the infinite, which when analytically considered by Russell leads to his infamous paradox. Why the problem of paradox is of such significance rests on the fact that it created what was considered an

intractable problem of inconsistency within mathematics in addition to introducing the problem of infinity and infinite self reference within logic itself. The threat to the foundations of mathematics which this discovery introduced ensured that efforts to restore consistency were embarked upon with urgency. The culmination of this complex series of events surrounding the problem of paradox precipitated what is referred to as the crisis of mathematics wherein adherents to logicism (Russell and Frege), Intuitionism (Brouwer), and Formalism (Hilbert) sought in different ways to solve this problem by providing a consistent logical model which was strong enough to avoid the problem of self reference while at the same time accommodating the infinite as presented in the work of Cantor. All of these efforts ultimately failed as Kurt Gödel later demonstrated that proving a systems consistency from within that same system was logically impossible - a view which I suggest can be considered as implicit in the work of the later Wittgenstein.

The core of the problem - first identified by Cantor - centres on establishing consistent relations between sets. The aim is to achieve a bijective or one to one mapping between sets which treat of mathematical objects such as numbers. A set is denoted by the brackets, { }. For example, if we want to talk about the set containing the numbers 4 and 5, we would write {4,5}. If the set contains other sets, another set of brackets is introduced within the set. For example, {4,{5}} is the set which contains both the number 4 and the set containing 5. Cantors difficulty began when he tried to map the set of the natural numbers to the set of the real numbers. The natural numbers consist of ordinary counting numbers – 0,1,2,3,4, etc. As mathematician Richard Elwes notes “It might seem obvious that the set of even numbers {0,2,4,6,8,..} should be smaller than the set of natural numbers, aren’t there only half as many of them?”(Elwes 2008, p.2) What is surprising and counter intuitive is that both sets can easily be matched up “0 to 0, 1 to 2, 2 to 4, 3 to 6, and so on, just multiplying by 2 every time.”(Elwes 2008, p.2) In Cantors sense both sets are “actually the same size.” When it comes to the set of real numbers the matter becomes much more complex. The set of real numbers includes whole numbers – 0,1,2,3, - rational numbers such as 0.125, 0.344, and

irrational numbers such as π , $\sqrt{3}$, etc. As Elwes points out the set of real numbers “can be thought of as the collection of all infinite decimal expansions (such as 19.0000000... or 1.23456789101112...) You can also think of the real numbers as being the points on an infinitely long line with no holes in it. For this reason the set of real numbers is sometimes called the continuum.” (Elwes 2008, p.2) Cantor proved that any effort to match the set of the real numbers with the set of the natural numbers will fail – “there will always be some real numbers which get missed out. So we are forced to conclude that the infinity of real numbers is bigger than that of the natural numbers.” (Elwes 2008, p.2) In terms of the Cantor set the problem of paradox emerges as follows:

In his revolutionary works, Cantor, building on tentative beginnings by Bolzano, had begun to work with the general notion of a class or set and had established that sets with infinitely many members need not all have the same size (cardinality) or number of elements. In particular the size of the continuum, that of all numbers on a continuous line, is greater than the size of the set of all finite natural numbers. (Irvine 2009, pp.291-310)

Cantor introduced the logical concept of cardinality to discriminate between sets which were and were not isomorphic or bijective. For example while the sets {4, 6, 8, 10} and {3, 4, 5, 6} are not equal they have the same cardinality - four. As such two sets have the same cardinality only if there is a clear isomorphism between the elements of the two sets. While this is unproblematic in the case of finite sets, in the case of infinite sets, the isomorphic principle of mapping breaks down. In the case of the set of real numbers we are dealing with an infinite set and as such an infinite notion of cardinality. Cantor’s discovery was the spark which led to the crisis within mathematics, and to this day remains a controversial subject in mathematics.

Prior to Cantor it was assumed that all of mathematics was consistent and crucially based on consistent axioms. What Cantor had discovered shattered this assumption and called into question the very basis of mathematics itself. The culmination of the problem of the infinite, the problem of paradox, self reference and their visual counterparts – fractals – created the perfect logical storm precipitating the crisis in mathematics. The following reveals Cantor’s shock on discovery of the Cantor set – the first fractal ‘monster’ in the crisis of mathematics:

In a letter to Dedekind at the very beginning of the 1875-1925 crisis in mathematics, Cantor is overwhelmed by amazement at his own findings and slips from German to French to exclaim that “to see is not to believe”. And as if on cue mathematics seeks to avoid being misled by the graven images of monsters. (Mandelbrot 1973, p. 21)

Simultaneously the surety and consistency of both Euclidian geometry and the logical foundations of mathematics, no longer seemed infallible logical models, but rather seemed to possess some inherent logical contradiction. It seemed there was not one geometry but “different and equally valid geometries – where by a geometry is meant a theory of properties of abstract points and lines.” (Hofstadter 1979, p.19) While it was assumed that progress in geometry would be achieved through an extension of the work of Euclid this would prove impossible in just as decisive a manner as the logicist ambitions of Frege and Russell:

This idea was shattered by the roughly simultaneous discovery of non Euclidian geometry by several people – a discovery that shocked the mathematics community, because it deeply challenged the idea that mathematics studies the real world. How could there be many different kinds of “points” and “lines” in one single reality? (Hofstadter 1979, p.20)

Cantors work – and the work of other mathematicians – revealed that logical inconsistencies were thus present in both strictly logical models in addition to geometric or visual mathematical models.

As mentioned the problems of the infinite, paradox, contradiction and self reference are revealed theoretically in Cantors work concerning the greatest cardinal number. The concept of the infinite is further expressed in Russell's paradox which arises through a consideration of Cantors logical concept of the infinite in the context of a consideration of the greatest cardinal number. Visually the concept of the infinite is revealed in the Cantor set. The problem of the infinite is expressed in Cantors set theoretic paradox by a consideration of the "universal set V , i.e. the set of all (pure) sets" and its "power set $P(V)$." (Berto 2009, p.35). In effect, given Cantors theorem " $P(V)$ is bigger than V . This is inconsistent with the fact that V , by definition is inclusive of all sets: and would mean logically that V would have to be bigger than itself!"(Berto 2009, p.35)

The very generality of the notion of size or cardinality of a set led to that curious result: there could not be a largest set, because if there were, by the diagonalization argument, there would have to be one larger still, contradicting the original assumption that there was a largest. Hence there could be no such set as the set of all things, for it would by definition have the largest cardinality. (Irvine 2009, pp.291-310)

Unlike Wittgenstein who bans the concept of the infinite in its entirety from his closed model of logic in the Tractatus, Russell would try to accommodate it within the open logical model of type theory.

Prior to the publication of what was to be Frege's magnum opus Russell discovered his infamous paradox:

Russell's paradox though the clearest and the most damaging, was but one of a cluster of paradoxes which had begun to infect post Cantorian mathematics. The general atmosphere conveyed by the rash of paradoxes coming to light was that modern mathematics was in a crisis. The paradoxes dramatically highlighted the importance of ensuring that mathematical theories are consistent. (Irvine 2009, pp.291-310)

The essential aim of logicism had been to demonstrate that logic could provide consistent foundations which proved that all of mathematics is ultimately rooted in and derived from logic. The problem which Cantor introduced into mathematics seemed to make this ambition impossible, as the infinite is by definition a domain which *cannot be quantified over*:

The sticking point in establishing the consistency of geometry, analysis and number theory had always been the infinite. Any attempt to transmit consistency from finite cases to all cases by a recursive procedure, such as that sketched by Hilbert in 1905 was subject to Poincare's criticism that the consistency of inductive principles was being assumed, so that a vicious circularity was involved. (Irvine 2009, pp.291-310)

Professor Douglas Lackey, who wrote the introductions to Russell's contributions in the book *Essays in Analysis*, refers to the mammoth theoretical difficulties which mathematical paradox and self reference exerted within the fields of logic and mathematics:

The discovery of the paradoxes must rank as one of the most interesting chapters in the history of science. In the space of a few years, intuitions which had been the basis of a whole generation of development in logic and mathematics were called into question. (Russell and Lackey 1973, p.127)

It was Russell's analysis of Cantor's proof that there is no greatest cardinal number - as infinite cardinality logically excludes that there is a largest - which led to the paradox. Here we see that Russell's difficulty emerges in attempting to quantify or range over the domain of sets which has infinity at its core. As he remarks:

So long as we confine ourselves to finite numbers, there are no important formal differences between ordinal and cardinal; but when we allow infinite numbers, the differences becomes important due to the failure of the commutative law. (Russell 2007, p. 98)

The infinite and a definable totality are in this sense mutually opposing logical concepts and herein resides the heart of the problem. In considering Cantors proof "that there is no greatest cardinal number" Russell was led to "the consideration of a very peculiar class..it seemed to me that a class sometimes is and sometimes is not a member of itself"(Russell 2007, p.58):

The class of teaspoons for example is not another teaspoon, but the class of things that are not teaspoons, is one of the things that are not teaspoons. There seemed to be instances that are not negative: for example, the class of all classes is a class. The application of Cantors argument led me to consider the classes that are not members of themselves; and these, it

seemed, must form a class. I asked myself whether this class is a member of itself or not. If it is a member of itself, it must possess the defining property of the class, which is to be not a member of itself. If it is not a member of itself, it must not possess the defining property of the class, and therefore must be a member of itself. Thus each alternative leads to its opposite and there is a contradiction. (Russell 2007, p. 58)

As evidenced in the above the paradox emerges from a consideration of “the classes that are not members of themselves” considered as a defined totality. If they form a total and definable class – which Russell maintains they ought to - then the question raised is: can this class which ranges over classes which are not members of themselves, be in turn considered as a member of itself? If it is a member of itself it must possess the logical quality of not being a member of itself, therefore it is not a member of itself. Alternatively if it is not a member of itself then it must not possess the logical quality of the class and must be a member of itself:

Hence either hypothesis, that it is or that it is not a member of itself, leads to its contradiction. If it is a member of itself, it is not, and if it is not, it is. (Russell 2007, p.261)

During his early work Russell thought of mathematics as “not primarily a tool for understanding and manipulating the sensible world, but as an abstract edifice subsisting in a Platonic heaven and only reaching the world of sense in an impure and degraded form.” (Russell 2007, p. 209) His discovery of the paradox was to put an abrupt “end to the logical honeymoon” (Russell 2007, p.75) he had been enjoying. Russell’s exposition of the paradox in the above remark explains the

logical impasse with which he was confronted. At the heart of the problem of paradox and self reference lurked the seemingly unavoidable problem of the infinite:

I was led to this contradiction by considering Cantors proof that there is no greatest cardinal number. I thought, in all my innocence, that the number of all the things there are in the world must be the greatest possible number, and I applied his proof to this number to see what would happen.(Russell 2007, p.75)

There seemed in both Russell's and Cantor's experience to be something very amiss with our intuitive idea of a set:

What is wrong with our intuitive idea of 'set'? Can we make a rigorous theory of sets which corresponds closely with our intuitions, but which skirts the paradoxes? Here as in number theory and geometry, the problem is in trying to line up intuition with formalised or axiomatized reasoning systems. (Hofstadter 1979, p.28)

While Russell believed in the possibility of a strictly logical language which was not rooted in the physical world of intuition, his efforts to uncover the logical problem with our intuitive idea of sets begins in the world of intuition. He begins with the question raised by Cantors work of whether or not there is a greatest cardinal number. He observes that, following the path of intuition, the "plain man would suppose you could not get a larger class than the class of all the things there are in the world."(Russell 2007, p.259) However within set theory the problem of the infinite soon emerges to shatter this assumption:

It is very easy to prove that if you take selections of some of the members of a class, making those selections in every conceivable way that you can, the number of different selections that you make is greater than the original number of terms. (Russell 2007, p.259)

Russell provides an example of the impossibility of avoiding the infinite when we define the world through the lens of set theory:

This is easy to see with small numbers. Suppose you have a class with just three numbers, a, b and c. The first selection that you can make is the selection of no terms. The next of a alone, b alone and c alone. Then bc, ca, ab, abc, which makes in all 8 selections. Generally speaking if you have n terms, you can always make 2^n selections. It is very easy to prove that 2^n is always greater than n, whether n happens to be finite or not. So you find that the total number of things in the world is not so great as the number of classes that can be made up out of those things. (Russell 2007, p.260)

The results of the application of set theory lead to the conclusion that “the total number of things in the world is by no means the greatest number.”(Russell 2007, p. 260) It is precisely at this point where the infinite emerges, that the problem of paradox, contradiction and self reference emerge, denying the possibility of providing a closed logical model:

On the contrary, there is a hierarchy of numbers greater than that. That on the face of it seems to land you in a contradiction. (Russell 2007, p. 260)

In turning to a consideration of “classes that are not members of themselves” (Russell 2007, p. 260) the infinite operates as the catalyst from which Russell’s particular paradox emerged. Russell describes the logical difficulty of self reference inherent in trying to avoid the problem of the infinite as generated by the seemingly inherent and unavoidable issue of linguistic self reference, in the construction of a consistent and closed logical model:

The process is like trying to jump onto the shadow of your head. We can illustrate this most simply by the paradox of the liar. The liar says ‘everything that I assert is false’. This is, in fact, an assertion which he makes, but it refers to the totality of his assertions and it is only by including it in that totality that a paradox results.(Russell 2007, p.82)

In discovering the paradox logic seems to be confronted with the problem of an impossible object something which can simultaneously claim to be and not be a member of itself. According to Russell, the problem within his own and Cantor’s paradox “lay in logic rather than mathematics” (Russell 2007, p.59) concluding that “it was logic which would have to be reformed.” (Russell 2007, p.59)

Believing the problem of paradox to be soluble within logic Russell initially thought a simple error underpinned the paradox:

At first I thought there must be some trivial error in my reasoning. I inspected each step under a logical microscope, but I could not discover anything wrong. (Russell 2007, p. 5)

Russell's discovery of the paradox had devastating consequences for Frege. While the paradox which Cantor identified called the consistency of mathematics into question, for theological reasons he was "happy to accept that there were a pluralities of things too numerous to be collected together into a set: he called them inconsistent totalities" (Irvine 2009, pp.291-310) The view of Russell and Frege however stood in sharp contrast and understandably so:

The same indifference could not apply to Frege, whose logical system required him to quantify over all objects, including all sets, and for whom sets were included among the objects. (Irvine 2009, 291-310)

The problem which now faced Russell was not simply one of reducing mathematics to logic but rather one which demanded that the infinite be logically accounted for and accommodated within logicism if, paradox, self reference and contradiction were to be eliminated. To accommodate the infinite logicism would in effect have to construct a logical model wherein its language was capable of being mapped outside of the infinite in order to include the infinite within the logical model of logicism considered as a totality. This was the only way consistency could be restored to mathematics and by extension to logic. More critically it seemed the only way by means of which paradox, contradiction and self reference could be removed from logic. Clearly this was an impossible task.

2.2 Russell's Type Theory considered as an exemplar of non referential secondary language use within the process of logical analysis

In the Principles of Mathematics Russell had developed what he describes as a “crude form” (Russell 2007, p.78) of the theory of types which while effective to some degree in blocking the paradox, was still incapable of considering logic as a totality given the role which the infinite assumed within set theory and which would now have to be accommodated in any logical model which sought to show that mathematics can be derived from purely logical principles:

To sum up it appears that the special contradiction of Chapter X is solved by the doctrine of types, but that there is at least one closely analogous contradiction which is probably not soluble by this doctrine. The totality of all logical objects, or of all propositions, involves, it would seem a fundamental logical difficulty. (Russell 2007, p. 79)

As Russell makes clear in the above the problem of paradox and the associated issues of self reference and contradiction originate in the concept of infinity as originally discovered by Cantor. If there could not be a “set of all things” then the ambitions of logicism which was to prove that “the principles of mathematics are logical in nature, and can be demonstrated to follow from logical principles alone” (Simons 2009, p.12) was in jeopardy, as given both Cantors results and Russell's independent discovery of the same problem within Frege's system, it would now be necessary to accommodate the infinite within their logical model, which could no longer be a closed and therefore consistent model. It seemed logic was confronted with a problem language seemed incapable of resolving. Most critically it indicated that any logical model which sought to resolve and accommodate the infinite and its associated problems of paradox, self reference and contradiction,

would have to be open ended in nature. It now seemed certain that consistency could not be achieved by way a closed logical model which could range over logic in its entirety, while also including the infinite, as Russell and Frege had envisaged.

In the Preface to his 1879 *Begriffsschrift* Frege makes an important reference to the limits of language when attempting to “prevent anything intuitive” from entering into the logical model. His remarks highlight the difficulty of creating a closed logical model which can accommodate the infinite:

I found the inadequacy of language to be an obstacle; no matter how unwieldy the expressions I was ready to accept, I was less and less able, as the relations became more and more complex, to attain the precision that my purpose required. (Black and Geach 1960, p.234).

The problem of containing the infinite within a closed logical model would ultimately lead to the demise of Frege’s logicist project, when Russell identified that the problem of paradox was inherent in Frege’s Axiom V. In his response to Russell, Frege remarks:

And even now I do not see how arithmetic can be scientifically established; how numbers can be apprehended as logical objects, and brought under review; unless we are permitted to pass from a concept to its extension. (Black and Geach 1960, p. 234)

Significantly Frege recognised that the paradox was not only relevant to logicism, but to the entire domain of logical analysis, commenting that “everybody who has made use in his proofs of extensions of concepts, classes, sets, is in the same position as I. What is in question is not just my particular way of establishing

arithmetic but whether arithmetic can possibly be given a logical foundation at all. (Black and Geach 1960, p.234)

Within the domain of mathematics and independent of Wittgenstein's criticisms of the theory of types, Kurt Gödel's incompleteness theorem dealt the ultimate death knell to Frege and Russell's ambitions demonstrating that a perfectly consistent logical notation was logically impossible. Gödel's incompleteness theorem ultimately proved that what Russell and Frege had been attempting was logically impossible. Crucially the reason why the discovery of such a notation was impossible was rooted in the self referential and iterative nature of language in its non referential use, whereby what was being referred to by the notation was incorrectly assumed to be independent of the logical model of which it was a part:

Gödel showed that in any suitable formal system expressively powerful enough to formulate the arithmetic of natural numbers with addition and multiplication, if the system is consistent, then it cannot be proved consistent using the means of the system itself. (Irvine 2009, pp.291-310)

Solutions to the problem of the infinite and the generated problem of paradox were reached by Zermelo – who discovered the paradox independently of Russell in 1900. In light of these developments Russell's theory of types became obsolete and approaches such as that of Zermelo-Frankel set theory took its place. However while in Russell's eyes this initially was considered a failure, Gödel's work would cast a more complex light on this perspective. In proving that no system or logical model can prove itself consistent from within its own system or logical model, Russell's work can be seen as signalling, not a failure but rather a significant contribution to our understanding of what is involved in the process of logical analysis and its ability to create new models.

Principia Mathematica represents Russell's efforts to provide an alternative logical notation similar to Frege's but crucially one which avoided the paradox. This effort centred on creating a logical model which could accommodate the infinite, and which would in turn remedy the linguistic problems of paradox, self reference and contradiction. Russell termed this new logical model the theory of types. The idea of a logical notation or concept script and the frenzied focus on discovering *the one* which would definitively prove that mathematics was derived from logic, and which was devoid of inherent paradox and self reference, developed in response to the perceived inability of ordinary language to represent complex issues without simultaneously becoming victim to the problem of self reference and paradox. This view of language was central to Frege's logicist programme:

"If the task of philosophy is to break the domination of words over the human mind [...], then my concept notation, being developed for these purposes, can be a useful instrument for philosophers [...] I believe the cause of logic has been advanced already by the invention of this concept notation." (Black and Geach 1960, p.254)

For Russell it was an absolute necessity that the problem of paradox or the "contradictions should disappear" (Russell 2007, p.61) once the correct logical notation was discovered. His efforts were in vain, as he admits in My Philosophical Development, noting "The contradiction discussed proves that something is amiss but what this is I have hitherto failed to discover." (Russell 2007, p.62)

It will be found that in all the logical paradoxes there is a kind of reflexive self reference which is to be condemned on the same ground: viz. That it includes, as a member of a totality, something referring to that

totality which can only have a definite meaning if the totality is already fixed. (Russell 2007, p.63)

The difficulties Russell encountered in developing a new logical model originally which could accommodate the infinite and thereby remove the paradox, became “increasingly disagreeable”. (Russell 2007, p.79):

In the first place the whole problem struck me as trivial and I hated having to concentrate attention on something that did not seem intrinsically interesting. In the second place try as I would I could make no progress. (Russell 2007, p.79)

In *On Denoting* published in 1905, Russell would make a key logical distinction between knowledge by acquaintance and knowledge by description. This work would prove central in the formulation of his theory of types and later to his theory of judgment, noting retrospectively that it signalled “his first success” in remedying the issues of the infinite and paradox:

This was apparently not connected with the contradictions but in time an unsuspected connection emerged. In the end it became entirely clear to me that some form of the doctrine of types is essential. I lay no stress on the particular form of that doctrine which is embodied in *Principia Mathematica*, but I remain wholly convinced that without some form of the doctrine the paradoxes cannot be resolved. (Russell 2007, p.79)

It would prove that his work in *On Denoting* would not only lead to the theory of types but also to a very diverse implicit theory of visual perception:

In perception we have acquaintance with the objects of perception and in thought we have acquaintance with objects of a more logical abstract character; All thinking has to start from acquaintance but it succeeds in thinking about many things with which we have no acquaintance. (Russell 2007, pp. 41-42)

It is because we do not visually perceive the logical entities in question – e.g. the infinite - as objects of perception that the difficulty arises. Here language becomes a barrier to logical knowledge, in so far as it merely *refers* to an absent object or entity. However Russell's position also places an absolute faith in language such that if we *can* articulate something then it must exist:

The distinction between acquaintance and knowledge about is the distinction between the things we have presentations of, and the things we only reach by denoting phrases. (Russell 2007, p.41)

Russell maintained that the problem encountered by the absence of a corresponding object to denoting propositions can be solved through the development of an effective logical language, situated within an effective and purposeful logical model. The construction of such a logical model would in effect take the place of the absent object which visual perception takes for granted:

In the discussion of indefinables – which forms the chief part of philosophic logic – is the endeavour

to see clearly, and to make others see clearly, the entities concerned, in order that the mind may have that kind of acquaintance with them which it has with redness or the taste of a pineapple. (Russell 2007, p.81)

Methodologically this demanded applying the principles of set theory to objects known by acquaintance and objects known by denoting. Thus we can group objects with which we have direct acquaintance into classes or sets quite easily. However when it comes to grouping denoting propositions – which do not have a corresponding object and therefore constitute non referential language use– into classes or types it becomes more problematic. He reflects that this was what the theory of types attempted to do, and inherent in this task was the need to accommodate the infinite, and thereby to resolve the problem of self reference:

For example the set of *propositions* will be supposed to contain a proposition stating that ‘all propositions are either true or false’. It would seem however that such a statement could not be legitimate unless ‘all propositions’ referred to some already definite collection, which it cannot do if new propositions are created by statements about ‘all propositions’. (Russell 2007, p. 263)

Russell clearly recognised the inherent problem raised by the logical concept of infinity, noting “it seems certain that we shall not think of more than a finite number of arithmetical facts in the course of our lives, and we know the total number of arithmetical facts is infinite.” (Russell 2007, p.137) concluding that “We shall therefore have to say that statements about ‘all propositions’ are meaningless” (Russell 2007, p.216).

His approach to taming the infinite and the resulting paradox proves dynamic and complex when contrasted with Wittgenstein's one dimensional logic of showing as presented in the Tractatus. By allowing the infinite to reside within logic itself Russell explains how the theory of types will operate as an open logical model:

By saying that a set has no total, we mean primarily that no significant statement can be made about all its members. In such cases it is necessary to break up our set into smaller sets, each of which is capable of a total which does not generate self reference. This is what the theory of type's aims at effecting. (Russell 1989, p. 216)

Russell's open logical model would in effect set limits to language through the rule of type restriction and as such would crucially - while not claiming to range over or remove the concept of infinity - would seek to accommodate it structurally as a means of resolving the problem of paradox, self reference and contradiction:

A set of the lowest type could only contain objects as members not sets. A set of the next type up could only contain objects, or sets of the lowest type. In general a set of a given type could only contain sets of lower type. Clearly no set could contain itself because it would have to belong to a type higher than its own type. (Hofstadter 1979, p.29)

While Wittgenstein would regard Russell's efforts as futile, Russell believed that type structure imposed on the non referential use of language was successful in baring the paradox and as such was a logically tenable solution. In the following Russell uses the visual field as an analogy of the non referential use of language in

logical analysis, reflecting on what we mean when for example instead of using the word Piccadilly, we employ a secondary language such as that of set theory:

So that you would find that the logical status of Piccadilly is bound up with the logical status of series and classes and if you are going to hold Piccadilly as real you must hold that series of classes are real. (Russell 1912, p.51)

The extension of the referential use of language from the physical visual world – where we assert that Piccadilly is real – to that of a logical world –where we assert by extension that “classes are real” – is considered entirely unproblematic by Russell. His view that both a referential and non referential language are equally necessary and valid, is a view Wittgenstein would reject on the grounds that logical languages cannot talk about language or logic without resulting in self reflexive nonsense:

The whole question of the meaning of words is very full of complexities and *ambiguities* in ordinary language. When one person uses a word, he does not mean by it the same thing as another person means by it. (Russell 1912, p. 56)

Although Russell acknowledges that communicating in a logical language would not be effective in everyday discourse, he maintains that a logical language which overcomes the conceptual problems of meaning within the process of logical analysis, allowing meaning to change is necessary. This point is articulated in the implicit theory of visual perception it implies where Russell notes the following regarding those who have never physically been to Piccadilly:

They will know Piccadilly as an important street in London; they many

know a lot about it, but they will not know just the things one knows when one is walking along it. If you were to insist on language which was unambiguous you would be unable to tell people at home what you had seen in foreign parts. (Russell 1912, p.56)

These remarks reveal a much more dynamic and complex theory of visual perception than Wittgenstein offers in the *Tractatus*. These remarks imply that as the visual field is so diverse, ordinary language must also reflect this diversity of visual experience. It is he argues because our ordinary language is full of words which are “all ambiguous” that a “logically perfect language” is demanded in order to clarify meaning in a secondary manner within the process of logical analysis:

It is a language which has only syntax and no vocabulary whatsoever...That is one reason why logic is so very backward as a science, because the needs of logic are so extraordinarily different from the needs of daily life. One wants a language in both. (Russell 2007, p.198)

Significantly Russell’s understanding of a logical language or non referential language use, also demands diversity in particular when logical meaning is not a straight forward issue, which is particularly evident in the case of the infinite:

If one says ‘This is white’ it will do for about as simple a fact as you can get hold of. There you have a whole infinite hierarchy of facts – facts in which you have a thing and a quality, two things and a relation, three things and a relation, four things and a relation, and so on. That whole hierarchy constitutes what I call atomic facts. (Russell 2007, p.199)

While in the open logical model of type theory, self reference is ‘replaced’ by endless type restriction which are essentially self referential, as each type restriction is only valid in reference to all the others, Russell’s work reveals more about the process of logical analysis than Wittgenstein’s closed logical model. However it is only because self reference is regarded as logical Armageddon that this sentiment of removing self reference dominates:

For instance I may say ‘All atomic propositions are either true or false’, but that itself will not be an atomic proposition. If you try to say ‘All propositions are either true or false’, without qualification, you are uttering nonsense, because if it were not nonsense it would have to be itself a proposition and one of those included in its own scope..(Russell 2007, p. 199)

Type theory, as expressed in the open logical model of type theory allows for continuous linguistic self reference between types, which would prove the central logical component of Wittgenstein’s later work on aspect seeing:

You have to cut propositions up into different types, and you can start with atomic propositions, or if you like, you can start with those propositions that do not refer to sets of propositions at all. Then you will take next those that refer to sets of propositions of that sort that you had first. Those that refer to sets of propositions of the first type, you may call the second type and so on. (Russell 2007, p.263)

The critical difference in relation to Russell’s positioning of atomic propositions and Wittgenstein’s is that it is possible in the open logical model of type theory to

begin with atomic propositions and to progress from there to more complex types in a hierarchical manner. However as we will see in the closed logical model of the Tractatus it was essential that atomic propositions signalled an internal terminus of logical analysis itself.

According to Russell's early work the process of logical analysis through non referential language use, by means of which we access the world of logic was via "the contemplation of what is non human, the discovery that our minds are capable of dealing with material not created by them. (Russell 2007, p.211) Logic therefore indicated for Russell a domain where "pure thought can dwell as in its natural home" and where most significantly we can "escape from the dreary exile of the actual world."(Russell 2007, p.210) While the failure of the logicist project altered this idyllic perception of the process of logical analysis, Russell maintained commitment to the essential creativity of non referential language use in a logical context.

In his endeavour to bar the problem of paradox from logic through the open logic model of type theory, Russell acknowledged that the issue of infinite self reference and infinite expansion of types was unavoidable. However Russell also maintained that any logical model and as such language itself must always be capable of internal self reference so that "further structure would always be made explicit":

A logical language as I conceived it would be one in which everything that we might wish to say in the way of propositions that are intelligible to us, could be said, and in which further structure would always be made explicit. We should need in such a language words expressing structure, but we should also need words denoting the terms that have the structure. (Russell 2007, p.123)

This open logical structure of type theory is revealed in his analysis of how type theory operates. In order to avoid the paradox and accommodate the infinite it was necessary to “distinguish between propositions that refer to some totality of propositions and those that do not” (Russell 2007, p. 82):

Those that refer to some totality of propositions can never be members of that totality. We may define first order propositions as those referring to no totality of propositions; second order propositions as those referring to totalities of first order propositions; and so on ad infinitum. (Russell 2007, p.82)

By proceeding within this open ended logical model Russell remarks that the liar in the liar paradox will now have to say:

I am asserting a false proposition of the first order which is false. But this is itself a proposition of the second order. He is thus not asserting any proposition of the first order. What he says is thus simply false, and the argument that it is also true collapses. Exactly the same argument applies to any proposition of higher order. (Russell 2007, p. 83)

The later Russell reflects that the inherent difficulty of dealing with the problem of the infinite can also be seen in the finite world of objects. He remarks that what “gives unity to class is solely the intension which is common and peculiar to its members. This is obvious whenever we are dealing with a class whose members we cannot enumerate.” (Russell 2007, p. 87) However while:

In the case of infinite classes, the impossibility of enumeration is obvious; it is equally true of most finite classes. Who for example, can

enumerate all the members of the class of earwigs? (Russell 2007, p.87)

Russell reflects that originally the Epimenides liar paradox was treated as a puzzle or a joke “until it was found that it had to do with such important and practical problems as whether there is a greatest cardinal or ordinal number.” (Russell 2007, p. 262)

The solution which the open logical model of type theory provided in dealing with the problem of the infinite is summed up by Russell as follows:

When you ask yourself the question ‘Is there or is there not a greatest cardinal number?’ the answer depends entirely upon whether you are confining yourself within some one type, or whether you are not. Within any given type there is a greatest cardinal number, namely the number of objects of that type, but you will always be able to get a larger number by going up to the next type. (Russell 2007, p.264)

Central to how the open logical model of type theory accommodated the infinite in a comprehensive manner, in so far as the possibility of the infinite was inherent in the open structure of the logical model itself, was Russell’s parallel understanding of meaning, language use, and the process of logical analysis itself:

There is not one single concept of ‘meaning’ as one ordinarily thinks there is, so that you can say in a uniform sense ‘All symbols have meaning’, but there are infinite numbers of different ways of meaning...That is the real philosophical truth which is at the bottom of the theory of types.(Russell 2007, p. 269)

In stark contrast to the early Wittgenstein's closed logical model, Russell ultimately considers the question of whether logical analysis must arrive at a final atomic structure as unimportant remarking:

Whether there must be units incapable of analysis because they are destitute of parts is a question there seems no way of deciding. Nor is it important (Russell 2007, p.222)

This instinct of safeguarding the infinite potential of language to reveal itself in new and infinite ways proved central to his view of what logical analysis consists in. He remarks that the reason why the aforementioned question – which is the *raison d'être* of the *Tractatus* – is not important rests on the linguistic and logical reality regarding non referential logical language use, which for Russell implies that:

There is nothing erroneous in an account of structure which starts from units that are afterwards found to be themselves complex. For example, points may be defined as classes of events, but that does not falsify anything in traditional geometry, which treated points as simples. Every account of structure is relative to certain units which are, for the time being, treated as if they were devoid of structure, but it must never be assumed that these units will not, in another context, have a structure which it is important to recognise. (Russell 2007, p. 153)

In these reflection's Russell addresses the issue of how Wittgenstein's view of logical atoms as presented in the closed logical model of the *Tractatus* now appear

to him. In the following remark concerning Wittgenstein, it is clear that Russell was so influenced by the logical atomism of the Tractatus that he was prepared to follow Wittgenstein's new logical ambition:

The reason that I call my doctrine *logical atomism* is because the atoms that I wish to arrive at as the sort of last residue in analysis are logical atoms and not physical atoms. (Russell 2007, p. 179)

Russell reflects that this was "the only point on which at the time I most nearly agreed with Wittgenstein" However almost agreeing is no agreement at all and Russell points out that even in his introduction to the Tractatus he maintained the following position regarding the ability of language to constantly reveal new expressions via the process of logical analysis and non referential use of language:

Although in any given language there are things which that language cannot express, it is yet always possible to construct a language of *higher order in which these things can be said*. (Russell 2007, p.85)

He maintains that all logical advances are made through understanding the process and activity of logical analysis as an open ended logical model, wherein:

Advances consist in the recognition that what has been thought simple is complex; for example molecules are composed of atoms, and atoms have a structure which has been becoming known in recent years. So long as we abstain from asserting that the thing we are considering is simple, nothing that we say about it need be falsified by the subsequent discovery of complexity. It follows that the whole question whether there are simples to

be reached by analysis is unnecessary.
(Russell 2007, p.166)

For Russell “the ultimate units so far reached may at any moment turn out to be capable of further analysis.”(Russell 2007, p.222), a position which the early Wittgenstein would reject in an absolute sense. The view that language has the inherent capability to outrun the burden of self reference clearly illustrates the core characteristic of an open model of logic. Most significantly it is clear that for Russell the very problems of the infinite and of self reference can act as generative and creative forces in critical and logical analysis, a view which the early Wittgenstein vehemently opposes.

Critically and strikingly – in light of Wittgenstein’s later work - the problem of self reference inherent in the non referential use of language, and in its necessarily open logical model, is entirely unproblematic for Russell as although:

There will in the new language, still be things which it cannot say” these “can be said in the next language, and so on ad infinitum. (Russell 2007, p.85)

While Russell clearly acknowledges that the problem of self reference will necessarily emerge when the logician attempts to talk about language and logic as a totality, this is for Russell an unavoidable dimension of language – one which will not be resolved by imposing an eternal logical silence on the issue. Moreover he does not consider this a restriction within the process of logical analysis or the use of non referential language. He thus remarks that the problem of paradox and self reference only emerges in an untenable manner when we try to talk about language or any other entity as a totality:

After stating some of the paradoxes of logic, we found that all of them arise from the fact that an expression referring to *all* of some collection, may itself appear to denote one of the collection; as, for example, ‘ all propositions are either true or false’ appears to be itself a proposition. We decided that when this appears we are dealing with a false totality, and that in fact nothing whatsoever can be said about *all* of the supposed collection. (Russell 2007, p.101)

Within Russell’s open logical model self reference is therefore an essential ladder of non referential language use within the process of logical analysis, and not one which is to be discarded as nonsense. Critically non referential or the secondary use of language within the Russell’s type theory displays all the characteristics of an open ended language game which cannot be limited – a logical quality which Wittgenstein would employ in his later work on aspect seeing. Even if a logically perfect language can never be achieved as a final closed logical model, Russell maintains that the logical pursuit is more important than the idea of a final fixed logical model, and is therefore entirely unproblematic. Consequently he maintains his principle of analysis holds, even if it continues ad infinitum:

A sentence is a series of words....A word is a class of similar noises....A sentence is also a class of noises.....Each instance of a word is a complex sound..Behind the phonetic analysis there is a further stage: the physiological process of uttering or hearing a single word. Behind the physiological analysis is the analysis of physics and from this point on analysis proceeds...ad infinitum. (Russell 2007, p. 153)

Chapter Three

The First Stage of Evolution in Wittgenstein's logical analysis of the Infinite

Removal of the Infinite, the Fractal Connection, and Russell' Paradox in Wittgenstein's Closed Logical Model of the Tractatus

3.1 The Influence of Set theory and the Augustinian Picture of Language in the Tractatus

Wittgenstein's objection to Russell's theory of types as a solution to the problem of the infinite and the problem of paradox, hinges on a rejection of the view that non referential language use can access the logical or metaphysical realm of the infinite in a logically referential manner. As Monk remarks:

This view would require the Platonic assumption of the objective existence of forms – the assumption that there exists, not only individuals, but also abstract entities such as morality. Such an assumption is of course made by Russell in his Theory of Types with which Wittgenstein became increasingly dissatisfied. (Monk 1991, p.70)

That Wittgenstein's rejection of Russell's type theory was a rejection of all and any type theory is highlighted by James Davant in his 1975 paper *Wittgenstein on Russell's Theory of Types*:

Wittgenstein's criticisms of the theory of types would apply to any theory of types, given the metaphysics and theory of logic in the Tractatus. (Davant 1975, p.1)

Rather than Russell's type theory considered in isolation, it is the very structure of an open logical model which the theory of types epitomises, which Wittgenstein most despises. Inherent in the early Wittgenstein's closed logical model of the Tractatus and in the work of Frege and Russell is the Augustinian picture of language. This picture of language which would later be rejected by Wittgenstein as it is used in the Tractatus plays a critical role in how Wittgenstein would treat the problem of infinity and Russell's paradox in his early, middle and later periods of work. The picture of language it presents is that "every genuine name has a meaning, and its meaning is the object for which it stands." (Hacker 1996, p.23) In essence it epitomises a referential model of language use which is central to the closed logical model of the Tractatus:

The Tractatus everywhere displays the force of this mesmerising picture of the essential nature of language and of the relation between language and reality. It was part of the unquestioned framework of the Fregean and Russellian philosophies, and Wittgenstein, in his confrontation with his predecessors, worked within this framework. (Hacker 1996, p.23)

While this referential picture of language use informed the work of both Russell and Wittgenstein in their respective responses to the concept of the infinite and the issue of paradox as derived from set theory, Russell's open logical model of type theory would ultimately prove far more successful than the closed logical model of the Tractatus. This would prove particularly true regarding the status of logical analysis itself as treated within their respective logical models. Unlike Wittgenstein the influence of this referential language model within Russell's work does not lead to a limitative role of language use within the process of logical analysis itself. Ultimately Russell's work remains entirely uninhibited by the referential model of

language, while Wittgenstein's logical model ultimately proves redundant as it condemns logical analysis itself to an eternal silence.

Reflecting on the influence which set theory and the famous paradox exerted on the young Wittgenstein, Russell makes a critical and astute remark regarding Wittgenstein's methodological resolution of the paradox:

It was not by the paradoxes that he wished to be known, but by a suave evasion of paradoxes. (Russell 2007, p. 214)

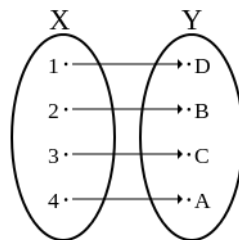
Both Russell's and Wittgenstein's involvement in the ensuing mathematical controversy sparked by Cantor's work during the early 1900's resulted in the development of diametrically opposed logical models.

The influence of set theory in Wittgenstein's approach to resolving the problem of the infinite and Russell's paradox is of immense significance and is one of the central logical pictures the Tractatus rejects. However its influence is rarely singled out as such. While the rejection of set theory would remain a constant feature in Wittgenstein's work, its logical principles were utilised in his creation of the closed model of logic we encounter in the Tractatus, allowing him to effect "a suave evasion" of Russell's paradox. In Wittgenstein and the Turning point in Mathematics S.G. Shanker highlights the central role of set theory and the concept of the infinite in both Wittgenstein's early and later work:

The pattern which emerges from these interlocking quotations reveals a decidedly intuitionistic hostility to set theory running from the Tractatus to the Philosophical Remarks, gathering force in the later work and subsequently blossoming into a frontal assault on transfinite set theory

and the Cantorean notion of the
'actual infinite'.(Shanker 1987, p.163)

While Wittgenstein rejected set theory in all its guises he utilised that same rejection in two distinct ways. In this regard the one logical rule which Wittgenstein borrows from set theory is the principle of bivalence or isomorphism which is utilised in his closed logical model to ban the concept of infinity, and as a consequence resolve Russell's paradox by making it 'vanish'. Firstly this principle is employed to refute set theory as it appears in the guise of logicism. Here its rejection hinges on the grounds that inconsistency within logicism is rooted in its inability to establish an isomorphic relation to any logical objects – particularly infinite sets - therefore forfeiting any potential to either verify or falsify its claims. The efforts of the logicist project of Frege and Russell are thus classified within the logic of the Tractatus as nonsense. Secondly the principle of isomorphism is employed in the Tractatus to bolster Wittgenstein's unique variant of logical atomism where the closed logical model of showing is argued to assume a logical precedence over open logical models such as Russell's type theory. The logical rule of isomorphism within set theory illustrated below would prove pivotal in its success.



The numbering system Wittgenstein employs in the Tractatus also reflects the fundamentals of set theory. In effect each of the seven central propositions

constitutes a logical set. Within each set, with the exception of number seven, there is an analysis of the subsets or parts of that set. Proposition number seven is in this sense the most significant and can be considered as applicable to the process of traditional non referential language use within logical analysis itself:

7 Whereof one cannot speak thereof
one must be silent. (Wittgenstein and
Ogden 1999, p. 108)

This final proposition crucially does not proceed with further commentary and as such remains without parts or subsets, signalling the internal limits of the closed logical model. Here we meet the elementary or atomic proposition. Problematically Wittgenstein fails to explicate the methodology or logical process he will employ to discover this internal logical bolt, nor does he provide examples. At most we can assume it will be achieved through the logic of showing. More problematically, if one agrees with Wittgenstein's position regarding the non referential use of language, it signals the end of logical analysis as a viable and tenable activity, leaving the entire discipline in a very precarious situation. In essence this conclusion brings with it a denial of the dual aspect in language use – as we encounter in the experience of aspect seeing. Furthermore it invalidates the tenability of non referential or logical language use as we encounter in Russell's open logical model - which would later be referred to by Wittgenstein as the secondary use of language – and therefore the validity of the process of logical analysis itself. As logical analysis is dependent on the non referential use of language, this use is exiled from the closed model of the Tractatus as nonsense. As I will show it was Wittgenstein's approach to the concept of the infinite both within logic itself and within the visual field which led to this intolerable position regarding Wittgenstein's final conclusion on the impotence of logical analysis to say anything. Retrospectively considered this position would be rejected in his middle and later periods through revisiting the concept of the infinite which had served as the catalyst for the closed logical model we encounter in the Tractatus.

The link between Wittgenstein, the work of Cantor and Peano and its influence on the logic of the Tractatus is noted by James K. Feibleman in his book *Inside the Great Mirror*. In the following Feibleman points out the link between specific remarks from the Tractatus and their relation to the work of both Cantor and Peano. Here we see the centrality which the problem of the infinite assumed in the Tractatus:

[4.2211] Infinity does not exclude discreteness nor a correspondence between propositions and facts or objects. The analogy here is *Cantors* definition of the infinite as the one-one correspondence between the elements of a set and those of a proper subset.

[5.42] Logical symbols indicate operations of a logical sort, and not logical constants, not logical things.

[5.43] A finite set of axioms can generate an infinite number of theorems, as for instance from *Peano's postulates*. (Feibleman 1973, pp. 87-95)

Feibleman's commentary in the above relates to the identified remarks from the Tractatus where Wittgenstein directly addresses the problem of the infinite as encountered by Cantor, Peano, and Russell, noting that the problem of the infinite can only be addressed by recognising the limits of language in so far as such propositions "say nothing":

5.43 That from a fact p an infinite number of others should follow, namely, $\sim p$, $\sim\sim p$, etc.

But the propositions of logic say the same thing. That is, nothing.

5.452 The introduction of a new expedient in the symbolism of logic must always be an event full of consequences. No new symbolism may be introduced in logic in brackets or in the margin – with so to speak, an entirely innocent face. (Wittgenstein and Ogden 1999, pp 74-75)

Crucial to Wittgenstein's removal of the concept of infinity from the field of logic was his reaction to the emerging mathematical objects which paralleled the work of Cantor. The relevance of these mathematical objects or pictures was the inherent logical principle of infinity which they displayed. Known today as examples of fractals these new figures feature in an implicit manner in the Notebooks, but would feature in a more explicit manner in Wittgenstein's middle period of work. Wittgenstein's implicit use of these fractal pictures not only adds a new dimension to the picture theory of meaning but also serves as an essential link in his later journey from the closed logical model of the Tractatus to the open logical model of aspect seeing.

3.2 The role of the infinite considered via the fractal Connection

Removing the Infinite from the Visual Field in the resolution of paradox and self reference

Using puzzles to address logical problems of philosophical analysis was a method encouraged by Russell:

A logical theory may be tested by its capacity for dealing with puzzles, and it is a wholesome plan, in thinking about logic, to stock the mind with as many puzzles as possible, since these serve much the same purpose as is served by experiments in physical science. (Russell 1989, p.484)

This particular facet of Russell's methodology left a lasting influence on Wittgenstein. However Wittgenstein's interest in puzzles was not solely concerned with grammatical and logical puzzles as was Russell's, but was equally concerned with visual puzzles, such as the Necker cube and the emerging ambiguous figures generated by mathematical paradox. It is Wittgenstein's early recognition of visual puzzles, which underpinned his early proficiency in "seeing connections", a logical approach which would extend to his later work on aspect seeing. As far back as the notebooks Wittgenstein had penned a prophetic remark concerning his later work on aspect seeing and its association with visual puzzles:

Puzzle pictures and the seeing of situations. (Wittgenstein 1998, p.28)

Prior to the Tractatus, he posed a crucial question which anticipated the entire orientation of his later work on aspect seeing:

Is seeing an activity? (Wittgenstein
1998, p.77)

Retrospectively considered it would indeed prove to be the case that seeing is an activity in a linguistic context - where in his later work on aspect seeing - this linguistic seeing would be formally expressed as the secondary use of language. Moreover it would constitute a language game which was itself infinite in its logical essentials thereby mirroring the principle of an open logical model which the Tractatus rejects. However Wittgenstein's logical position on the role of the infinite would first be expressed in a diametrically opposed manner in the closed logical model of the Tractatus.

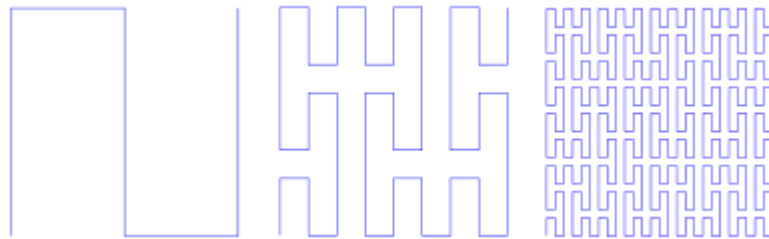
The concept of the infinite is most explicitly shown as opposed to being stated or expressed in the visual images of the Cantor set and the space filling curves of Peano and Hilbert. The principle of logic underpinning fractals centres on their fragmented geometric shape that can be split into parts, each of which is a reduced sized copy of the whole, a property called self similarity. The term was coined by Benoit Mandelbrot in 1975 and was derived from the Latin word 'fractious' meaning broken or fractured. Mathematicians originally referred to the Cantor set and the Peano and Hilbert curves as a "gallery of monsters" (Mandlebrot 1973, p.9) entirely unsure of how to logically account for them.

The two most significant characteristics displayed by what we now know as fractals, are their pictorial or logical form and the nature of whole part relations therein. Crucially the impossibility of moving from a combination of parts to the whole set in its entirety – i.e. the infinite set – is most powerfully displayed in its pictorial or logical form, visually displaying the same logical problem encountered by Russell in his efforts to quantify over logic as a totality. The Cantor set clearly illustrates how the set through constant self reference, self replicates over and over ad infinitum. Visually the Cantor set can be considered as representing either a

closed or an open logical model. The concept of aspect seeing – seeing the visual image now one way and now another – is inherent in these fractal images and as such illustrates the tension between the concepts of an open and closed model of logic. The Cantor set is an infinite set which emerges on the continuum or number line.



Cantor G. Cantor Set (1883)



Peano, G. Space Filling Curve (1890) (three iterations)

The work of Cantor, Peano, and others, now stand as the historical forerunners of what Mandelbrot would later define as fractals. Below is a synopsis of the historical antecedents of fractals by F.H. Dyson:

Fractal is a word invented by Mandelbrot to bring together under one heading a large class of objects that have [played] ..an historical role...in the development of pure mathematics. A great revolution of ideas separates the classical mathematics of the 19th century

from the modern mathematics of the 20th. Classical mathematics had its roots in the regular geometric structures of Euclid and the continuously evolving dynamics of Newton. Modern mathematics began with Cantor's set theory and Peano's space filling curve. Historically the revolution was forced by the discovery of mathematical structures that did not fit the patterns of Euclid and Newton. (Mandelbrot 1973, p.3)

As the above synopsis indicates the fact that fractals – just as the theoretical concept of the infinite in set theory – failed to 'fit' any known pattern, proved extremely problematic. However Wittgenstein recognised a crucial logical characteristic of self reference in this gallery of monsters which he would utilise in two very different ways in both his early and later models of logic. In his early work these images are banned in addition to the formal logical concept of the infinite as expressed in Cantor's set theory, and accommodated within type theory.

While such logical objects are banned from the Tractatus they prove methodologically significant in considering the picture theory of meaning and its evolution to aspect seeing from a new perspective. In effect the visual images show how the logical or pictorial form – understood as the basic visual building block of each image – self replicates or is mirrored throughout the figure. Thus while the image *looks* different at various iterations or stages of self replication the same logical form of the whole subsists throughout. Because of this internal self replication, logically separating the logical form of the whole from its constituent parts becomes impossible as both are reciprocally mirrored in each whole part construction – a logical concept which Wittgenstein utilises in the Tractarian logic of showing. The principle of construction underpinning these counter intuitive geometric curves, thus extends to produce not a finite object but an infinite and as such an impossible object.

The following key remark from the Tractatus reveals an inherent structural and conceptual similarity between the logic of showing and these mathematical fractal objects:

4.011 At first glance the proposition – say as it stands on printed paper – does not seem to be a picture of the reality of which it treats. But nor does the musical score appear at first sight to be a picture of a musical piece; nor does our phonetic spelling (letters) seem to be a picture of our spoken language. (Wittgenstein and Ogden 1999, p. 45)

Just as the different iterations look entirely different at each stage of construction, so too the isomorphic logical mapping between language and world share this characteristic. Closer examination reveals a crucial internal relation which shows the isomorphic logic informing both pictures:

4.014 The gramophone record, the musical thought, the score, the waves of sound, all stand to one another in that pictorial internal relation, which holds between language and the world. To all them the logical structure is common. (Wittgenstein and Ogden 1999, p.46)

While Wittgenstein identifies an internal self referential relation, this internal self reference is firstly a mirroring of the same logical form in different representations, and secondly is a limited self reference which Wittgenstein argues terminates with elementary or atomic propositions. Here the internal self referencing is one of sameness and not difference as would prove to be the case in his later work on aspect seeing. Mandelbrot provides a snapshot of the reception to the Peano curve by both Peano and others:

Everything had come unstrung! It is difficult to put into words the effect that [Giuseppe] Peano's result had on the mathematical world. It seemed that everything was in ruins, that all the basic mathematical concepts had lost their meaning" (Vilenkin 1965) "[Peano motion] cannot possibly be grasped by intuition: it can only be understood by logical analysis" (Hahn 1956) "Some mathematical objects, like the Peano curve, are totally non intuitive,...extravagant."(Mandelbrot 1973, p. 59)

Mandelbrot would go on to show that the reaction to this gallery of monsters as being counter intuitive was incorrect and moreover that the initial tendency of both mathematicians and logicians to appeal to some metalogical realm to explain such figures was based upon entirely false premises. He thus remarks:

I claim that the preceding quotes merely prove that no mathematician ever examined a good Peano graph with care. An unkind observer could say these quotes demonstrate a lack of geometric imagination. I assert to the contrary that, after Peano teragons are observed attentively, letting ones thoughts wander about, it becomes very difficult not to associate them with diverse aspects of Nature. (Mandelbrot 1973, p.59)

What is most striking about Mandelbrot's remarks in this context, is the reciprocal intellectual sentiment which Wittgenstein expresses in his later work when addressing the concept of aspect seeing, and the importance he attaches to these images which at that time were considered meaningless oddities. Just like Mandelbrot the later Wittgenstein would identify through such mathematical objects the experience of aspect seeing, and the ability of language to hold the

logician captive. This experience of being held captive by language has the potential to condemn the logician – as indeed was the case with Wittgenstein – to seeing language in only one dimension or aspect:

115 A picture held us captive. And we couldn't get outside it, for it lay in our language, and language seemed only to repeat to us inexorably. (Wittgenstein 2009, p.53)

The necessary “geometric imagination” Mandelbrot refers to proved the exception rather than the rule. As far back as 1901 when this “gallery of monsters” began its evolution, Russell regarded them with logical contempt, claiming that “by banishing the figure it becomes possible to discover all the axioms that are needed...” (Russell 1985, p.72).

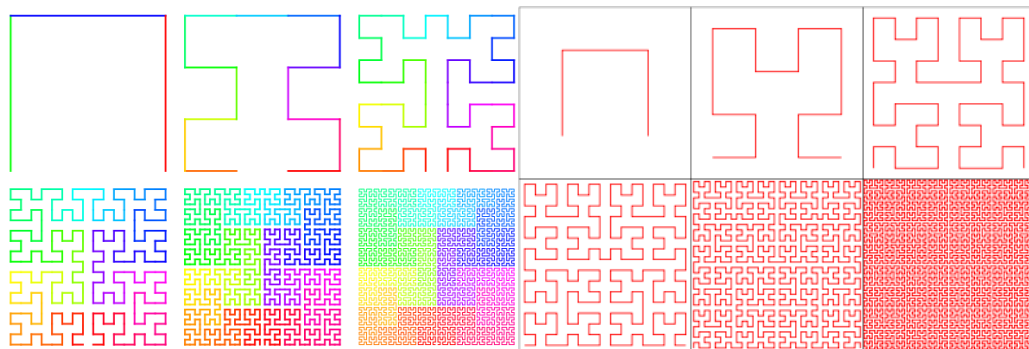
In addition to the matters that I have already mentioned, there were other things that delighted me in the work of Peano and his disciples. I liked the way in which they developed geometry, *without* the use of figures... (Russell 2007, pg.56)

It is unsurprising that Peano opted to exclude a visual representation of the mathematical curve he discovered in its original publication. Peano's omission of the visual graphics was no doubt informed by the less than enthusiastic reception these developments were receiving within academia. For Cantor however the discovery- while giving rise to the problem of inconsistency – had opened a door to a new logical paradise:

These new structures were regarded as ‘pathological’... as a ‘gallery of monsters’, The mathematicians who created the monsters regarded them as important in showing that the world of pure mathematics contains a richness

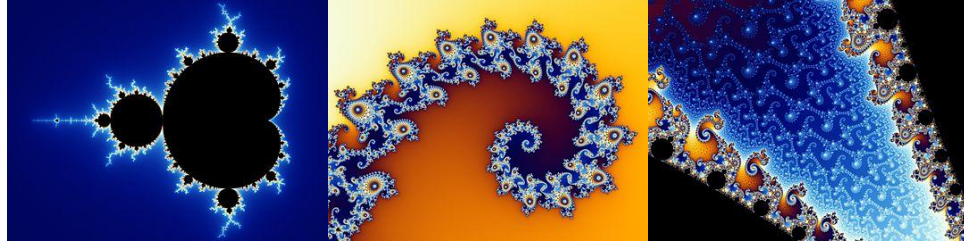
of possibilities going far beyond the simple structures that they saw in nature. Twentieth century mathematics flowered in the belief that it had transcended completely the limitations imposed by its natural origins. (Mandelbrot 1973, p.3)

As F.H. Dyson highlights in the above the most significant result of the revolution in mathematics instigated by Cantor was the belief that the world had effectively been transcended – that what was now within the domain of mathematical discovery was an infinite realm which was not comparable with the contents of the visual world. A year after the Peano space filling curve was identified, a second curve named the Hilbert curve (pictured below) was discovered by David Hilbert the leader of the Formalist tradition:



Moore E.H, Moore Curve (Six Iterations) Hilbert, D. Hilbert Curve (1891)

While Russell’s recognition of a “supreme beauty” (Russell 2007, p.210) in mathematics is ironically illustrated in the very images he regarded as irrelevant, he unlike Wittgenstein recognised the recursive, iterative and self referential nature of language to constantly move beyond itself as a crucial logical principle within the process of logical analysis – just as it is in these fractal images:



Mandelbrot, B.B., The Mandelbrot Set (1980)

First published in 1975 – five years after Russell’s death – Mandelbrot’s work would certainly have been of immense interest to Russell, bringing together the work of mathematicians from Cantor, Peano, Hilbert and many others who had a profound and lasting influence on Russell during his career. Historically the significance of fractals is a narrative which only assumes a clear logic retrospectively. With the work of Mandelbrot it would transpire that fractals, composed of infinite self-replicating patterns, do not just exist in the abstract world of mathematical logic but in nature, in the human body, the human brain, DNA, and today in mobile phones and computer software. While initially regarded as pathological, the gallery of monsters would prove to occupy a fundamental role in the modern world, their pictorial fascination signalling not some metaphysical world of the infinite but rather the ordinary everyday world we inhabit:

Now as Mandelbrot points out.....Nature has played a joke on the mathematicians. The 19th century mathematicians may have been lacking in imagination, but Nature was not. The same pathological structures that the mathematicians invented to break loose from 19th

century naturalism turn out to be
inherent in familiar objects all
around us. (Mandelbrot 1973, p.4)

While the logic of the Tractatus shares a similarity with the pictorial form in these early fractal images, Wittgenstein denied them any logical status in the Tractatus. In contrast to Russell, Wittgenstein's approach to the issue of the infinite - both theoretically and visually - was to exclude it in totality from the logical model which would become the Tractatus, thereby setting the groundwork for what he believed would be a closed and internally consistent model. Central to the development of his closed logical model was approaching the concept of infinity through an analysis of perception within the visual field. Firstly Wittgenstein is decisive that the visual field, perception and therefore representation of the visual field through language is limited to the isomorphic mapping format of the show say distinction. A finite and closed logical grammar, therefore dominates from the outset.

The closed logical model of the Tractatus is ultimately dependent on denying language a dual aspect where both referential and non referential uses of language are acceptable. In turn the necessity of non referential language use within the process of logical analysis is rejected in its entirety. The view of Russell that the ladder of language can be scaled to heights which allow the logician to look beyond isomorphic representation via referential language use is rejected. For Wittgenstein the efforts of Russell and Frege to “4.1237 express in conceptual notation the general proposition....contains a vicious circle” (Wittgenstein and Ogden 1999, p.57) as “4.442 it is quite impossible for a proposition to state that it itself is true” (Wittgenstein and Ogden 1999, p.62) In order to avoid infinite regress “finding the measure of amount that is said” becomes critical. (Wittgenstein 1998, p.54)

For what I should now have to do is to
find an expression in the language of
this theory for HOW MUCH a

proposition says. (Wittgenstein 1998, p.54)

Just as the concept of the infinite as applied to the visual field, it would prove for Wittgenstein the very lack of referential coordination in logic and logical language which gives logical propositions their tautological character. It is this tautological character which in turn both informs the logic of showing and leads Wittgenstein to a position which regards non referential language use as logically untenable. If he argues, we try to say what logical propositions show, we will introduce a dual aspect into language use which the Tractatus bans on the following basis. Firstly we cannot speak of language and logic let alone speak of it as a totality, less still an infinite one. Secondly if we do we are inevitably going to become embroiled in the problem of paradox and self reference as language in its non referential use is unable to operate in a referential manner and therefore will only produce nonsense. The problem with this position is that it demands an austere one dimensional referential view of language which would ultimately prove just as untenable as the open logical models he rejects.

3.3 Spatial, logical and linguistic limits

The Fractal connection as revealed through the complex spatial objects of the Notebooks

Arriving at the positions which informed the closed logical model of the Tractatus was preceded by a detailed reflection on, and analysis of the infinite in the Notebooks. Here Wittgenstein's analysis of the infinite is used to reject the non referential use of language in the process of logical analysis, and by extension to reject Russell's paradox and the problem of self reference. The infinite is first address via complex spatial objects. These mathematical objects – while rejected by Wittgenstein as irrelevant in the Tractatus - serve as a crucial explanatory device. This geometric dimension of the Tractatus is referred to by Russell in his introduction to the Tractatus:

He compares linguistic expression to ***projection in geometry***. A geometrical figure may be projected in many ways: each of these ways corresponds to a different language, but the projective properties of the original figure remain unchanged whichever of these ways may be adopted. These projective properties correspond to that which in his theory the proposition and the fact must have in common if the proposition is to assert the fact. (Wittgenstein 1999, p.9)

In the following Wittgenstein makes specific reference to what he describes as complex spatial objects referencing what today we know as fractals. In the first move in establishing a closed logical model Wittgenstein poses the following question where he connects the logic of pictorial and linguistic representation:

Is spatial complexity also logical complexity? It surely seems to be.
(Wittgenstein 1998, p.45)

In first aligning spatial and logical complexity as seen via the theoretical and visual instances of the infinite, Wittgenstein observes that uniformity between language and the visual field is a logical necessity, concluding that the idea of infinite complexity is not present in either:

When the proposition is just as complex as its reference, then it is completely analysed. But the reference of our propositions is not infinitely complicated. (Wittgenstein 1998, p.46)

In denying infinity any logical status Wittgenstein makes his first step in creating a closed logical model which was essential if it was to offer a fundamentally different logic from that of Russell. Ultimately this closed finite logical model would lead to the logical concept of atomic or elementary propositions which operate as the logical bolt of the closed logical model. From the outset Wittgenstein denies any infinite complexity to the visual field insisting that the “reference of our propositions is not infinitely complex.” (Wittgenstein 1998, p.46) Extending the logic of spatial complexity to language Wittgenstein establishes the isomorphic or bivalent dimension of the logic of showing, which stipulates that the proposition must be a picture of what it represents in order to be able to speak of it. Being acutely aware of the role of the infinite as the generating mechanism Russell’s paradox Wittgenstein, maintains the objective world “is not infinitely complicated”. (Wittgenstein 1998, p.46) The central concern for Wittgenstein is the picture of the infinite which had held the world of logic captive since Cantors work first appeared, both in terms of the infinite itself and the problems of paradox, self reference and contradiction which it generated.

As the following remark makes clear Wittgenstein's central concern in removing the infinite, is to remove the possibility of language referring to its own logical foundations from the domain of what can be articulated, such that, regardless of what the proposition pictures, its logical form "will not depend on me":

The proposition is the picture of the fact. I can devise different pictures of the fact. But what is characteristic of the fact in all of these pictures will not depend on me. (Wittgenstein 1998, p.47)

In order to forcefully debar the infinite from the Tractatus Wittgenstein turns his focus to "the general concepts (a) of portrayal and (b) of co ordinates." (Wittgenstein 1998, p.47). Wittgenstein locates the solution through combining pictorial representation in conjunction with the co ordinates of that which the proposition pictures - a process which operates by way of a logical geometrical bivalence or isomorphism. If this principle is taken as the absolute benchmark for his closed logical model then it becomes impossible for the infinite to exist therein as the infinite cannot by definition be co-ordinated with what it portrays. Another question regarding complex spatial objects follows:

Can we regard a part of space as a thing? In a certain sense we obviously always do this when we talk of spatial things. (Wittgenstein 1998, p.47)

In the following Wittgenstein comments on the efforts of logicians and mathematicians to evade the problem of the infinite, by conferring reality on this unreality, by way of naming, thereby suggesting something metaphysical right in the sphere of the public visual field:

For it seems – at least so far as I can see at present – that the matter is not settled by getting rid of names by means of definitions: *complex spatial objects, for example, seem to me in some sense to be essentially things* – I as it were see them as things. – And the designation of them by means of names seems to be more than a mere trick of language. *Spatial complex objects – for example – really so it seems do appear as things.* (Wittgenstein 1998, p.47)

The closed logical model is further extended in the following where the principle of isomorphism or bivalence so central to set theory is utilised concluding that any “proposition dealing with a complex” will “not be nonsensical if the complex doesn’t exist but simply false”:

At any rate that we quite instinctively designate those objects by means of *names*. –

So much is clear, that a complex can only be given by means of its description; and this description will hold or not hold.

The proposition dealing with a complex will not be nonsensical if the complex does not exist, but simply false. (Wittgenstein 1998, p.48)

Continuing his reflections on complex spatial objects Wittgenstein regards their reality as an impossible one remarking:

When I see space do I see all its points?

It is no more possible to present something “contradicting logic” in language than to present a figure contradicting the laws of space in geometry by means of its co ordinates, or, say, to give the co ordinates of a point that does not exist. (Wittgenstein 1998, p.48)

His remark that it is impossible to “present a figure contradicting the laws of space” clearly indicates that such objects cannot exist in the closed logical model of the Tractatus. He reflects further that if it were possible to position ourselves outside pictorial representation - where language could refer to its logical foundations through an objective mapping – we would then be in a position outside the logical form of the world, and would be able to represent what we say about logic with what logic shows us in an isomorphic manner:

If there were propositions asserting the existence of proto pictures they would be unique and would be a kind of “logical propositions” and the set of these propositions would give logic an *impossible reality*. There would be co ordination in logic. (Wittgenstein 1998, p.48)

However for Wittgenstein it is the very lack of co ordination which gives logical propositions their tautological character. To use a visual illustration the task is akin to the effort to observe or look at oneself, looking at oneself in a mirror. As Wittgenstein assumes a one dimensional approach to both language and the visual field, this is considered a logical impossibility. The following makes specific reference to the presentation of the infinite in Cantors work on the infinite series:

A complex just is a thing!

We can quite well give a spatial representation of a set of circumstances which contradict the

laws of physics, but not of one contradicting the laws of geometry.

The mathematical notation for the infinite series like “ $1+x/1+1+x/2+1+x/3+\dots$ ” together with the dots is an example of that extended generality. A law is given and the terms that are written down serve as an illustration. (Wittgenstein 1998, p.49)

In aligning the logic of mathematics and language, the logic of showing denies the mathematics of the infinite any special logical status or access to logical form. Rather in trying to define the infinite the problem of self reference is merely perpetuated and as such no logical terminus can be achieved:

Remember that the “propositions about *infinite* numbers” are all represented by means of *finite* signs.

The propositions dealing with infinite numbers, like all propositions of logic, can be got by calculating the signs themselves (for at no point does a foreign element get added to the original primitive signs). So here too, the signs must themselves possess all the logical properties of what they represent.

Logic takes care of itself; all we have to do is to *look and see* how it does it. (Wittgenstein 1998, p.11)

Wittgenstein’s conclusions are clear and definitive. His position is that what logic shows us, cannot be reinterpreted by language as a means of providing a consistent framework whereby we can talk about the logical foundations of that framework. Rather we must accept that $2+2=4$ for example, shows its logical form. Attempting to reinterpret what logic in such instances shows by means of non referential language use, can only lead to nonsense:

In logic (mathematics) process and result are equivalent (Hence no surprises), pg. 42 Notebooks)

Now everything turns on the fact that I apply numbers to ordinary things, etc. Which in fact says no more than numbers occur in quite ordinary sentences. (Wittgenstein 1998, p.67)

Mathematical logic just like linguistic logic demands that logical form is a precondition of and to experience. Its reference must therefore be confined to public objects and not metaphysical logical entities:

The logical identity between sign and thing signified consists in its not being permissible to recognise more or less in the sign than in what it signifies. If sign and thing signified were not identical in respect of their total logical content then there would have to be something still more fundamental than logic. (Wittgenstein 1998, p.4)

As he observes in the Tractatus, mathematical propositions do not picture anything. They are just as logical propositions tautological - showing their logical form which cannot in turn be reinterpreted or re expressed:

6.2321 And that the propositions of mathematics can be proved means nothing else than that their correctness can be seen without our having to compare what they express with the facts as regards their correctness.

6.22 The logic of the world which the propositions of logic show in tautologies, mathematics shows in equations. (Wittgenstein and Ogden 1999, pp.98-99)

In denying the infinite any logical status in the Tractatus Wittgenstein concludes that the “world has a fixed structure”(Wittgenstein 1998, p.62) Wittgenstein provides a visual example of “a patch in the visual field which is to the right of the line” which he contrasts with the assumption “that every patch in our visual field is infinitely complex”(Wittgenstein 1998, p.64) Focusing on complex objects in the visual field as a means of unifying the closed logic of showing and visual perception he remarks that the sense in which “complex objects do not exist” (Wittgenstein 1998, p.63) is: rooted in the nature of the proposition such that “It must be clear in the proposition how the object is composed, so far as it is possible for us to speak of its complexity at all.” (Wittgenstein 1998, p.63) The question now to be resolved is whether in analysing the visual field we “arrive at parts that cannot be further analysed” (Wittgenstein 1998, p.62) or not:

This question is a logical one and the complexity of spatial objects is a logical complexity, for to say of one thing that it is part of another is always a tautology. (Wittgenstein 1998, p.62)

In critiquing the concept of infinity in the visual field, the logical backdrop of infinity as the generative locus of self reference in logical models is addressed. He first observes that in determining how the object is composed, language itself serves as the limiting device, and as such this limitation must also be inherent in the visual field which it represents. In this sense the logic of language and the visual must reflect each other perfectly. If we try to reinterpret what logic shows – as he criticises Russell for doing in the theory of types – we generate a nonsense structure which no longer displays the original fixed logic of showing:

The sense of the proposition must appear in the proposition as divided

into its simple components - . And these parts are then actually indivisible, for further divided they just would not be THESE. In other words the proposition can no longer be replaced by one that has more components, but any that has more components also does not have this sense. (Wittgenstein 1998, p.63)

In addressing the problem of the infinite as expressed in complex spatial objects – such as the Cantor set and the Hilbert and Peano curve, Wittgenstein determines that all such representations are flawed and are merely “subsequent constructions” (Wittgenstein 1998, p.64) of an original fixed limit. This logical point is now also applied to the propositions which claim to be able to quantify over or refer to logical concepts such as the infinite, via logical language which is used in what he claims is an untenable referential manner:

Now it seems of course perfectly possible that in reality infinitely many different propositions do not follow from such a proposition because our visual field perhaps – or probably – does not consist of infinitely many parts – but continuous visual space is only a subsequent construction - ; and in that case only a finite number of propositions follow from the one known and it itself is finite in every sense. (Wittgenstein 1998, p.64)

In so far as there are theoretical and visual representations of an infinite this is only possible because there is an original fixed limit which we erroneously imagine we can reach beyond by “subsequent constructions”. Such “subsequent constructions” as we are presented with by the non referential use of language in the process of logical analysis – such as in type theory for instance – are therefore considered nonsense. The original fixed limit is that of isomorphic representation:

4.04 In the proposition there must be exactly as many things distinguishable as there are in the state of affairs which it represents. They must possess the same logical multiplicity.
4.041 This mathematical multiplicity naturally cannot in its turn be represented. One cannot get outside it in the representation. (Wittgenstein and Ogden 1999, p.49)

The logician cannot therefore establish further constructions for logical objects or concepts such as the infinite, as they have no referent with the “same logical multiplicity”. Such subsequent constructions are for Wittgenstein merely self referential, reflecting not a new logical or infinite content, but the limits of both language and the visual field:

When the sense of the proposition is completely expressed in the proposition itself, the proposition is always divided into its simple components – no further division is possible and an apparent one is superfluous. (Wittgenstein 1998, p.63)

Wittgenstein’s identification of the development of “further division” in logic, is essentially a recognition of an inherent inventiveness and creativity in language as we see in type theory for instance, where non referential language use establishes a further division within language to expose new structural and conceptual connections. However such creative logical constructions are rejected in the closed logical model and are regarded by Wittgenstein as meaningless nonsense. This position would eventually be entirely overthrown in his later work where creativity and inventiveness becomes essential to logic. The unification of the logical and visual field, allows Wittgenstein to accommodate the problem of the infinite in a novel and unique manner, albeit it occupies a negative position in so far as it does not picture anything in the world:

I notice only that the spot has finite extension, and this of itself seems to presuppose an infinitely complex structure...It is certain that I do not see all the parts of my *theoretical* visual field. Who knows *whether* I see infinitely many points? (Wittgenstein 1998, p.65)

In the Tractatus the possibility of referential ostensive showing therefore removes what he regards as the inherent ambiguity of language. In asserting “I know what I mean, I mean just THIS”, pointing to the appropriate complex with my finger” (Wittgenstein 1998, p.70), the problem of resolving the referential content is simple as language can map itself to its bivalent or isomorphic counterpart. However, when addressing logic itself the matter is entirely different. While the “the outward form” of the proposition “this watch is lying on the table” is “clear and simple”, Wittgenstein regards this simplicity as “only constructed.”(Wittgenstein 1998, p.69) The underlying logic or “sense is more complicated than the proposition itself” and is explained by the fact that:

The conventions of our language are extraordinarily complicated. There is enormously much added in thought to each proposition and not said. (Wittgenstein 1998, p.70)

As the above suggests what is going on in thought and language— which is determined by the logical form of reality – is essentially hidden and “not said” but shown by the bivalence of the proposition and its objective referent. Wittgenstein believed this logical mapping was also maintained between the primary and secondary levels of language. At the primary level of language we reach the internal

limits of showing - the assumed atomic propositions - thereby closing the set of the logic of showing as presented in the Tractatus.

Within Wittgenstein's closed logical model it thus becomes logically impossible for language to refer to its own foundations as a totality, and therefore falling into the trap of self reference and paradox cannot occur. Its occurrence only arises if we try to use non referential language in a referential manner and attempt to quantify over language and logic as a totality. Of course he maintains when the logician does this language is only operating in a nonsense capacity. He thus establishes a much more achievable benchmark of consistency which relates only to "the sense of the proposition" (Wittgenstein 1998, p.24):

The knowledge of the representing relation *must* be founded only on knowledge of the component parts of the situation. (Wittgenstein 1998, p.24)

The concern of logic is thus firmly situated and limited within the closed logical model or set and specifically within the relation of language to what it represents, such that we attain knowledge of "an internal relation" (Wittgenstein 1998, p.24) This internal relation between the a priori foundation of language and logic and that which it pictures and refers to in the world, is essentially linear in its construction and not hierarchical like Russell's type theory – highlighting their respective differences regarding the status and function of non referential language use. This central structural difference in turn reflects the opposing dynamics of language use in their respective logical models. Furthermore the contrast between Wittgenstein's linear closed logical model, and Russell's open hierarchical logical model, highlights their respective position on the recursive and iterative nature of logic and language use. For Wittgenstein it is fundamentally closed and limited whereas for Russell there is an inherent potential for logic and language to reinvent itself in an unlimited manner. In this sense the internal relation between types within Russell's

logical model implies the possibility of iterative and self reflexive extension whereas in Wittgenstein's case this possibility is denied. For Wittgenstein the internal relation terminates with the discovery of atomic or elementary propositions, signalling the internal limit and thereby closing the logical model:

What the completely general propositions describe are indeed in a certain sense structural properties of the world. Nevertheless these propositions can still be true or false. According as they make sense the world still has that permanent range. And the range which is left to its structure by the TOTALITY of all elementary propositions is just the one that is *bounded by the completely general propositions*. (Wittgenstein 1998, p. 20)

For Wittgenstein this implies that the consistency or sense of any logical model will either be congruent or non congruent by virtue of whether or not reality or the objective world corresponds through a bivalent mapping:

The proposition itself sunders what it congruent with it from what is not congruent. For example if the proposition is given, and congruent, then the proposition is true if the situation is congruent with it.

What can be said can only be said by means of a proposition and so nothing that is necessary for the understanding of *all* propositions can be said. (Wittgenstein 1998, p. 25)

In the above Wittgenstein claims that the proposition which is needed to resolve the dilemma which logicism faced in its efforts to quantify over the infinite and to resolve the subsequent problem of paradox self reference and contradiction that issue as a result is impossible to express. He thus asserts “that which is necessary for the understanding of *all* propositions” cannot be articulated by language specifically the secondary or non referential use of logical language operative in the process of logical analysis. There must he asserts be a logical equivalence between different types of things, an equivalence he maintains is absent in Russell’s type theory, precisely because he goes beyond the limits of language:

It is clear that neither a pencil stroke nor a steamship is simple. Is there really a logical equivalence between the two?

“Laws” like the law of sufficient reason, etc. , deal with the network not with what the network describes.(Wittgenstein 1998, p.43)

In removing the possibility of dealing with the infinite in a logical context in the sense in which logic and language can be quantified over as a totality by language, the Tractatus ensures that “nothing that is essential for the understanding of all propositions can be said”. (Wittgenstein 1998, p. 25) This position thus demands that logical form or the foundation of logic is positioned as prior to experience and language, therefore access to it is logically denied from the outset:

A statement cannot be concerned with the logical structure of the world, for in order for a statement to be possible at all, in order for a proposition to be CAPABLE of making sense, the world must already have just the logical structure it has. The logic of the world is prior to all truth and falsehood. (Wittgenstein 1998, p.14)

The logical form of showing is thus closed and denies the possibility of being represented or of being spoken of:

But might there not be something which cannot be expressed by a proposition? (and which is also not an object)?” In that case this could not be expressed by means of language; and it is also impossible for us to ask about it. (Wittgenstein 1998, p.51)

The efforts of trying to articulate or speak of the logical form or foundations of logic as attempted in logicism, is now considered by Wittgenstein as “something which cannot be expressed by a proposition”.(Wittgenstein 1998, p.24) Referring to set theory Wittgenstein observes that it is an inherently contradictory logical model which confirms the existence of the infinite through finite means:

This theory treats of propositions exclusively, so to speak, as a world on their own and not in connection with what they present.

It is not a picture of reality, in the sense that it does not present anything; it is what all – mutually contradictory – *pictures* have in common. (Wittgenstein 1998, pp.55-56)

For Wittgenstein what set theory tries to say lacks isomorphic bivalence and as such does not represent anything, but rather illustrates the process of logical analysis wherein non referential language use and meaning runs up against its own limits of representation. The open logical model of type theory as a response to the problem of the infinite and of avoiding the paradox it generates, is thus rejected by

Wittgenstein on the basis of its complexity and infinite extension, which similar to set theory fails to represent anything:

5.554 The enumeration of any special forms would be entirely arbitrary.

5.5541 How could we decide a priori whether for example I can get into a situation in which I need to symbolise with a sign of a 27 termed relation?

5.5542 May we then ask this at all? Can we set out a sign form and not know whether anything can correspond to it?

Has the question sense: what must there be in order that anything be the case?

5.555 And how would it be possible that I should have to deal with forms in logic which I can *invent*: but I must have to deal with that which makes it possible for me to invent them.

5.556 There cannot be a hierarchy of the forms of the elementary propositions. (Wittgenstein and Ogden 1999, p.87)

The above remarks are pivotal in the context of Wittgenstein's rejection of the theory of types, suggesting that the entire process of logical analysis in Russell's work rests on his own invention of forms which, as it has no isomorphic referent, is nonsense. Critically it is Russell's accommodation of the infinite within his open logical model allowing for the creation of a 27 termed relation and any other subsequent construction, which Wittgenstein specifically rejects on the basis that such constructions are mere invention. In contrast Wittgenstein's rejection of the infinite as a concept of logical significance means no such logical accommodation is required. Operating as the logical bolt of the Tractatus, Wittgenstein remains steadfast that his conception of elementary propositions ensure "empirical reality is

limited". (5.5561 Wittgenstein and Ogden, 1999, p.87) From Wittgenstein's perspective, the expulsion of non referential language use and its replacement with a new process of logical analysis thus ensures a closed logical model, avoiding the problem of self reference and denying traditional logical analysis any significant role. Russell's efforts therefore are merely a "subsequent construction" of an original fixed limit representing the impossible effort of trying to use language to refer to its own foundations. On these grounds, the very ambition of logicism to quantify over the entire domain of logic as a means of providing consistent foundations for mathematics is rejected by Wittgenstein as untenable. According to Wittgenstein the perspective which logicism demands if it was to be successful in its endeavours, is that the logician must occupy a position outside of logic and language, and as such an impossible perspective is required:

4.12 Propositions can represent the whole reality but they cannot represent what they must have in common with reality in order to be able to represent it – the logical form. To be able to represent the logical form, we should have to be able to put ourselves with the propositions outside logic, that is outside the world. (Wittgenstein and Ogden 1999, p.53)

Wittgenstein's central contention is that logical form as the frame and foundation of logic and language, and as the mechanism which allows language to picture reality *shows* itself in terms of how pictorial sense and cohesion operate. Language - by means of which we represent the world - cannot in turn be used to refer to its own logical foundation or logical form without the problem of paradox, self reference and contradiction emerging. As Wittgenstein denies the infinite in both the visual field and language any logical status the problem of having to quantify over the infinite and accommodate it is absent. As there is no infinite in Wittgenstein's Tractatus, there is no problem regarding an infinite totality, and consequently no problem regarding the generated problem of paradox self reference and

contradiction. In removing the idea of logic as a totality over which language can quantify, claiming instead that logic – and hence the logical form of language and all else in the world – operates through a process of logical mirroring, which is closed by elementary propositions, Wittgenstein believed that the linguistic loop which had initially led to Russell's paradox can be closed entirely:

4.121 Propositions cannot represent the logical form: this mirrors itself in the propositions.

That which mirrors itself in language, language cannot represent.

6.5 For an answer that which cannot be expressed the question too cannot be expressed. The riddle does not exist. If a question can be put at all then it can also be answered. (Wittgenstein and Ogden 1999, pp.53-107)

While for Russell there was the potential for the logician to experience logic itself via access through non referential language use, this is prohibited in Wittgenstein's closed logical model. In contrast Wittgenstein denies the possibility of any logical experience, which ensures that logical objects such as the infinite which cannot be encountered in a manner parallel to objective contents in the world, are denied any logical status. This position further implies that the non referential use of language in the process of logical analysis cannot be used in a referential manner and therefore is redundant:

5.552 The 'experience' which we need to understand logic is not that such and such is the case, but that something *is*, and that is *no* experience.

Logic precedes every experience – that something is so.

It is before the How, not before the
What.(Wittgenstein and Ogden
1999,p.86)

Unlike Russell Wittgenstein denies any possibility of logical experience which as an a priori condition of experience is not part of experience. Unlike Wittgenstein, Russell's model of type restrictions carried with it a parallel assumption that the infinite could be accounted for and accommodated, allowing the problem of paradox and self reference to be resolved through establishing hierarchical type restrictions. In the following remark from the Notebooks Wittgenstein expresses his early opposition to such type restriction, an opposition which would eventually set in place elementary propositions as an alternative structural device in his closed logical model:

I think there cannot be different Types of things! In other words whatever can be symbolised by a simple proper name must belong *to one type*. And further every theory of types must be rendered superfluous by a *proper theory of symbolism*. (Wittgenstein 1998, p.122)

For Wittgenstein the logical notation of a 27 termed type restriction for instance defines nothing at all as it pictures nothing at all. As non referential language use inevitably runs up against the limits of linguistic representation, such a logical approach continues to generate the same problems of self reference wherein as the structure being built is self reflexive, it is of necessity involved in an endless self reflexive loop. In condemning the process and activity of logical analysis itself as operative in the non referential use of language, Wittgenstein's following position signals its terminus as a viable process:

4.462 Tautology and contradiction are not pictures of reality. They present no possible state of affairs. For the

one allows every possible state of affairs, the other none.

In the tautology the conditions of agreement with the world – the presenting relations – cancel one another, so that it stands in non presenting relation to reality. (Wittgenstein and Ogden 1999, p.63)

In place of the non referential use of language in logical analysis Wittgenstein's new logic of showing requires acceptance of the failure of non referential language use to achieve its goal – which is to refer to its own logical foundations as a totality.

4.1212 What can be shown cannot be said. (Wittgenstein and Ogden 1999, p.53)

The implicit role of Wittgenstein's early primary secondary language use distinction can be seen as reflected in the following remark whereby elementary propositions constituting primary language allow for the generation of secondary language use. Here we see how the closed logical model and its implicit primary secondary language distinction, resolves the problem of the infinite considered as a totality and the generated problem of self reference, by locating the totality within the closed logical model:

The propositions are everything which follows from the totality of all elementary propositions (of course also from the fact that it is the *totality of them all*) (So in some sense one could say that *all* propositions are generalisations of the elementary propositions.) (Wittgenstein and Ogden 1999, p.65)

As we can no longer speak of logical foundations the burden on language to refer to itself and talk about itself is removed. However Wittgenstein fails to address any specifics of what an elementary proposition is, or of the new methodology of showing which would take the place of non referential language use in its efforts to reveal this hidden matrix of primary language – crucially without speaking of what cannot be spoken of. As Russell prophetically remarks in his introduction to the Tractatus regarding his hesitation in accepting the logical model of the Tractatus:

What causes hesitation is the fact that after all, Mr. Wittgenstein manages to say a good deal about what cannot be said, thus suggesting to the sceptical reader that possibly there may be some loophole through a hierarchy of languages, or by some other exit. (Wittgenstein and Ogden 1999, p.22)

Having banished the infinite and the generated problem of paradox and self reference, the closed logical model of the Tractatus concludes with the following position; “6.1222 logical propositions can no more be empirically confirmed than they can be refuted” and the laws of logic – logical form – “6.123 cannot themselves obey further logical laws” (Wittgenstein and Ogden 1999, p. 95). Wittgenstein’s new logic of showing consequently leaves the activity and process of logical analysis in the following position:

6.121 The propositions of logic demonstrate the logical properties of propositions, by combining them into propositions which say nothing.

This method could be called a zero method. In a logical proposition propositions are brought into equilibrium with one another, and the state of equilibrium, then shows how these propositions must be logically constructed.

6.1222 This throws light on the question why logical propositions can no more be *empirically confirmed than they can be empirically refuted*. Not only must a proposition of logic be incapable of being contradicted by any possible experience, but it must also be incapable of being confirmed by any such.

6.123 It is clear that the laws of logic *cannot themselves obey further logical laws*. (Wittgenstein and Ogden 1999, pp.94-95)

In criticising Russell's type theory Wittgenstein had observed that the inclusion of more and more types needs a justification which can only be provided for within that open logical model, by the addition of more and more type restrictions. Just as the visual images of fractals so splendidly show, such a logical model is considered by Wittgenstein as epitomising a process which as it continues ad infinitum, is trying unsuccessfully to make the laws of logic obey further logical laws – in this case the rule or law of type restriction. Thus the non referential use of language which allows for the creation of a new use and sense of language is condemned on the basis that “5.452 No new symbol may be introduced in logic in brackets or in the margin – with so to speak an entirely innocent face”(Wittgenstein and Ogden 1999, p.75):

5.452 Thus in the Principia Mathematica of Russell and Whitehead, there occur definitions and primitive propositions in words. Why suddenly words here? This would need a justification. There was none and can be none for the process is not actually allowed. (Wittgenstein and Ogden 1999, p.76)

Here Wittgenstein's disdain for the non referential use of language within the process of logical analysis is definitive and considered as an essential rejection in validating his closed logical model. Most significantly he follows this remark by criticising the paralysis exerted upon logicians and mathematicians alike by the fascination which infinite numbers – considered pre-eminent – had cast over the entire domain of logic during the crisis in mathematics:

5.453 All numbers in logic must be capable of justification.

Or rather it must become plain that there are no numbers in logic.

There are no pre-eminent numbers.

5.454 In logic there is no side by side, there can be no classification.

In logic there cannot be a more general and a more special.

5.4541 The solution of logical problems must be neat for they set the standard for neatness. (Wittgenstein and Ogden 1999, p.76)

Russell's paradox and the problem of self reference it introduces, are now unable to emerge within the set of the Tractatus firstly because the concept of the infinite has been removed by denying any numbers – in this case the infinite – a special logical status. In asserting that there are no numbers in logic, Wittgenstein makes explicit the central difference between his and Russell's logical models – in so far as Russell's attempts to accommodate the infinite while his excludes them in totality. In removing the infinite and mathematics in general form the sphere of logical analysis he thus remarks:

6.2 Mathematics is a logical method. The propositions of mathematics are equations, and therefore pseudo-propositions.

6.21 Mathematical propositions express no thoughts.

6.22 The logic of the world which the propositions of logic show in tautologies, mathematics shows in equations. (Wittgenstein and Ogden 1999, p98)

As the infinite has been removed as a logical problem which must be dealt with by logic, the issue of self reference and paradox- when it does emerge – signals, not the inability of the logician to quantify over the infinite because the correct model has not yet been found, but rather the limits of the isomorphic representing ability of language. Rather the problem is rooted in the impotence of language to refer to itself thereby representing its own logic. For Wittgenstein this is a problem of self reference which cannot be avoided but rather is perpetuated when language tries to breach the limits of isomorphic representation through the use of non referential language in a logical context:

Suppose there is something outside the facts? Which our propositions are impotent to express?

What cannot be expressed we do not express. – And how try to ask whether THAT can be expressed which cannot be EXPRESSED? (Wittgenstein 1998, p.52)

For Wittgenstein the error of logicism and of Russell's type theory is thus rooted in the effort to identify and talk about something more "in the sign than what it signifies." - i.e., that which allows the sign to represent in the first instance. What logic shows can thus not in turn be restated by language as logic is it's a priori condition:

4.122 It is impossible to assert by means of propositions that such

internal properties and relations obtain: rather, this makes it manifest in the propositions that represent the relevant states of affairs and are connected with the relevant objects. (Wittgenstein and Ogden 1999, p.54)

Wittgenstein's linear internal mirroring logic of internal relations closes the logical model of showing, and any efforts of non referential language use to go outside this limit are therefore logically redundant. Rather than referring to anything efforts to define logic itself simply reveal a logical limit of language as opposed to a metaphysical or metalogical object. According to Wittgenstein efforts to do precisely this rest on asking senseless questions which are only raised as logicians don't understand the logic of language:

4.0030 Most propositions and questions that have been written about philosophical matters are not false, but senseless. We cannot therefore answer questions of this kind only state their senselessness. Most questions and propositions of the philosophers result from the fact that we do not understand the logic of our language" (Wittgenstein and Ogden 1999, p.45)

Having removed the concept of the infinite as a valid logical concept in the Notebooks from the visual field, and having aligned the concept of the infinite with the logical and linguistic problem of self reference, the groundwork was completed for the closed logical model of the Tractatus.

3.4 Wittgenstein's Rejection of Russell's Paradox

How Wittgenstein's analysis of the Infinite ensured Russell's paradox vanished

In the Tractatus Wittgenstein specifically criticises the problem of self reference in the work of Russell and Frege on the basis that they are trying to include in the function (the empty frame) the frame itself as a proof or argument of its own logical consistency. This is a feat which for Wittgenstein is now logically impossible:

3.332 Russell's error is shown by the fact that in drawing up his symbolic rules he has to speak about the things his signs mean. No proposition can say anything about itself, because the propositional sign cannot be contained in itself (that is the whole theory of types).

3.333 A function cannot be its own argument, because the functional sign already contains the prototype of its own argument and it cannot contain itself. (Wittgenstein and Ogden 1999, p.42)

Wittgenstein's rejection here is an extension of banishing the concept of the infinite, where the focus now turns to a rejection of the assumption that either language or logic can somehow represent itself objectively – or quantify over itself - by referring to itself within a given logical model.

The show dimension of the Tractatus is now intended to bypass the problem of self reference which emerges when the logician uses non referential language to speak about itself or more specifically its own logical foundations. Once we throw away the logical ladder of non referential language use which the logician believes can allow us to logical quantify over issues such as the infinite, we will Wittgenstein

assures be lead to a position of “seeing the world rightly”. It is the inherent absence of a public object and as such a public referent for judgment of a given logical model – viz a viz an assumed logical or metaphysical object specifically the infinite – which is the central point of difficulty which Wittgenstein identifies, remarking in the Tractatus that:

4.1272 It is senseless to speak of the number of all objects.(Wittgenstein and Ogden 1999, p.56)

The rejection of the idea of ranging over logic in its totality – specifically the notion of the infinite - is in effect is a wholesale rejection of logicism and type theory, and their efforts to say what Wittgenstein argues cannot be said. While understanding how referential language use can occur when dealing with finite objects or numbers is unproblematic, the case is entirely different when dealing with logical concepts such as the infinite. Attempting to use language in a non referential context *as if* it was still functioning in a referential context, is considered by Wittgenstein as an attempt to assume a position external to language.

Russell’s use of type theory to resolve the issue of paradox and self reference as generated through a consideration of the problem of the infinite, is regarded by Wittgenstein as an illegitimate and illogical extension of language to logical objects, with the implied belief that logical objectivity over concepts and as such over logic itself could be achieved. Wittgenstein understood quite clearly that no public criteria in terms of verifying or falsifying such efforts could be provided and it is in this sense that such efforts are to be regarded as nonsense. Logical objects in their entirety are thus banned from the Tractatus:

4.441 It is clear that to the complex “F” and “T” no objects (or complex of

objects) corresponds; any more than to horizontal or vertical lines or to brackets etc. There are no “logical objects”.(Wittgenstein and Ogden 1999, p.61)

In the following Wittgenstein criticises Frege and Russell’s efforts to prove that mathematics was deducible to purely logical concepts, by using language as if it functioned in a referential manner within a logical or metaphysical context:

4.1272 Wherever the word “object” (“thing” “entity”, etc.) is rightly used, it is expressed in the logical symbolism by the variable name. Wherever it is used otherwise, i.e. as a proper concept word, there arises senseless pseudo propositions. ...And it is senseless to speak of the number of all objects. The same holds of the words Complex, Fact, Function, Number etc. They all signify formal concepts and are presented in logical symbolism by variables, not by functions or classes (as Frege and Russell thought).(Wittgenstein and Ogden 1999, p.56)

In asserting that it is senseless or nonsense to “speak of the number of all objects” the concept of infinity is targeted specifically. On the basis that “a function cannot be its own argument” because “no proposition can say anything about itself”, Wittgenstein rejects Russell’s open logical model of type theory:

3.333 Russell’s error is shown by the fact that in drawing up his symbolic rules he has to speak about the things his signs mean. No proposition can say anything about itself because the propositional sign cannot be contained in itself (that is the “whole theory of types”).

A function cannot be its own argument because the functional sign already contains the prototype of its own argument and it cannot contain itself.

“Herewith Russell’s paradox vanishes” (Wittgenstein and Ogden 1999, p.42)

In removing the possibility of quantifying over logic and language as a totality wherein non referential logical language is used in an untenable referential manner to refer to itself, the possibility of self reference within the field of logical analysis is made impossible, and Wittgenstein casually announces that Russell’s paradox has vanished. This is easily achieved from Wittgenstein’s perspective as self reference is made inadmissible within the Tractatus by denying language the possibility of talking about itself and as such about its logical foundations. As using non referential language to talk about logical objects – i.e. classes, the infinite etc. as totalities - and about language and logic as such, is identified by Wittgenstein as the very source of Russell’s paradox, the possibility of its emergence in the closed logical model of the Tractatus is impossible, as both the infinite and the non referential use of language which claims to refer to such logical objects, is logically prohibited. Put in another way the closed logical model of the Tractatus does not allow the logician to talk about logical concepts in the manner in which Russell’s open logical model of type theory does. Here the great contrast between the two logicians is clear. Wittgenstein on the one hand demands logical silence in return for rigour and finality, and Russell on the other hand demands constant articulation of logic through a logical model of ever increasing complexity and possibility, where he sees no logical loss in this same process of non referential language use.

The problem of self reference acutely shown in the case of Russell’s paradox extends for Wittgenstein to all non referential logical language use, which he

maintains can only operate by violating the rules of isomorphism and in so doing of talking about that which can only be shown. For Wittgenstein if language is allowed to talk about itself in this way, any logical model can be presented as the definitive correct model, without any public criteria to either verify or falsify its claims:

4.441 It is clear that to the complex of the signs “F” and “T” no object (or complex of objects) corresponds; any more than to horizontal and vertical lines or to brackets. 4.442 There are no “logical objects”..... Frege’s assertion sign is altogether meaningless; in Frege and Russell it only shows that these authors hold as true the propositions marked in this way. A proposition cannot possibly assert of itself that it is true.(Wittgenstein and Ogden 1999,p.61)

In condemning the work of Russell and others on the basis of self reference or having to talk about itself, so too Wittgenstein would have to condemn the Tractatus on the same grounds, declaring that anyone who understands the propositions of the Tractatus “6.54 finally recognises them as senseless”, a recognition which in turn demands throwing away the ladder of traditional non referential logical language and accepting in its place the closed logical model of showing, which of course signals the end of the traditional process of logical analysis.

In validating his rejection of non referential language use and the transgression of the mathematical concept of the infinite into the field of logic, Wittgenstein places his closed and limitative conception of both language and seeing as the source of logical intuition:

6.2 Mathematical expressions express no thoughts.

6.233 The the question whether we need intuition for the solution of mathematical problems it must be answered that language itself here supplies the necessary intuition.

6.2341 The essential of mathematical method is working with equations. On this method depends the fact that every proposition of mathematics must be self evident. (Wittgenstein and Ogden 1999, p.99)

In this sense returning to the world of intuition is to abandon the non referential use of logical language, and the illusion that language can be used to access and explicitly state its own logical form in a referential manner – either in logic or mathematics. Language cannot accommodate this demand and the result of attempting to do so is the attempt to go beyond the limits of language resulting in nonsense and ultimately Russell's paradox:

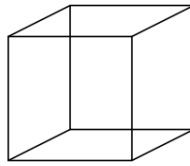
6.124 The logical propositions describe the scaffolding of the world, or rather they present it. They "treat" of nothing. They presuppose that names have meaning and that elementary propositions have sense. And this is their connection with the world. (Wittgenstein and Ogden 1999, p.96)

Wittgenstein's rejection of the infinite and the subsequent generation of paradox and self reference in the visual field can be considered as dealt within the Tractatus via the Necker Cube. Here the problem of self reference as expressed in Russell's paradox is addressed in its visual context within the Tractatus via the Necker cube. Its appearance in the Tractatus is connected to the idea of logical nonsense which Wittgenstein levelled specifically at Russell's type theory. Here we see its

extension to the visual field which was critical if a public isomorphic criterion between showing and saying which ensured logical tenability was to hold:

5.5442 The correct explanation of the form of the proposition “A judges p” must show that it is impossible to judge a nonsense. (Russell’s theory does not satisfy this condition.)

5.5423 To perceive a complex means to perceive that its constituents are combined in such and such a way. This perhaps explains that the figure,



can be seen in two ways as a cube; and all similar phenomenon. For we really see two different facts. (Wittgenstein and Ogden 1999,p.85)

In the above Wittgenstein briefly mentions the visual phenomena of ambiguous perception which is crucially derived from the logical internal self reference of two potential objects of sight within a given singular content. The function of its brief inclusion here is I contend, to reinforce the closed logical model which the Tractatus presents, and to re-orientate logic from the metaphysical nonsense of traditional non referential language use within logical analysis, to a closed logical model where there is no longer a need for such non referential language use. In presenting the experience of aspect seeing as simply seeing two different facts, Wittgenstein ultimately bypasses the most central issue which is the paradoxical experience of the aspect switch when looking at the image. The fact that the aspect switch is dependent on self reference and reflexivity within the object is ultimately not addressed in any meaningful manner by Wittgenstein, no doubt as it would threaten the closed logical model of the visual field which the Tractatus demands. For Wittgenstein if self reference and contradiction is disallowed in the field of

language and logic, then it cannot be allowed any special status in the visual field of public perception. Wittgenstein thus ‘resolves’ the issue of visual self reference inherent in the Necker cube by stating that “we really see two different facts”, as opposed to a phenomena within which self reference is essential. This is a critical point as if absolute independence was to be attributed to atomic or elementary propositions, then this inclusion was essential. If the issue was unaddressed it would have served as an illustration wherein self reference between atomic facts was necessary. Furthermore if this phenomenon was granted any logical status then the possibility of visual reality being limited as a closed set would prove impossible, as infinite ways of seeing reality would have to be allowed, and it would not be possible to set internal limits to seeing or to language use, as self reference in both the visual field and language would hold logical precedence. As evident in the following a finite closed logical grammar in logic, language and the visual field was essential:

5.5561 Empirical reality is limited by
the totality of objects. The boundary
again appears in the totality of
elementary propositions.
(Wittgenstein and Ogden 1999, p.87)

Crucially in the following remark regarding the primary secondary language distinction which feature in Philosophical Remarks from his middle period of work, Wittgenstein focuses specifically on the visual phenomena of ambiguous perception or optical illusions, linking it to this critical distinction.

We talk for instance of an *optical illusion* and associate this expression with the idea of a mistake, although of course it isn’t essential that there should be any mistake; and if appearance were normally more important in our lives than the results of measurement, then language would also show a different attitude to this phenomenon.

There is not – as I used to believe – a *primary language as opposed to our ordinary language, the ‘secondary’ one*. But one could speak of a primary language as opposed to ours in so far as the former would not permit any way of expressing a preference for certain phenomena over others; it would have to be, so to speak, absolutely *impartial*. (Wittgenstein 1998, p. 84)

The alignment of the issue of optical illusions, later formally described by Wittgenstein as visual instances of aspect seeing - the experience of seeing something as something else e.g. Jastrow’s duck rabbit illustration – with the concept of a primary and secondary language is of immense significance, and supports the analysis I present. *Philosophical Remarks* represents Wittgenstein’s middle period marking the transition from his early to later work. These remarks clearly indicate that Wittgenstein did adhere to a very particular form of logical atomism in the *Tractatus* which represents as I have argued a closed logical model. While in this remark he rejects the Tractarian position that such a language exists, he crucially aligns the possibility of a primary language (a logical language) as a means of neutralising the problem of nonsense – later classified as aspect seeing - as presented in optical illusions noting that such expressions do not necessarily indicate an error or mistake, and that moreover they could be neutralised by an “impartial” primary language as he envisaged in the *Tractatus*.

In terms of how the Necker cube relates to Russell, Wittgenstein’s central criticism of Russell had been his understanding of logical form or the logical foundations of language. His specific criticisms of type theory and his infamous paradox consisted in denying language the possibility of referring to itself. Therefore acquaintance with logical objects – and as such logical experience itself - is according to Wittgenstein impossible.

While he thought our knowledge of them (propositions of logic) is independent of knowledge of any specific empirical facts, he also held that all knowledge was knowledge either by acquaintance or by description (and the latter reducible on analysis to the former). So he held that logical knowledge involved acquaintance with logical objects or as he sometimes put it 'logical experience'. (Hacker 1996, p. 15)

Russell's open logical model of type theory can be considered in the context of the later Wittgenstein's work on aspect seeing. Unlike Wittgenstein's closed logical model, the theory of types as an open logical model, allows for the evolution of logical assumptions. Inherent in this possibility, the ability to see a logical model or concept in a new way or under a new aspect is essential. Here the idea of being acquainted with logical objects assumes an entirely different character if we consider the experience of logical objects as a *linguistic experience* and not a metaphysical or meta-logical one as Wittgenstein does in the Tractatus:

5.552 The "experience" which we need to understand logic is not that such and such is the case, but that something *is*; but that is no experience. Logic *precedes* every experience – that something is *so*. (Wittgenstein and Ogden 1999, p.86)

What the above remark reveals in a retrospective context is one of the fundamental flaws of the Tractatus, to the extent that Wittgenstein fails to question how he can assert and talk of that which he simultaneously claims cannot be spoken of. This position leads directly to the second central logical flaw which is that, as a closed logical model it demands a one dimensional view of logic and the visual field. The isomorphic logic of the Tractatus thus ensures that the visual field cannot have a

dual aspect or aspect switch, such as we see in the Necker cube, in Russell's paradox, or in Russell's type theory, which allows sets to be conceived under new aspects or types. For Wittgenstein Russell's theory allowed for the possibility of judging nonsense, precisely because logical form was one the constituents of the proposition to be judged – i.e. it is considered a part within the set and not an a priori condition outside it:

5.5422 The correct explanation of the form of the proposition 'A judges p' must show that it is impossible to judge a nonsense. (Russell's theory does not satisfy this requirement). (Wittgenstein and Ogden 1999, p.85)

Wittgenstein's observations regarding the Necker cube thus prove pivotal in two ways. Firstly his position regarding our perception of ambiguous figures, and his solution to the phenomena of "two possible ways of seeing the figure" is that we "really see two different facts". For the Tractarian Wittgenstein the experience in question is not one of seeing the same object or fact in two different ways – an experience later identified by Wittgenstein as aspect seeing - but rather of seeing two entirely different and crucially *independent* facts. If the possibility of aspect seeing was allowed to operate in the Tractatus it would necessitate the introduction of the concept of infinity, as there could be no logical limit to the possibility of aspect perception. The common shared pictorial form in all propositions of judgment - where the proposition is bivalent to what it refers – thus ensures for Wittgenstein that correct *public checks* are in place when judging the visual world.

Hintikka offers an account of Wittgenstein's treatment of the Necker cube in the Tractatus which recognises the problem of logical form in his rejection of Russell's theory of judgment:

Russell postulates two classes of objects of acquaintance: (i) the

concrete objects of acquaintance and (ii) the abstract objects of acquaintance viz. Logical forms. Now the philosophy of the Tractatus can be described quite simply. What Wittgenstein did was to reject the second class. (Hintikka 1986, p.37)

As Hintikka observes phenomenological objects must take precedence to logical objects for Wittgenstein and as such non referential language use which claims to quantify over logical objects is rejected. In effect the proposition must relate directly to a picture in the world which everyone can see and which can be assessed on the basis of this public criterion. However, Wittgenstein is clear that “3.03 one cannot think anything illogical, for otherwise we should have to think illogically. (Wittgenstein and Ogden 1999, p.35) With this assertion Wittgenstein allocates to nonsense statements a crucial *logical status*. The importance of this distinction between the possibility of nonsense propositions and the impossibility of illogical thoughts – and consequently illogical propositions - is crucial as in effect it means that while nonsense propositions and nonsense images are possible, and as such self reference and paradox, they are *also resolvable* within the closed logical model of the Tractatus, on the basis that they don’t refer to anything. It is in this sense that "Logic takes care of itself; all we have to do is to look and see how it does it. (Wittgenstein 1998, p. 11) However in the context of his later interest in ambiguous figures the limits the Tractatus sets to language, to sense and to pictorial representation would be radically reconfigured.

If language cannot by virtue of a priori necessity refer to itself and its own foundations, without the intractable problem of self reference emerging, then logical form and the foundations of logic as such, must be framed within an entirely new logical model which ensures that it is not treated as a content of any sort. In ensuring the new logical model dissolves the problem of self reference and paradox in both language and the visual field, the necessary framework must be an

entirely closed one and of necessity a limitative one, devoid of all and any metaphysical or meta-logical content. As he remarks:

The limit can therefore only be drawn in language and what lies on the other side of the limit will simply be nonsense. (Wittgenstein and Ogden, 1999, p. 27)

Logical efforts to refer to what is on the other side of language, now represent for Wittgenstein “4.126 the whole of old logic” (Wittgenstein and Ogden 1999, p.55) and are to be replaced with the new logic of showing – albeit this is problematic regarding both the assumed methodologically and conclusions i.e. the discovery of atomic propositions within a matrix of primary language:

4.126 That anything falls under a formal concept as an object belonging to it, cannot be expressed by a proposition. But it is shown in the symbol for the object itself. (The name shows that it signifies an object, the numerical sign that it signifies a number etc.)(Wittgenstein and Ogden 1999, p.55)

It is in this sense that the ladder of the Tractatus represents the efforts of non referential language use to range over itself in order to gain a perspicuous view of its own logical foundations. As this is impossible according to Wittgenstein, the ladder of non referential logical language as traditionally employed must be abandoned. The essential argument of the Tractarian closed logical model can now be considered as the entirely simple one that “the name shows that it signifies an object, the numerical sign that it signifies a number”. Wittgenstein’s conception of elementary or atomic propositions within the Tractatus thus reveal a closed logical model, ensured by his rejection of the infinite, all open logical models which

attempt to accommodate it, and his rejection of the visual field as infinitely complex:

1.12 The world is determined by the facts, and by these being all the facts.

For the totality of facts determines both what is the case and also all that is not the case.

4.51 Suppose *all* elementary propositions were given me: then we can simply ask: what propositions I can build out of them. And these are *all* propositions, and *so* they are limited. (Wittgenstein and Ogden 1999, pp. 29-65)

If the world of facts is limited, and if pictorial representation must follow an isomorphic logic such that every proposition must refer in an isomorphic and geometric manner to the state of affairs it asserts, instances of ambiguous perception, self reference and paradox as illustrated by the Necker cube are accountable for, but significantly they are also banned as nonsense if we regard them as any more than he states regarding the Necker cube – i.e. they are two independent facts. This in turn places a logical restriction on what Wittgenstein would later refer to as the experience of seeing as or aspect seeing. The central reason why it was essential to remove the problem of paradox from the visual field relates directly to his project of logical atomism which he believed could use the logic of showing to reveal– not say – what the simplest atomic structure of a proposition was. This project demanded that facts in the visual field – the totality of objects – were independent of one another. Resolving the problem of the infinite, thus required liberating logic from the recursive loop of self reference which the paradoxes had created, and Wittgenstein believed the Tractatus did just

that. Rather than a system of unrelenting complexity as in Russell's theory of types - an infinite and open set - simplicity was tantamount.

Rather for Wittgenstein the logical problems with which logicism and type theory are concerned are essentially problems of our own creation, and are specifically the result of attempting to go beyond the limits of language or the limits of what logic shows, with the crucial understanding that what is shown cannot in turn be represented or reinterpreted by a proposition in an objective manner:

4.121 Propositions cannot represent the logical form: this mirrors itself in the propositions.

4.1212 What can be shown cannot be said

5.552 Logic precedes every experience – that something is *so*. It is before the How not before the What. (Wittgenstein and Ogden 1999, pp.53-86)

Wittgenstein remarks towards the conclusion of the *Tractatus* that “6.53 The right method of philosophy would be this. To say nothing except what can be said, i.e. the propositions of natural science.” (Wittgenstein and Ogden 1999, p.107) This of course leaves the traditional process of logical analysis and the essential non referential use of language therein in an impossible position.

Russell's reflections on this impossible position which the Tractarian closed model of logic heralded as a new dawn in logic are revealed in the following remark:

He himself, as usual, is ocular and emits his opinion as if it were a Czars ukase, but humbler folk can hardly content themselves with this procedure. (pg. 118 MPD)

This section has revealed that parallel and intrinsic to this early understanding of the logical position of a primary and secondary language is the logical status which Wittgenstein allocates to the concept of the infinite and the generated problem of Russell's paradox which is addressed in a very flippant manner in the Tractatus. In the closed model of the Tractatus it is essential that both the concept of the infinite and Russell's paradox must disappear and with it the problems of self reference and contradiction. His claim in the Tractatus that within his closed logical model of showing Russell's paradox vanishes as "3.332 No proposition can say anything about itself. 3.333 Herewith Russell's paradox vanishes. (Wittgenstein and Ogden 1999, p.42) is I have shown directly rooted in his rejection of the infinite.

That the propositional sign cannot be contained in itself, in effect states that language cannot refer to itself without the problem of self reference emerging. That a function cannot be its own argument is again a grammatical and logical rule which the logic of showing sets down as essential, as if a function is allowed to be its own argument, linguistic self reference and self reflexivity has to be accepted as logically tenable. For the early Wittgenstein there is only one way to approach logic if self reference is to be avoided, and that is through the logic of showing, not the logic of saying via non referential language use:

4.121 Propositions cannot represent
the logical form: this mirrors itself
in the propositions.

That which mirrors itself in
language, language cannot represent.

That which expresses itself in
language, we cannot express by
language. (Wittgenstein and Ogden
1999, p.53)

In conclusion Wittgenstein utilises the concepts of the infinite and the related problem of paradox and self reference in a negative and limitative way to guarantee their disappearance from the field of logic, and to thereby ensure that the

Tractatus is a closed logical model. The significance of these influences in the Tractarian closed logical model, is in turn utilised to ensure that neither language nor logic operates according to a dual aspect. Logical form or logical foundations are revealed in one way – through the logic of showing – and this cannot be reinterpreted via the non referential use of language to say what he contends can only be shown. The revelation of logical form in one way only is ‘ensured’ by Wittgenstein’s insistence that logic terminates with the logical bolt of elementary or atomic propositions which limits both language, the visual field. Most importantly it denies aspect seeing any logical status outside of the observation that the dual aspect of the Necker cube illusion can be accounted for by asserting we are seeing two different facts which are in turn rooted in the terminus of elementary or atomic propositions, as opposed to seeing one fact under a different or new aspect. Wittgenstein’s rejection of traditional non referential use of language in the process of logical analysis ultimately renders this process obsolete – *if* we accept his account of language and logic. In accepting Wittgenstein’s position we accept that having elucidated the problems of philosophical analysis in their entirety we are left with a zero method which shackles logical analysis to the framework of tautology and contradiction as limiting devices:

4.112 A philosophical work consists of essentially of elucidations.
(Wittgenstein and Ogden, 1999, p.52)

6.121 The propositions of logic demonstrate the logical properties of propositions, by combining them into propositions which say nothing. This could be called a zero method.
(Wittgenstein and Ogden, 1999, p.94)

Here the framework of showing assumes dominance over language itself determining that any efforts to move beyond what language shows us is nothing more than an infinite self reflexive loop of mere nonsense. Retrospectively considered Wittgenstein’s early efforts to contain the infinite within the closed

logical model by means of banishing it was logically impossible. Even though Wittgenstein rejected the infinite, and as such does not believe he is accommodating it, the infinite remains structurally present in the secondary use and meaning of language by means of which he has generated the closed logical model in the first instance. As the later Wittgenstein identifies the infinite as central to the language game of aspect seeing in the context of the activity of logical analysis, trying to define or exclude it within a given logical model is impossible. Rather than a logically tenable closed logical model, what we do encounter is the illustration of both aspect perception and an impossible object which I suggest is inherent in the Tractarian concept of logical form. I suggest that logical form or the logical foundations of language which assumes the paradoxical position that while it can be shown it cannot be spoken of, gravitates on the very duality of an aspect switch which the Tractatus expressly rejects. As illustrated we encounter this perplexing concept of logical form, as something which is shown but cannot be spoken of. Like all impossible objects or paradoxes its local geometry makes sense but when considered as a totality it is counter intuitive and for that very reason becomes a source of bafflement and fascination. The reason Wittgenstein could not complete the closed logical model in the manner he envisaged – i.e. by discovering atomic or elementary propositions – is simply because he failed to see that he was playing a language game which was itself infinite and therefore impossible to close. What remains of significance regarding the concept of logical form is the aspect switch which is inherent in its sense – allowing us to consider it in one way, and then another but not both simultaneously in a similar manner to Russell’s paradox. However it is only in its retrospective context that this logical flaw can be identified. Wittgenstein’s rejection of the concept of the infinite as a logical concept unworthy of serious consideration, and its use in turn to banish Russell’s paradox and with it the problem of self reference, retrospectively considered proves inherently flawed. His comment from the Investigations casts an insightful retrospective eye on this fundamental issue:

If I say I have locked the man up in the room – there is only one door left open – then I simply haven't locked him up at all; his being locked up is a sham...An enclosure with a hole in it is as good as none. (Wittgenstein 2009, p.50)

The colour exclusion problem raised by Frank Ramsey would prove the means by which this inherent flaw became explicit, and the means by which the problem of the infinite, Russell's paradox and self reference would escape from the unsecured enclosure of the Tractatus.

Chapter Four

The beginnings of a logical self reflexive loop within Wittgenstein's work

4.1 How the Failure and Rejection of the Closed Logical model of the Tractatus

Orientated Wittgenstein logic back to the infinite

If the closed logical model of showing was to hold there had to be *a reason* why this isomorphic relation between language and the world was correct. The reason it was correct in Wittgenstein's view was ultimately based on his belief at the time that a uniform logic – primary language consisting of elementary or atomic propositions - lay behind secondary or ordinary language, accounting for its ability to represent the world. However misguided, Wittgenstein's ambition was not to articulate or say what this primary language was but to somehow show it – hence avoiding the problem of self reference. Clearly this approach is in itself flawed and indeed paradoxical. We can of course appreciate the logical orientation and sentiment of the Tractatus regarding logical form – we can linguistically see what Wittgenstein means. However what can we really do with such a logical model? How can it be applied in any meaningful way? The truthful answer is that the model does not have any applicative potential and is essentially redundant unless we also subscribe to the views it sets forth. What Wittgenstein failed to realise is that no logician has a monopoly over language, not even one as ingenious as Wittgenstein.

The closed logical model of showing operated like a pedagogical tool meant to show logicians that their metaphysics and conceptual system building was based on a misunderstanding of the logic of language. The dramatic change which sees Wittgenstein abandoning this view is clear in the Blue and Brown Books. The new emphasis Wittgenstein places on pedagogical concerns has become central, indicating that his teaching experience post the Tractatus certainly had a profound influence upon his later thought. These reflections on being held captive by a

certain picture of language can be considered in the context of his early logical position on the primary secondary language distinction:

When we are worried about the nature of thinking, the puzzlement which we wrongly interpret to be about the nature of a medium is a puzzlement caused by the mystifying use of our language. This kind of mistake recurs again and again in philosophy; e.g. when we are puzzled about the nature of time, when time seems to us a *queer thing*. We are most strongly tempted to think that here are things hidden, something we can see from the outside but which we can't look into. (Wittgenstein 1965, p. 6)

Ian Proops highlights Wittgenstein's retrospective criticism of the Tractatus in the *New Wittgenstein – A Critique*, where he quotes the following from a lecture given by Wittgenstein:

If you look at Russell and at the Tractatus you may notice something very queer – i.e. a lack of examples. They talk of individuals and atomic propositions but give no examples. Both of us in different ways pushed the issue of examples to one side. (Proops 2001, p.392)

His idea that language concealed a logical matrix generated a picture which positioned secondary or ordinary language as the outer and its underlying primary language as something hidden from view which the logic of showing could bring to light is now firmly rejected. This position led Wittgenstein to some key false conclusions in the Tractatus. Firstly a primary language of showing was the object of analysis and secondly Wittgenstein maintained that accessing this logic could be achieved by looking at – or into – language itself – all justified by the fact that

logical form was not being referred to but revealed through the logic of showing. Of course ironically none of this logic could exist without the use of non referential language.

The true death knell for the closed logical model of the Tractatus originates with Frank Ramsey's criticism of the colour exclusion problem, to which his 1929 paper *Some Remarks on Logical Form* was a response. Wittgenstein was fundamentally unhappy with the logical argument – or lack thereof – in this final effort to salvage the Tractatus. While it has the “distinction of being the only piece of philosophical writing he published after the Tractatus” (Monk 1991, p.272) and was originally intended to be delivered at the 1929 Annual Joint Session of the Aristotelian Society and Mind Association in Nottingham, this would never materialise. The reason why Wittgenstein never delivered *Some Remarks on Logical form* is a significant one. What is even more compelling and intriguing however is the paper he chose to deliver in its place:

It is a mark of how quickly his thought was developing at this time, however, that almost as soon as he had sent it off to be printed he disowned it as worthless, and at the meeting of which it supposedly forms part of the proceedings, read something quite different - a paper on the concept of infinity in mathematics, which has, consequently, been lost to posterity. (Monk 1991, p. 273)

Some Remarks on logical form “can be seen as an attempt to answer these criticisms” of Ramsey regarding the colour exclusion problem. (Monk 1991, p .273) Monk observes that “In proposition 6.375 of the Tractatus Wittgenstein had insisted: ‘Just as the only necessity that exists is logical necessity, so too the only impossibility that exists is logical impossibility’”. Wittgenstein “had gone on in the

following proposition to apply this to the impossibility of something's being say both red and blue.”(Monk 1991, p.273):

6.3751 For two colours e.g. to be at one place in the visual field, is impossible, logically impossible, for it is excluded by the logical structure of colour (Wittgenstein and Ogden 1999, p.104)

As Monk points out:

The problem here is that, if this is so, then the statement ‘This is red’ cannot be an atomic proposition. In the Tractatus it is claimed that atomic propositions are logically independent of one another, with ‘This is red’ quite clearly not being independent of ‘This is blue’: the truth of one implies the falsehood of the other. (Monk 1991, p.273)

What is logically significant about the colour exclusion problem is the similarity it shares with Russell’s paradox and the Necker cube, in so far as the truth of one proposition implies the falsehood of the other, revealing an inherent self reflexivity which could not be logically aligned to the idea that atomic or elementary propositions were independent of each other. While colour is clearly not an ambiguous figure or a paradox, it is Wittgenstein’s restriction of language which is the shared problem. Just as Russell’s paradox and the Necker cube exist only by virtue of self reference between two conflicting propositions about the same content, so too colour exists by virtue of the fact that one colour is discriminated against another, through self reference amongst all colours. Just as paradox is dependent on self reference to a contradictory proposition in order to be a paradox, so too colour is dependent on self reference to all colour in order to be individuated as a colour. There is it seems a logical vicious circle at the heart of the closed

logical model. Wittgenstein realised that the colour exclusion problem conclusively demonstrated that the logic of the Tractatus itself was internally inconsistent. His masterpiece was now subject to the same criticism which he had directed at Russell and was indeed a reality which was difficult to face and impossible to overcome within the confines of the Tractatus. In Some remarks on logical form Wittgenstein recognised that the truth of one statement of colour implied the falsehood of another – thereby indicating that such statements are not logically independent of one another – but rather *dependent* on one another. In his later work on aspect seeing this crucial logical insight finds expression in Wittgenstein's presentation of language and language games as being interdependent as opposed to independent, replacing this austere and ultimately unworkable logical model which the colour exclusion problem had revealed. In this sense language games in his later work are presented as entities which only acquire logical sense via their self reflexive reference to other language games, in a manner similar to the individuated concept of colour. At this point however the logical bolt of the closed logical model had become entirely undone. The potential solution he presents in that paper is numerical ordering – which in effect constitutes putting in place a logical notation akin to type theory and very similar to Gödel numbering. It was clear to Wittgenstein that the vicious circle which had emerged within the heart of the Tractatus could not be remedied by any such numerical addition, as this would in turn have to be justified. In his 1929 paper Wittgenstein addresses the colour exclusion problem:

The propositions, "Brown now sits in this chair" and "Jones now sits in this chair" each, in a sense, try to set their subject term on the chair. But the logical product of these propositions will put them both there at once. and this leads to a collision, a mutual exclusion of these terms. ...but this is nonsense, as the top line, "T T F," gives the proposition a greater logical multiplicity than that of the actual possibilities. (Wittgenstein 1929)

In so far as a greater logical multiplicity emerges, the parallel between Russell's position regarding the greatest cardinal number and its generation of the paradox is clear and insightful. Just as Russell was, Wittgenstein was now faced with a problem which involved both the infinite – in so far as the problem generated a greater multiplicity – and self reference. He concludes that it is “a deficiency of our notation that it does not prevent the formation of such nonsensical constructions, and a perfect notation will have to exclude such structures by definite rules of syntax.” However such a perfect notation remains entirely absent:

Such rules, however, cannot be laid down until we have actually reached the ultimate analysis of the phenomena in question. This, as we all know has not yet been achieved.(Wittgenstein 1929)

The problems inherent in his own work were no different than the problems inherent in Russell's of which Wittgenstein had been a vocal critic. That his concerns now turned to mathematics and to the problem of infinity is a reflection of how deeply the problem of the infinite had influenced the logic which had now fallen apart. Abandoning the position of the Tractatus was further “prompted by his discussions with Sraffa” (Monk 1991, p.274) accepting that “the idea that there had to be a commonality of structure between the world and language” was logically untenable. The point at which he abandons the closed logical model of showing is also as Monk observes:

the point at which he decided he could not read this paper before the conference. For the paper does not present the solution to the problem raised by Ramsey so much as an admission that, within the terms of the Tractatus Wittgenstein had no solution.(Monk 1991, p. 274)

The discussion to which Monk refers in the above concerns a conversation between Wittgenstein and Sraffa – an Italian economist -in which:

Wittgenstein insisted that a proposition and that which it describes must have the same 'logical form'. To this idea Sraffa made a Neapolitan gesture of brushing his chin with his fingertips, asking: 'What is the logical form of that?' This according to the story broke the hold on Wittgenstein of the Tractarian idea that a proposition 'must' be a picture of the reality it describes.(Monk 1991, p.262)

As Monk remarks this conversation “goes some way to explain why Sraffa is credited as having such an important influence” and why he is therefore acknowledged in the preface to the *Philosophical Investigations*. For Wittgenstein the solution to the problems of the *Tractatus* proved not a metaphysical one but a rather mundane and decidedly un-metaphysical one - the human acquisition of a language. The flaw in Wittgenstein's logic was the result of an overly mechanised conception of language which he would later recognise.

The idea that the logic of showing could reveal an implicit logical structure of “primary language” which was concealed by “secondary” or ordinary language thus proved just as flawed as the logic which the *Tractatus* rejected. Wittgenstein's careful positioning of logical form outside of the bounds of language had seemed to him to be a panacea to the problems of paradox which had plagued Russell's theory of types and been the cause of the crisis in mathematics. However as atomic propositions are essentially contained in or hidden by ordinary language, trying to ‘find’ them involves creating a logical language albeit Wittgenstein believed quite paradoxically that the logic of showing was not involved in such a process:

The *Tractatus* as he remarked to Elizabeth Anscombe, is not all wrong:

it is not like a bag of junk professing to be a clock, but like a clock that does not tell the right time. (Hacker 2001)

After completing the *Tractatus* Wittgenstein trained as a primary school teacher and took up a post in a small rural school south of Vienna at this point satisfied that the problems of paradox had been definitively addressed by the *Tractatus*. This period would end badly for Wittgenstein after accusations of corporal punishment against students were made. (Monk 1991, p.193) A recent publication *Showing and Doing – Wittgenstein as a Pedagogical Philosopher* (Michael Peters, N. Burbules and P. Smeyers) argues the case that Wittgenstein's teacher training and his experience of teaching exerted a fundamental influence on his later work on aspect seeing. The arguments these scholars present focus specifically on C.J.B Macmillan's "argument about Wittgenstein's pedagogical turn" (Peters et al 2008, p.2):

We often find him turning from a consideration of the meanings of a term or concept to ask 'How was this learned?' or 'How would you teach it?' (Macmillan 1984) (Peters et al 2008, p.2)

After abandoning his career as a primary school teacher Wittgenstein returned to Vienna in 1927 bringing him back into the ongoing debates on the foundations of mathematics. Here he met with a group of academics which would later evolve into the Vienna Circle to discuss "the foundations of mathematics and science" (Monk 1991, p.242). Through an encounter with Brower his thought would soon be forced to confront some of the certainties which he held in the *Tractatus*:

In March 1928 Brower came to Vienna to deliver a lecture entitled 'Mathematics, Science and Language'

which Wittgenstein attended with Waismann and Feigl. (Monk 1991, p.249)

This lecture was to have a profound effect on Wittgenstein. According to Feigl:

It was fascinating to behold the change that came over Wittgenstein that evening...he become extremely voluble and began sketching ideas that were the beginnings of his later writings...that evening marked the return of Wittgenstein to strong philosophical interest and activities.(Monk 1991, p. 249)

As Monk makes clear these remarks do not mean that Wittgenstein was suddenly converted to Brouwer's theory of intuitionism, albeit there certainly were fundamental points on which he agreed with Brouwer. Like Brouwer Wittgenstein "rejected the idea that mathematics either could or needed to be grounded in logic" in addition to rejecting the "notion that consistency proofs were essential in mathematics." (Monk 1991, p.250) Wittgenstein shared in common with Brouwer a rejection of the objectivity' of mathematics in the sense that it is usually understood – i.e. for Brouwer there is no mind independent mathematical reality about which mathematicians make discoveries:

The mathematician on Brouwer's view is not a discoverer but a creator: mathematics is not a body of facts but a construction of the human mind. (Monk 1991, p.250)

What is clear however is that Brouwer's lecture compelled Wittgenstein to return to logical analysis, a return which would see focus directed against his own magnum opus the Tractatus:

Wittgenstein was not interested in reconstructing mathematics; his interest lay in extracting the philosophical root from which confusion about mathematics grew. (Monk 1991, p.260)

Despite Ramsey's criticisms he admired Wittgenstein, describing him as a "philosophic genius of a different order" and also singling out his focus on infinity as a potential solution to the flawed logic of the *Tractatus* as particularly significant:

He began with certain questions in the analysis of propositions which have now led him to problems about infinity which lie at the root of current controversies on the foundations of Mathematics. (Monk 1991, p.271)

At this point in 1929 Wittgenstein was now categorical in his view that the logic of the *Tractatus* was inconsistent:

Once I wrote [TLP 2.1512] 'A proposition is laid against reality like a ruler....' I now prefer to say that a system of propositions is laid against reality like a ruler. What I mean by this is the following. If I lay a ruler against a spatial object, I lay all the graduating lines against it at the same time. (Monk 1991, p.285)

What is crucial about these observations is that they signal an entire rejection of the uniform homogenous approach to the logic of language as presented in the *Tractatus*, resulting in Wittgenstein abandoning his view that *all language operates in the same way*. In laying "all the graduating lines" against reality we in effect lay all the different ways of seeing against reality. Any logical analysis of language

would now have to take account of this non uniformity, which the logic of the Tractatus had failed to do.

The concept of infinity was central in Wittgenstein's mind during this period of intellectual and logical change. A final crucial event which was a direct result of the crisis in mathematics was to occur in the summer of 1930 which would have a profound effect on the evolution already underway in Wittgenstein's logic. This final event was the unveiling of Gödel's incompleteness theorem to the world at a conference on the theory of knowledge. Waismann had intended to read a paper on mathematics prepared with Wittgenstein. However this was overshadowed by Gödel's famous incompleteness theorem. (Monk 1991)

His writings after this period are the first time Wittgenstein addresses in an explicit and formal manner the concept of aspect seeing – which would later become the central concern of his work. It is in *Philosophical Grammar* that the concept of aspect seeing first makes its appearance and this significantly is in the context of mathematical problems:

We don't see that something can be looked at in a certain way until it is so looked at.

We don't see that an aspect is possible until it is there.

We might have operated a calculus with cubes without having had the idea of putting them together to make prisms. (Wittgenstein 2005, p.445)

Monk notes that it was precisely the “philosophical fog” surrounding the emerging mathematical contradictions or paradoxes which “drew him into philosophy in the first place” (Monk 1991, p.306) However the “philosophical fog” which remained after the failure of the Tractatus

Wittgenstein considered his work on mathematics as the most essential part of his work, in the sense that it was “an attempt to undermine the idea that mathematics needs foundations. (Monk 1991, p. 326) One of the first formal references to aspect seeing post the Tractatus and the Notebooks where it features in an implicit manner features, in Philosophical Grammar which was compiled between 1930 and 1932. Significantly Wittgenstein’s early formal references to aspect seeing occur in a mathematical context, one chapter entitled “Seeing or viewing a sign in a particular manner. Discovering an aspect of a mathematical expression. “Seeing an expression in a particular way.” (Wittgenstein 2005, p. 437) His early development of the concept of aspect seeing reveals a deep intellectual and logical struggle, a struggle which no doubt was also influenced by the mistakes he had made in the Tractatus:

Now you try and say what is involved in seeing something as something’, Wittgenstein challenged Drury; ‘it is not easy. These thoughts I am now working on are as hard as granite.’ (Monk 1991, p.537)

The above correspondence between Wittgenstein and Drury reveals the immense difficulties of expression which Wittgenstein faced as he began to formulate his ideas on aspect seeing. Wittgenstein died two years later:

During the remaining two years of his life, although he continued to write philosophy, he made no further attempt to restructure his book in the way that he had intended. Philosophical Investigations has therefore reached us in the somewhat transitory state in which it was left in the summer of 1949. (Monk 1991, p. 550)

Stylistically Wittgenstein would never again complete a work on a par with the Tractatus. All of his work after the Tractatus lacked the completion and polish so splendidly illustrated in that seminal work. This was not however a flaw or an indication of an intellectual height Wittgenstein failed to reach in his later work. Rather the very idea of logical analysis being bound as a complete internally consistent whole – a closed logical model - decreeing the final word on all of logic not only became an aim which was abhorrent to him but one which he came to regard as both logically impossible and undesirable. In the Tractatus Wittgenstein's writing style followed a methodology which was rigid and fixed devoid of any fluidity while his later work in contrast is far less confined and sterile. The remarks in his later work are now open ended in nature rather than closed and rigid. His approach now considers logical concepts as moving parts of a language game, and not as concepts which can lead to a set of timeless results of logic which the Tractatus had attempted to do.

The following is taken from Wittgenstein's Lectures from G.E. Moore around 1930 not long after he had realised the failure of the Tractatus, clearly illustrating that the solution to the failure of the Tractatus was considered by Wittgenstein to reside in the mathematical problems which the Tractatus had attempted to resolve:

...Plainly therefore, he thought that Sheffer, though he admitted that Sheffer had actually defined " p/q " as meaning " $\sim p. \sim q$ " had done something else. But what else? He said that Sheffer's "discovery" consisted in finding a **"new aspect" of certain expressions**. But I am sorry to say I did not and do not understand what he meant by this. (Wittgenstein 1993, p.92)

These early observations and explicit references to aspect seeing would find a more final and comprehensive expression in the Philosophical Investigations,

wherein the Tractatus serves as a constant retrospective reference point to the new logic of aspect seeing:

Tractatus Logico-Philosophicus (4.5)
“The general form of proposition is:
This is how things are.” – That is the
kind of proposition one repeats to
oneself countless times. One thinks
that one is tracing nature over and
over again, and one is merely tracing
round the frame through which we
look at it.(Wittgenstein 2009, p. 53)

The above remark highlights both themes of aspect blindness and aspect seeing which assume centre stage in his later work. In “tracing round the frame through which” one looks at something or sees it as something, there comes the unavoidable tendency to be held captive by a certain picture or way of looking at the subject matter in question. The key question is what type of logical model could adequately deal with the logical problems of the infinite and the generated problems of paradox and self reference. As I will show the successful logical model would prove to be the open ended logical model which Russell first pioneered in the theory of types. Wittgenstein’s initial recognition of this via his return to the concept of the infinite would ultimately generate a self reflexive feedback loop through the open logical model of aspect seeing, allowing both Wittgenstein and us a privileged insight into the workings of secondary language use and meaning as expressed in the logical journey he would now undertake.

*4.2 The inversion of the primary secondary language distinction considered
through Wittgenstein's changed logical position on non referential language use
in the process of logical analysis*

In the Philosophical Investigations Wittgenstein's position on the status of and relation between a primary and secondary language has changed radically, where the closed logical model of the Tractatus is replaced by what I contend is the open and infinite logical model of aspect seeing and language games. Here Wittgenstein's remarks on the significance of a primary and secondary language have evolved considerably, appearing in an inverted form, such that what would have been deemed absolute nonsense in the Tractatus now has a formal status, which is central to the language game of aspect seeing. In the following sequence of remarks from the Investigations which lead up to the introduction of the concept of a secondary use of language Wittgenstein firstly identifies the language game of aspect seeing:

But the question then remains why in connection with this game of experiencing a word, we also speak of 'the meaning' and of 'meaning it'. – This is a different kind of question. – It is a characteristic feature of this language game that in this situation we use the expression 'We pronounced the word with this meaning' and take this expression over from that other language game. Call it a dream it does not change anything. (Wittgenstein 2009, p.227)

Wittgenstein then provides an illustration of aspect seeing in a linguistic context using a non referential nonsense proposition to introduce the primary secondary distinction. Here the issues of paradox, self reference and contradiction are pronounced in a manner distinct from visual instances of aspect seeing, such that it is

directed towards how individuals see the public world differently in a linguistic context. His illustration expressing this difference of *seeing as* is an example situated within the public language game, where crucially there is no public criterion to verify or falsify what is said. That there is no criterion – unlike the isomorphic criterion of the Tractatus - is however no longer considered problematic but rather an intrinsic part of the language game of linguistic aspect seeing. Analogously the use of particular words in a non referential logical context – the word type or logical form for example - are initially encountered in non logical contexts and are clearly not initially acquired in the language game with the attached meaning and sense as used by either Russell or Wittgenstein. Despite this, Wittgenstein’s position indicates that using words in secondary ways allows logical analysis to create new uses and new language games by means of new logical models:

Given the two concepts ‘fat’ and ‘lean’ would you be inclined to say that Wednesday was fat and Tuesday lean or the other way round? (I am strongly inclined towards the former) Asked “What do you really mean here by ‘fat’ and ‘lean’? I could not explain the meanings in the usual way. I could not point them out by using Tuesday and Wednesday as examples. Here one might speak of a ‘primary’ and ‘secondary’ meaning of a word. Only someone for whom the word has the former meaning uses it in the latter. (Wittgenstein 2009, p. 227)

Here the positions of a primary and secondary language have been inverted from Wittgenstein’s previous understanding. As such ordinary language or ostensive language use now occupies the role of a primary language indicating a repositioning of logical form within the framework of language acquisition. Secondary language use and the language game of aspect seeing is in turn considered as evolving from this primary position, indicating that the set constituted by the language game of

aspect seeing is an open and infinite one – as aspect seeing cannot by definition be defined or limited. What has been referred to as Wittgenstein’s pedagogical turn serves to underpin and reflect the logical process operative in the inversion of the primary secondary distinction:

Only to someone who has learnt to calculate – on paper or out loud – can one render intelligible, by means of this concept of calculation, what calculating in the head is.

The secondary meaning is not a ‘metaphorical’ meaning. If I say ‘For me the vowel e is yellow’, I do not mean ‘yellow’ in a metaphorical meaning – for I could not express what I want to say in any other way than by means of the concept yellow.(Wittgenstein 2009, p.227)

His early reference to the primary secondary language distinction in Philosophical Remarks is clearly diametrically opposed to the manner in which it is presented in the Philosophical Investigations. The key to understanding the change in status of the primary secondary use of language and Wittgenstein’s later acknowledgment of secondary meaning as a legitimate use of language – as indeed an illustration of the language game of aspect seeing – rests on a revision of the infinite, the role of paradox, self reference and contradiction. This logical revision ultimately allows Wittgenstein to move from a position in the Tractatus which deemed all logical models using non referential language use as nonsense, to one in his later work on aspect seeing where these logical models which epitomised nonsense propositions and the limits of language, are formally accepted as logically valid uses of language.

A clear illustration of aspect seeing in its embryonic form occurs in the following extract from the Notebooks and would later feature as a central remark in the Tractatus itself. Here Wittgenstein reflects on how “Newtonian mechanics brings

the description of the world into a *unitary form*” noting that “Different systems of describing the world correspond to the different nets”. Crucially he points out that visual expressions of theory are also valid observing “One might also allow the net to consist of a variety of figures.” Indicating that logical form is shown and cannot be expressed, Wittgenstein remarks:

That a configuration like that mentioned above can be described by means of a net of a given form asserts nothing about the configuration (for this holds for any such configuration).” (Wittgenstein 1998, p.36)

Giving the example of Newtonian mechanics he thus observes:

It also asserts something about the world, that it can be described more simply by means of one mechanics than by means of another.(Wittgenstein 1998, p.36)

Here Wittgenstein’s early conceptual instincts regarding what would later be termed aspect seeing are clearly evident. Using the analogy of a net - evoking the Joycean metaphor of the nets of language - his identification of the different ways in which the world can be described by various systems of mechanics is I suggest an early forerunner of aspect seeing, constituting the different ways in which the world can be seen and the different ways in which meaning can be created by language. In this case however his remarks are to underpin his belief that the closed logical model of the Tractatus provides the simplest, most universal and comprehensive model of logic, unlike the open logical model of Russell with its dependence on the use of non referential language. The problem with Wittgenstein’s closed model is that it claims to have solved all logical and philosophical problems by condemning logical analysis to an eternal silence.

Clearly Wittgenstein's pronouncement was not going to achieve this. Logical analysis was not going to suddenly end, be tamed or defined by the limits the Tractatus imposed. If we cannot speak of logical form without talking mere nonsense then all efforts of logical analysis are futile and we ought to be compelled to follow the final words of the Tractatus "Whereof one cannot speak, thereof one must be silent."

Retrospectively however the Tractatus can be seen as Wittgenstein's own exemplar of aspect seeing and the secondary non referential use of language, wherein the aspects of logic he deals with are expressed under the aspects of showing versus saying within a closed and limitative logical model. While he believed that the Tractatus had solved what others had failed to remarking that, "The truths of the thoughts communicated here seem to me unassailable and definitive. I am, therefore, of the opinion that the problems have in essentials been finally solved." (Wittgenstein and Ogden 1999, p.28), he came to reject this position, a rejection which I contend hinged on a radical reconsideration of the problems of the infinite, paradox and self reference.

While the concept of aspect seeing was implicit in the logic of the Tractatus, it would take a mammoth re assessment of self reference and paradox, to make this concept an explicit and formal one. In this sense I suggest that while the concept of aspect seeing is implicit in the Tractatus this is only the case from a retrospective perspective. In contrast to Juliet Floyd of the resolute tradition, I maintain it is not the case that Wittgenstein was using the concept of aspect seeing in any formal or intentional sense in the Tractatus, as this was a logical system which he was neither aware of nor had developed at this point. In order to resolve the problems of the Tractatus Wittgenstein would have to return to the concept of the infinite and the generated problems of Russell's paradox and self reference. Retrospectively considered Russell's observations of a potential loophole in the closed logical model therein would prove accurate. In fact the solution had already been indicated by Russell in his introduction to the Tractatus where he observed:

These difficulties suggest to my mind some such possibility as this: that every language has as Mr. Wittgenstein says, a structure concerning which, in the language nothing can be said, but that there may be another language dealing with the structure of the first language, and having itself a new structure, and that to this hierarchy of languages there may be no limit. (Wittgenstein and Ogden 1999, p.23)

The logical repercussion of rejecting the infinite and the non referential use of language in the process of logical analysis was that Wittgenstein had created a closed logical model which was unable to accommodate the diversity of language use as revealed by the colour exclusion problem. It is only in returning to a reassessment of the infinite could this be rectified.

Chapter Five

The Second stage of Evolution in Wittgenstein's analysis of the Infinite

The beginnings of Aspect Seeing through a Reconsideration of the role of the infinite through the fractal connection

5.1 Accommodating the Infinite – Infinite reality versus infinite possibility

Fractal Seeing and Aspect Seeing on the Continuum

251 We find certain things about seeing puzzling because we do not find the whole business of seeing puzzling enough. (Wittgenstein 2009, p.224)

Aspect seeing is presented in the Philosophical Investigations as “the somewhat queer phenomenon of seeing this way or that”. (Wittgenstein 1980, p.8) standing in sharp contrast to the closed atomic visual field of the Tractatus. In all experiences of aspect seeing whether objective instances such as the duck rabbit or linguistic instances such as seeing e as yellow “the optical picture in one sense remains the same, while something else which one might call ‘conception’ changes”.(Wittgenstein 1980, p.8) The logical significance of aspect seeing can be summarised as follows:

One doesn't *take* what one knows as a knife and fork at a meal *for* a knife and fork. (pg.70 LWPPVI) What reaction am I interested in? The one that shows that someone takes a bowl for a bowl? Or the one that shows that he observes a change and yet shows at the same time that nothing has altered in his optical picture? When I contemplate the objects around me I am not conscious of there being such

a thing as a visual
conception.(Wittgenstein 1980, p.8)

Wittgenstein's transition from the austere closed logical model of the Tractatus to his open logical model of aspect seeing demanded a new critique and logical positioning of the concept of the infinite both in its theoretical and visual expression. His earlier notion of the atomic proposition, had served as the quintessential logical bolt which was to keep the closed logical model internally consistent. When Wittgenstein realised that logical positioning of atomic propositions, as being both independent of one another, and as being the internal limit of the closed logical model, had failed spectacularly as a result of Ramsey's criticisms, the one dimensional view of both language and the visual field which had originally led to the supposition of atomic propositions had to be revisited. This would demand a reanalysis of both the concept of the infinite and the generated concepts of paradox, self reference and contradiction. I will firstly consider Wittgenstein's re assessment of the role of the infinite.

In the following remark from Remarks on the Foundations of Mathematics we encounter Wittgenstein revisiting the complex spatial objects of the Tractatus. Unlike in the closed logical model these complex spatial objects are no longer banished but rather become essential in laying the foundation for Wittgenstein's later work on aspect seeing. In the following Wittgenstein highlights the internal self reference which occurs between the equation of a curve, and its visual counterpart – such as the Peano or Hilbert curve - as it appears on the number line or continuum. It is this internal self reference between a proposition or equation and that which it pictures, which Wittgenstein now identifies as a process which allows us to see the equation in a new way or under a new aspect. Retrospectively considered, the translation of one into the other, where one atomic proposition is now entirely dependent on another, would have been impossible in the confines of the Tractatus:

A new proof gives the proposition a place in a new system; here there is often a translation of one kind of operation into a quite different kind. As when we translate equations into curves. And then we realise something about curves, and by means of that about equations. (Wittgenstein 2001, p.368)

Central in reassessing the role of the infinite Wittgenstein relies heavily upon a decisive and consistent pedagogical strategy, making the following observation regarding the transformation of an equation into curve which grants language itself a dual aspect and which in turn allows us to see the given content in a dual aspect:

Our operations are not more remote from that object than is, say, dividing in the decimal system from sharing out nuts.(Wittgenstein 2001, p. 368)

This is most forcefully seen in the opening of the Investigations where he rejects the Augustinian picture of language and the referential model of language use it presents:

These words, it seems to me, give us a particular picture of the essence of human language. It is this: the words in language name objects – sentences are combinations of such names. – In this picture of language we find the roots of the following idea: Every word has a meaning. This meaning is correlated with the word. It is the object for which the word stands. (Wittgenstein 2009, p.1)

This referential model of language use is now rejected by Wittgenstein on the basis that “the philosophical notion of meaning” which it advances is based on a

“primitive idea of the way language functions” (Wittgenstein 2009, p.6) Such a model can only function within a “narrowly circumscribed area” (Wittgenstein 2009, p.6) which Wittgenstein clearly identifies as operative in the now redundant closed logical model of the Tractatus:

23 It is interesting to compare the diversity of the tools of language and the ways they are used, the diversity of kinds of word and sentence, with what logicians have said about the structure of language. (This includes the author of the Tractatus) (Wittgenstein 2009, p.15)

During Wittgenstein’s middle period we see both the infinite and Russell’s paradox occupy a central position as he grapples with the task of re constructing the logical ruins of the Tractatus. In Remarks on the Foundations of Mathematics, Philosophical Remarks and Philosophical Grammar the idea of aspect seeing can be seen to emerge from a reconsideration of complex infinite objects. Here the visual expression of the infinite is considered from the perspective of the continuum or number line, where fractal curves are considered as indicating new ways of seeing the continuum itself. This insight leads Wittgenstein to re consider the process of logical analysis itself, which is both inherent in mathematical application and in logical analysis. In the following Wittgenstein’s focus on the concept of an unlimited technique, anticipating what would later be formally expressed as language games, specifically the language game of aspect seeing:

To say that a technique is unlimited does not mean that it goes on without ever stopping – that it increases immeasurably, but that it lacks the institution of an end; that it is not finished off. As one may say of a playing field that is unlimited, when the rules of the game do not prescribe any boundaries – say by means of a line. (Wittgenstein 2001, p.138)

In stark opposition to the limits of the Tractatus, Wittgenstein now observes we are here dealing with “an unlimited technique” which “lacks the institution of an end”. (Wittgenstein 2001, p.138) While Wittgenstein notes that “following a rule is a human activity” he also now considers it possible to “give the rule an extension.”(Wittgenstein 2001, p.331), something which the closed logical model of the Tractatus had denounced as nonsense – specifically as seen in the work of Russell. In considering how we might see a rule in a new way or under a new aspect Wittgenstein explicitly focuses on the creation of mathematical fractal curves on the continuum or number line which allow a transformative process to ensue:

Might I say: See here if I follow the order I draw this line? Well in certain cases I shall say that. When for example I have constructed a curve according to an equation.
(Wittgenstein 2001, p.331)

Wittgenstein’s focus in the above centres on what he now refers to as the transformation of the continuum which is achieved by following the rule of a given equation. In such cases we are “following a rule of transformation”, which is considered as being no “more problematic than following the rule “keep on writing the same.”(Wittgenstein 2001, p.331) While Wittgenstein is still hostile to set theory, asserting that such objects do not prove that “visual space isn’t Euclidian”, he recognises that two different and opposing logical models can and do exist side by side:

Neither does the fact that a straight line drawn as a tangent to a circle gives the visual image of a straight line which for a stretch merges with the curve prove that our visual space

isn't Euclidian. (Wittgenstein 2001, p. 268)

Central to the transition from a closed to an open logical model and his return to the problem of the infinite was the question of whether or not language can quantify over the infinite domain:

Can the continuum be described? As Cantor and others tried to do. (Wittgenstein 1998, pg. 208)

In moving to answer this question fractal curves are employed as a response to the problem of quantifying over the infinite. Rather than rejecting the concept of the infinite in its totality, Wittgenstein's approach becomes more nuanced than it had been in the Tractatus. The construction of such fractal curves is now considered as an unlimited technique or process, which allows different representations of the continuum corresponding to two different ways of seeing the continuum. Firstly the continuum is described as the "whole material of observation" (Wittgenstein 1998, p.284) corresponding to which different fractal curves represent different ways of seeing the continuum:

The curve _____ is the actual course, so far as it is to be observed at all. The curves, ---, -.-.-, _._._, show different attempts to represent it that are based on a greater or lesser part of the whole material of observation. (Wittgenstein 1998, p.284)

Just as we have the number line or continuum in the process of mathematical analysis from which the problem of the infinite emerged

12345678910111213141516171819202122.....

So too we also have by analogy the number line of language in the process of logical analysis and its efforts to quantify over the former infinite domain.

A, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z,

Rather than seeing these as closed logical models, they now come to represent for Wittgenstein open logical models, signalling the key influence which his re assessment of the infinite exerts in his transitional period from a closed to open logical model. Just as infinite possibilities are inherent in the number line or the continuum, so too infinite possibilities of linguistic formation are possible by means of the alphabet.

Here the “whole material of observation” – the continuum – represents the primary way of seeing the line, in the same way as the acquisition of language and mathematical competency constitutes ones primary language, both of which are based on an original practice of rule following. On the continuum, the original line or continuum and the constructed curve self reference each other allowing the original or primary way of seeing the continuum to generate a secondary way of seeing via the curve’s construction. This insight would later be mirrored in the critical inversion of the primary secondary language model, such that primary language use and meaning allows for the generation of its secondary use and meaning.

The crucial element of the secondary way of seeing and in turn of representing the continuum is significantly rooted in a deviation from the original rules of representation. The visual phenomena of seeing the continuum according to different representations by means of different curves, which are constructed according to different equations or systems is considered as a way of “explaining a hypothesis by means of pictures.” (Wittgenstein 1998, p.284) Here Wittgenstein provides an example explaining the hypothesis “There is a book lying here” by way of different pictures which correspond to different ways of seeing the book. There

will be “pictures showing the book in plan, elevation, and various cross sections. Such a representation gives a law.” (Wittgenstein 1998, p.284)The phenomenon of infinite curves is thus considered in a parallel manner wherein “the equation of the curve gives a law, by means of which you may discover, the ordinates if you cut at different abscissa.” (Wittgenstein 1998, p.284)

If our experiences yield points lying on a straight line, the proposition that these experiences are *various views of a straight line* is an hypothesis.

The hypothesis is a way of representing this reality, for a new experience may tally with it or not, or possibly make it necessary to modify the hypothesis. (Wittgenstein 1998, p.285)

In the case of the continuum what has occurred is that we have through a given logical model, changed the way we see the line, generating a new picture or aspect parallel to a new method of representation or language game. In answer to the question “was defining this curve a piece of mathematics?” Wittgenstein answers that what is involved is more accurately described as the *creation of a new concept* – as opposed to the discovery of a metaphysical domain – which entails a deviation from strict rule following by means of the process of creating new rules such that we can assert “He did create a new concept.”(Wittgenstein 2005, p. 411);

Mathematics teaches us to operate with concepts in a new way. And hence it can be said to change the way we work with concepts. (Wittgenstein 2005, p.413)

Wittgenstein’s new approach to complex spatial objects generates a more much complex understanding of the infinite, signalling his realisation of the necessity of constructing an open logical model akin to Russell’s type theory as a means of

accommodating it. Thus the internal rules of the specific language game or system which considers the continuum in a certain way by seeing it in a new aspect – and as such seeing it in two opposing ways – is now granted a logical status, serving as a fundamental contrast to the Tractarian idea of closed and immutable elementary propositions. The most critical point which emerges from Wittgenstein’s reanalysis of the infinite is the new position he reaches via aligning this new perspective on the infinite –as seen through the construction of fractal curves - with the problem of paradox which it generates. He thus asserts that:

the geometrical process does not involve a vicious circle, since only an infinite possibility is presupposed by it, not an infinite reality.(Wittgenstein 1998, p. 285)

Most importantly, in identifying a qualitative distinction between infinite possibility and infinite reality, the contrast between the referential and non referential use of language, signals a new logical position regarding logical analysis itself. In seeing fractal curves as visual illustrations of seeing the continuum in different ways – relative to the curve which is created – the issue of non referential language use which the Tractatus banned as nonsense is now in an entirely new logical landscape. As such curves refer not to an “infinite reality” but rather constitute a rule of “infinite possibility” which consists in the ability of seeing the continuum and representing in different and opposing ways, the process which allows for this new way of seeing becomes more significant than the curves themselves, precisely because it reveals something new about language use within the process of logical analysis. Here language just as fractal curves continues to break off from a pre-established rule or pattern formation, delivering an infinite complex fractal on the one hand and on the other an infinite complex language game constituted by different logical models, which will in turn break off by the creation of new rules or new ways of seeing thereby allowing meaning and the process logical analysis itself to evolve in a similar fractal manner.

Now – unlike in the Tractatus - many different representations of what is seen are allowed to occupy the same visual and logical/linguistic space. Wittgenstein’s idea of infinite possibility providing a rule generator, as opposed to an isomorphic rule limit within the field of logical analysis and the non referential use of language, is further illustrated in the following. Here Wittgenstein observes that the continuum and its points can not only be seen as different curves, but as simply different coloured surfaces, highlighting the infinite possibility of the extension of rules in a general sense and the rule of infinite possibility specifically “Lines and points being given by the boundaries of coloured surfaces” (Wittgenstein 1998, p.219) This position is again reiterated in the following example of a fractal curve:

We can represent the equation of this curve:

A~~~~~
 ~~~~B

As the equation of a straight line with a variable parameter, whose course expresses the deviations from a straight line. It isn’t essential that these deviations should be ‘slight’. They can be so large that the *curve doesn’t look like a straight line at all*. ‘Straight line with deviations’ is *only one form* of description. It makes it possible for me to neglect a particular component of the description – if I so wish. (The form – ‘rule with exceptions’)

To say that the points yielded by this experiment distribute themselves around this curve – e.g. a straight line – means something like: seen from a certain distance, they appear to lie on a straight line.

If I state ‘That’s the rule’ that only has a sense as long as I have determined the maximum number of exceptions

I'll allow before knocking down the rule.

I can say of a curve -----^----  
that the general impression is one of a  
straight line, but not of the curve  
~^~^~^~^~^~

Even though it might be possible to see this stretch in the course of a long stretch of curve in which its divergence from a straight line would be swallowed up. (Wittgenstein 2005, p. 293)

The fact that we cannot see the continuum simultaneously as a straight line and as a space filling curve is a logical indication that the two pictures are generated by two different systems or logical models, allowing us to see the continuum in two alternate ways. The new position is that, just as we can look at the continuum in different ways, so too we can look at language itself in different ways, and as such the possibility of seeing the same content in different ways must be accommodated by language, a view which the Tractatus had ruled out as logically untenable. This insight which is the clear forerunner of aspect seeing would in turn inspire how Wittgenstein approached the related problems of paradox, self reference and contradiction. In focusing on the consistency question which had been at the core of the problems of logicism Wittgenstein now remarks:

Now if a case arose that a formula counted as having been proved on the basis of one method but refuted on the basis of another, then that wouldn't in the least imply that we now have a contradiction and are hopelessly lost; on the contrary we can say: the formula simply means different things. It belongs to two different calculi. In the one calculus it's proved, in the other refuted. (Wittgenstein 1998, p.344)

Wittgenstein thus asserts that the “whole series of confusions around the question of consistency”, can be accounted for by the fact that “we have two different formulas in front of us which by mere accident have their signs in common.”(Wittgenstein 1998, p.344). This ultimately amounts to an acceptance of the logical necessity of an open logical model which he had earlier dismissed as presented in Russell’s theory of type.

The idea that the infinite is somehow referred to by means of fractal curves, and that the true infinite is somehow ‘hidden’ in the line is perhaps a fascinating picture but one which Wittgenstein rejects as illogical. While this can be seen as a retention of the line of criticism that was present in the *Tractatus* regarding set theory, this is now situated in the new context of a distinction between a finite reality and an infinite possibility. As such this can be also considered as a retrospective meta-analysis and criticism of his own conviction that a primary language lay hidden within ordinary or secondary language. While the former distinction of an infinite reality is dismissed, the most crucial distinction of an infinite possibility is retained and now granted a logical status, contrasting sharply with his earlier vision of fixed and immutable elementary propositions. The concept of the infinite is therefore problematic *only* when we understand the idea of the infinite – considered as a fixed totality - as being hidden in the line, in the same sense as we understand an apple being hidden in a box – i.e. where of course there is the possibility of finding the object in question:

The curve exists independently of its individual points. This finds expression in the fact that I *construct* its highest point: that is derive it from a law and not by examining individual points. (Wittgenstein 1998, p. 209)

Rather than having discovered something which was hidden in a metaphysical sense, we have in the case of fractal curves “constructed” or created something according to a new rule which does not point to an infinite reality, but rather to the

infinite possibility of the language game of logical analysis itself. The crucial insight which Wittgenstein received from reconsidering the infinite via the fractal connection was that if visual perception has this logical dimension of self reference in the sense in which different logical models can construct different pictures of the same content, then language itself must also share this characteristic. The characteristic which he had now come to recognise as logically significant was that any logical analysis of language must take account of the open logical model inherent in the visual field, and moreover that the visual experience of aspect seeing itself signals the necessity of an open logical model. Retrospectively considered the influence of Russell's logical model of type theory which Wittgenstein had vehemently opposed as logically untenable can be seen to emerge in this second stage of the evolution of the infinite.

## *5.2 The Grammar of Infinite possibilities*

### *The role of pedagogical process as a methodological device in Wittgenstein's move from a closed to an open logical model*

Wittgenstein's analysis of the infinite in his middle period introduces the role of the pedagogical within the process of logical analysis itself, such that the infinite sets of Cantors paradise are no more mysterious or problematic than the Russian abacus. He thus remarks that, rather than disregarding the pedagogical, logic must look to it. (Wittgenstein 1998, p. 207) The pedagogical allows Wittgenstein to place a more fundamental logical weighting on the particular as opposed to the general or universal, which parallels the change from meaning as reference to meaning as creation of rules, a position antithetical to the closed logical model of the Tractatus. In this sense the use of non referential language use within the process of logical analysis begins to assume significance in the context of the infinite possibilities of language:

If in logic a question can be answered (1) generally and (2) in particular, the particular answer must always show itself to be a special case of the general answer; put differently: the general case must always include the particular as a possibility. (Wittgenstein 2005, p. 206)

Unlike the Tractatus wherein his closed model of logic operated by formulating the most general and all encompassing rules of limitation, Wittgenstein reverses his approach to allocate to the particular a key logical role. While retrospectively considered the Necker cube – operating as an illustration of the particular- was denied any logical status, the possibility of seeing as and of expressing such experiences in different and opposing ways is no longer considered logical Armageddon. The problems which had perplexed the early Wittgenstein and which resulted in denying any validity to the process of logical analysis itself in its efforts



to accommodate the particular, was based on an understanding of the infinite as a reality as opposed to a linguistic possibility:

Here again it is grammar which as always in the sphere of the infinite is playing tricks on us. (Wittgenstein 1998, p.208)

In transposing the logical principle of an infinite possibility to the process of logical analysis itself, Wittgenstein criticises the idea of finding the highest point of the curve, on the grounds that there is not a hidden highest point of infinity already existing and waiting to be found. Analogously there is not a final closed logical model which can quantify over logic and language itself, as he had once believed. Rather the logician decides the highest point by way of the rules of the equation which he has created, thereby exhibiting the infinite possibilities of grammar and language:

We say ‘the highest point of the curve’. But that cant mean ‘the highest point of all the points in the curve’ in the sense in which we talk of the largest of these three apples.

This is of course the same case as the one in which someone operates with the word ‘infinite’ as if it were a number word. (Wittgenstein 2005, p.209)

This radical reconsideration sees Wittgenstein reintroduce the concept of the infinite which the Tractatus had banished which now proves critical to restructuring the logic of the Tractatus and to critiquing his previous method of logical analysis. Through the invention of new ways of representation within the process of logical analysis we introduce and create new aspects, as opposed to referring to metalogical or metaphysical domains. New aspects and new ways of

seeing now begin to accommodate both the infinite and the non referential use of language. Here the intersection of logical models allows for the linguistic experience of aspect seeing, whereby through the process of logical analysis of a given logical model, a new aspect may be discovered:

Whoever invented calculation in the decimal notation surely made a mathematical discovery. But could he not have made this discovery all in Russellian symbols? He would so to speak have discovered a new *aspect*. (Wittgenstein 2001, p.177)

The discovery of a new aspect is now a conceptual and a linguistic discovery demanding both an understanding of rule following within different logical models, and the subsequent ability to create a new rule based logical model through the secondary use of language. Seeing another possible system within an existing one signals the introduction of “a new concept into our calculation. Here is a new aspect.”(Wittgenstein 2001, p.178) Wittgenstein highlights the apparent vagueness of the term aspect seeing noting:

The expression new aspect is vague. It means that we now look at the matter differently – but the question is: what is the essential, the important manifestation of this looking at it differently? (Wittgenstein 2001, p.180)

He continues to observe that it need not “have struck anyone that in certain products all the factors are equal”. (Wittgenstein 2001, p.180) The important manifestation of looking at something differently consists firstly in creating “a notation” where “I write  $a^2$  instead of ‘ $a \times a$ ’”, secondly in so doing Wittgenstein observes that one is thereby effectively “setting up a new connection”

(Wittgenstein2001,p.180):

But this means I refer to the series of numbers (allude to it) which did not happen before. So I am surely setting up a new connection! – A connection – between what objects? Between the technique of counting factors and the technique of multiplying”.

(Wittgenstein 2001, p.180)

In demoting the idea of metaphysical or logical objects to “different techniques” the concept of the metaphysical as a reality to which logic can refer, no longer has a logical fascination but is rather a part of a logical and linguistic language game. Unlike the Tractatus the idea of the metalogical or metaphysical is no longer considered a logical concept which must be banished on pain of talking nonsense. Self reflexively applied to Wittgenstein’s earlier work the very idea of “different techniques” as an acceptable logical model and process indicates clearly a complete break with the closed model of logic. Rather language accommodates the infinite possibility inherent in the language game. The language game of seeing things differently – aspect seeing – is thus concerned with “altering the aspect” of language and of therefore classifying “this expression with others, comparing it with others with which it was not compared before.”(Wittgenstein 2001, p.181)

The concept of aspect seeing is further expanded in Philosophical Remarks, indicating that diversity rather than uniformity – which had characterised the closed model of the Tractatus- is pivotal in the process of logical analysis:

If I see that a figure possesses an organization which previously I hadn’t noticed, I now see a different figure. Thus I can see IIIII as a special case of II II II or of III III or of I III I etc. This merely shows that that which we see isn’t as simple as it appears. (Wittgenstein 1998, p. 281)

Here: “understanding a Gregorian mode doesn’t mean getting used to the sequence of notes” rather “it means hearing something new, which I haven’t heard before, much in the same way – in fact it’s a complete analogy – as it would be if I were suddenly able to see 10 strokes IIIIIIIII, which I had hitherto only been able to see as twice five strokes, as a characteristic whole, or suddenly seeing the picture of a cube as 3 dimensional when I had previously only been able to see it as a flat pattern. (Wittgenstein 1998, .281)

The human tendency as illustrated here is to always seek out a pattern which persists in a given logical model be the model a visual pictorial series or a linguistic one. This tendency to privilege the iteration of a specific pattern or series is central to the language game of aspect seeing, where we experience this complex tendency to be held captive by a certain aspect or way of looking at something – as was Wittgenstein in the *Tractatus*. The fact that one individual has preference for a certain ordering of the concept in turn illustrates the tendency to be aspect blind to alternative ways of representation which deviate from a certain set of rules or pattern within a given logical model – as was the case in relation to Wittgenstein’s aspect blindness to the logical significance of Russell’s type theory.

In all such cases we have not discovered something hidden in language or hidden within the line analogous to discovery a physical object, but rather have a case of self reference between primary and secondary ways of seeing, and primary and secondary ways of using language and creating meaning. This generation and creation of new rules and new logical models, is now essential to the process of logical analysis, allowing us to appreciate the fundamental difference between Wittgenstein’s own linguistic experience of aspect seeing in his early and later periods.

Now the tension between closed and open logical models, is revealed in the tension and self reference between the concepts of infinite and finite, which is turn

reflected in the visual field and finally in the language game of linguistic aspect seeing. While this language game is an open and therefore infinite logical model – like type theory – we can only deal with each finite logical model one at a time not with the language game as a totality – just as in type theory, where its infinite open ended structure accommodates the finite structure of each type:

If I say that I can imagine a cylinder extended to infinity, that is already contained in its nature. So again, contained in the nature of the homogeneity of the cylinder and of the space in which it is – and the one of course presupposes the other – and this homogeneity is in the *finite* bit I see. (Wittgenstein 2005, p.160)

The concept of the infinite as used within the language game of aspect seeing thus becomes a grammatical rule, having of logical necessity endless or infinite application but crucially cannot itself be defined. It is in this way that the language game cannot be fixed or limited and as such no single logical model can define or limit any future potential process of logical analysis and secondary language use in the manner in which the Tractatus had. What is critical in a retrospective context is the clear similarity which this logical model shares with type theory as it also is an open logical model which cannot be defined or closed. Similarly logical models like types may be closed as individual entities or sets, but cannot determine either the hierarchy within type theory, or the process of logical analysis within the language game of linguistic aspect seeing or secondary language use. Aspect seeing as operative in the language game of logical analysis is thus concerned with the grammatical possibility of non referential language use, and not with a reality to which such models refer. In this sense the language game is a set which remains open and infinite due to the fact that the possibility of seeing as or seeing things differently is inherent in all such logical models:

And when (as in set theory) it tries to *express* their possibility, i.e. when it confuses them with their reality, we ought to cut it down to size.

*The infinite possibility in the symbol relates – i.e. refers – only to the essence of a finite extension, and this is its way of leaving its size open.*  
(Wittgenstein 2005, p.165)

While Wittgenstein levels this charge at set theory the same point is retrospectively equally applicable to his own closed logical model in the Tractatus. This fundamental insight would reappear in the Philosophical Investigations in relation to the secondary sense or use of language or the language game of aspect seeing which can in turn be considered as an infinite set. The secondary use of language cannot be defined and as such the language game cannot logically be limited, as language use within this language game is also dependent on any future possible way of seeing which cannot logically be limited or bound by a rule or system. This is once again re iterated in the following series of remarks where the concept of the infinite is presented as the possibility of expression within a given logical model generated by the process of logical analysis:

The infinite number series is only the infinite possibility of finite series of numbers. It is senseless to speak of the *whole* infinite number series, as if it too were an extension.

Corresponding to this is the fact that numbers – which of course are used to describe facts – are finite, whereas their possibility which corresponds with the *possibility* of facts is infinite. It finds expression as I've said in the *possibilities of the symbolism.*(Wittgenstein 2005, .164)

This remark signals clearly how the mathematical problem of the infinite functions as a catalyst allowing Wittgenstein to now only reassess his earlier position but to provide a new logical model where the possibilities of the symbolism of aspect seeing are infinite. The “possibilities of the symbolism” – of the language game of logical analysis or aspect seeing – stipulates a logical principle wherein representation of the same concept in different ways becomes essential. The possibility of representing lines and points by coloured boundaries for instance signals a new system or a new way of seeing lines and points, analogous to how representing the same concept by a logical model of different rules allows us *to see it in a new way*:

The generality of a Euclidian proof. We say, the demonstration is carried out for one triangle, but the proof holds for all triangles – or for an arbitrary triangle. And if I now measure the angles of a triangle and add them, I can’t in fact conclude that the sum of the angles in every other triangle will be the same. It is clear that the Euclidian proof *can say nothing about a totality of triangles*. A proof can’t go beyond itself. (Wittgenstein 2005, p. 152)

These views are further reflected in the Blue and Brown Books showing the evolution and retrospective significance of the mathematical concept of the infinite. In the Blue and Brown Books the crucial logical benchmark of an open logical model is now that “there is not one definitive class of features which characterise” (Wittgenstein 1965, p.19) our concepts. Using the concept of wishing as an illustration Wittgenstein remarks:

If on the other hand you wish to give a sharp definition of wishing, i.e. to draw a sharp boundary, then you are free to draw it as you like; and this boundary will never entirely coincide

with the actual usage, as this usage has no sharp boundary. The idea that in order to get clear about the meaning of a general term one has to find the common element in all its applications has shackled philosophical investigation; for it has not only led to no result, but also made the philosopher dismiss as irrelevant the concrete cases which alone could have helped him to understand the usage of the general term. (Wittgenstein 1965, p.19)

This evolution of the concept of the infinite allows Wittgenstein to now accept that while in the domain of language acquisition it is essential that an isomorphic relation between word and thing is established (e.g. an apple) creating a reference boundary, within the language game of logical analysis, it is essential that this is not the case (e.g. infinity). Consequently no logical boundary of either concepts or language game can be offered nor are they to be desired.

Logical form retrospectively considered, is now something we experience in the acquisition of a language, by means of which language evolves to its secondary use, rather than something we cannot logically speak of. Rather than the logical form of the Tractatus language acquisition – in taking its place – operates as the required ‘a priori’ condition required in order for anything to make sense – even that which the Tractatus had paradoxically asserted we cannot speak of. It is no longer a logical limit but a logical starting point. The logical relevance of the pedagogical world to the world of logical analysis is again reinforced in the following, where a logical model is considered as a world in its own right. In all such logical models, logical form is determined by the rules of the game, - retrospectively considering the Tractatus as an exemplar - which cannot be delimited from the perspective of logic in general or the system in general:

A system so to speak is a world.



A schoolboy equipped with the armoury of elementary trigonometry and asked to test the equation  $\sin x = x$  – etc. , simply wouldn't find what he needs to tackle the problem.

The system of rules determining a calculus thereby determines the 'meaning' of its signs too. Put more strictly: the form and the rules of syntax are equivalent. So if I change the rules – seemingly supplement them say – *then I change the form, the meaning.*

In mathematics we cannot talk of systems in general, but only *within* systems. (Wittgenstein 2005, p.179)

This crucial remark reflects the very point Russell offered in the introduction to the Tractatus highlighting just how significant his influence is. Moreover Wittgenstein's view that seeing an aspect and creating new rules is something which thus evolves through our experience of language is retrospectively evident in the process which we are presented with via his own meta-analysis of his earlier position which did try to condemn all logical analysis and all logical "systems in general" to an eternal silence. Without the ability to see as which is inherent in both a pedagogical and logical context Wittgenstein aspect blindness in the Tractatus can be considered analogously to the schoolboy in the above example who "couldn't merely not answer it, he couldn't even understand it. (It would be like the task the prince set the smith in the folk tale: Fetch me 'Fiddle-de-dee')" (Wittgenstein 2005, p.179)

For Wittgenstein "finding a system for solving problems which previously could only be solved one by one by separate methods isn't merely discovering a more convenient vehicle, but is something *completely new which we previously didn't have at all.*" (Wittgenstein 2005, p.182):

That is to say in my opinion, no way can be found in mathematics which isn't also a goal. You can't say: I already had all these results, now all I've done is find an even better way that leads to all of them. No: this way is a new place that we previously lacked. The new way amounts to a new system.

For only the group of rules defines the sense of our signs, and any alteration (e.g. supplementation) of the rules means an alteration of their sense.

Just as we can't alter the marks of a concept without altering the concept itself. (Frege)(Wittgenstein 2005, pp.182-183)

In this sense each new system within the language game of aspect seeing “is a new place that we previously lacked” which amounts to saying that “it is a new system”, which crucially has immense significance in terms of the validity of the process of logical analysis and the ability of language therein to create infinite new possibilities – possibilities denied to language in the Tractatus.

The crucial logical conclusion is that within the language game of aspect seeing it is illogical and impossible to single out any one logical model as holding a universal truth regarding and relative to all other logical models as Wittgenstein had claimed in the Tractatus:

Generality in mathematics is a direction, an arrow pointing along the series generated by an operation. And you can even say that the arrow points to infinity; but does that mean there is something – infinity – at which it points, as at a thing? Construed in that way, it must of course lead to endless nonsense.

A searchlight sends out light into infinite space and so illuminates everything in its direction, but you can't say it illuminates infinity. (Wittgenstein 2005, pp. 162-163)

While set theory can assert that “the arrow points to infinity” just as we can send “out light into infinite space”, it cannot define or limit the language game of aspect seeing, as in the language game of aspect seeing the “infinite is only in the rule” (Wittgenstein 2005, p.163) which amounts to saying that there are no rules. Similarly while Wittgenstein asserted that logic can only be shown and not spoken of, the illumination of the process of logical analysis within the closed logical model of the Tractatus did not illuminate a limited closed logical model as Wittgenstein had imagined. Retrospectively considered the above remark is applicable to the fact that Wittgenstein's isomorphic logic could not point to the imagined elementary propositions which would complete the closed logical model. Most importantly however in now firmly accepting that the “infinite is only in the rule” non referential language use now finally has a use and a crucial logical status.

### ***5.3 Replacing Elementary propositions with Infinite possibilities***

#### ***Non Referential Language use reconsidered through Rules of Reference and Rules of Invention in the process of Logical Analysis***

Wittgenstein's new position regarding the status of the infinite and its application to the non referential language use proves critical in both the development of the more formal concepts of language games and rule following, and in his later position on the process of logical analysis itself. Commenting on the process of logical analysis involved in the Hilbert curve, Wittgenstein contends that such points on the continuum indicate the construction or invention of a new logical model or calculus:

If someone were to describe the introduction of irrational numbers by saying he had discovered that between the rational points on a line there were yet more points, we would reply: '*Of course you haven't discovered new points between the old ones: you have constructed new points. So you have a new calculus before you.*'  
(Wittgenstein 1998, p. 339)

Ultimately it is the possibility of the open logical model of linguistic aspect seeing which allows for a new calculus or new logical model to be presented, crucial within which is the new free moving logical bolt of infinite possibilities as opposed to the fixed logical bolt of elementary propositions. Here we see Wittgenstein align this new logical bolt with the process of logical analysis within which the activity of seeing is singled out as being fundamental:

*It's a matter of seeing, not of proving.*  
No proposition corresponds to what I see – to the possibility of the system. Nothing is claimed and so neither is there anything I can prove.  
(Wittgenstein 1998, p.336)

In identifying the grammar of the infinite as being literally rooted in how we see the continuum in different ways through the construction of different infinite curves, and in turn aligning this with language use, Wittgenstein has reached a new logical perspective. In this new logical landscape his earlier logical position, that the picture of the infinite as a totality is impossible to represent, is now utilised by Wittgenstein to elucidate how the infinite operates in the grammatical rule of infinite possibility in the language game of aspect seeing. However the rule of infinite expansion as in the case of fractal curves can now be considered as a finite rule within the grammar of the language game – which allows us not to represent infinity but to create (in principle) an infinite number of systems – different ways of seeing - which is rooted in the possibilities of the game itself. He thus cautions that we must not forget that “mathematician’s discussions of the infinite are clearly finite discussions. By which I means they come to an end.”(Wittgenstein 1998, p.483) Here the activity of logical analysis is likened to the pedagogical process and is also presented as being derived from pedagogical practice, beginning with ones primary acquisition of a language both verbal and mathematical:

A number system is not something inferior – like a Russian abacus – that is only of interest to elementary schools, while the higher general discussion can afford to disregard it. (Wittgenstein 1998, p.207)

In using the pedagogical, which may seem at odds with the language game of logical analysis, Wittgenstein’s aim is to remove the metaphysical mystery surrounding the language game, such that it appears no more mysterious than “a Russian abacus”:

A proposition is completely logically analysed if its grammar is made completely clear: no matter what

idiom it may be written or expressed  
in. (Wittgenstein 1998, p.51)

More specifically the focus has been directed towards understanding the grammar of the infinite rather than banishing the concept in totality:

We have a grammatical class  
“infinite sequence” and equivalent  
with this expression a word whose  
grammar has (a certain) similarity  
with that of a numeral: “infinity” or  
“ $\aleph_0$ ”.(Wittgenstein 2001, p.136)

Of course the symbol for infinity was not discovered in any metaphysical sense but rather created to introduce a new rule, such that we now have a “grammatical class” as opposed to an “infinite class”. (Wittgenstein 2001, p.136)

Retrospectively these catalysing logical evolutions would find expression in the Investigations - specifically in relation to the process of logical analysis itself - where the movement from the fixed limitative logical bolt of elementary propositions is criticised using aspect seeing as a device of meta-analysis. In the Investigations he would thus remark “We see that what we call “proposition” “language” has not the formal unity that I imagined but is a family of structures more or less akin to one another.”(Wittgenstein 2009, p.51) In sacrificing formal unity he asks:

108 But what becomes of logic now?  
Its rigour seems to be giving way  
here. – But in that case doesn't logic  
altogether disappear? For how can  
logic lose its rigour? (Wittgenstein  
2009, p.51)

He retrospectively observes that what is removed in his new open logical model is not logic but “the preconception of crystalline purity” which his early approach was based upon:

107 The more closely we examine actual language the greater becomes the conflict between it and our requirement. (For the crystalline purity of logic was, of course not something I had discovered: it was a requirement)

108 The preconception of crystalline purity can only be removed by turning our whole enquiry around. (One might say: the enquiry must be turned around, put on the pivot of our real need.)(Wittgenstein 2009. P.51)

The misleading picture which had fascinated Wittgenstein - that of an ideal language which operates according to strict rules which are somehow hidden beneath secondary language – is now rejected in total allowing his new conception of the infinite to assume its place. Logic Wittgenstein retrospectively observes “does not treat of language or thought in the same sense in which natural science treats of natural phenomena” (Wittgenstein 2009, p.43) Rather than discovering anything, logic is now considered as an activity wherein “we construct ideal languages”. (Wittgenstein 2009, p.43) However the sense in which such languages are constructed is in stark contrast to the Tractarian understanding:

81 Here the word “ideal” is liable to mislead, for it sounds as if these languages were better more perfect than our everyday language; and as if it took a logician to show people at last what a proper sentence looks like.(Wittgenstein 2009, p.43)

It is only when we consider concepts such as “understanding, meaning something and thinking” in a new manner that it will “become clear what may mislead us (and did mislead me) into thinking that if anyone utters a sentence and means or understands it, he is thereby operating a calculus according to strict rules.”(Wittgenstein 2009, p.43)

The flaw is thus not in using the concept of the infinite or the concept of a totality but rather in failing to understand the grammar of the infinite, the nature of the language game being played, and the fact that language in such instances is not functioning in a referential sense. Most significant is Wittgenstein’s realisation that the non referential use of language as operative in the grammar of the infinite is essential to the process of logical analysis. The problem of the infinite as the catalyst for Russell’s paradox is now presented through a consideration of rule generation which changes according to how a given concept is used:

Now you can say, ‘A proposition cannot deal with all the numbers one by one, so it has to deal with them by means of the concept of ‘number’ as if this were a pis aller: ‘Because we can’t do it like this, we have to do it another way.’ But it’s not like that: of course it’s possible to deal with numbers one by one, but that doesn’t lead to the totality. For the totality is only given as a concept. (Wittgenstein 1998, p.147)

In his later work logical concepts thus operate on the basis of the new status which the infinite is allocated in his middle period, whereby logical concepts are granted a unique status in the language game of aspect seeing where they are considered in the following manner:

Operating with concepts permeates our life. I see some sort of analogy with a very general use of keys. If for instance one always had to open a lock



in order to move something around.  
(Wittgenstein 1982, p.51)

In this sense logical concepts become the lens through which linguistic aspect seeing or the secondary use and meaning of language within logical analysis itself operates – effectively allowing non referential language use the highest logical status it could be granted. In likening the concept of a logical model or calculus to a game during his middle period, the basis for a new fluid concept of a game would prove critical in Wittgenstein's efforts to remedy the problems of the *Tractatus* through his exposition of language games:

You could say arithmetic is a kind of geometry; i.e. what in geometry are constructions on paper, in arithmetic are calculations (on paper). You could say it is a more general kind of geometry.

And can't I say that in this sense chess (or any other *game*) is also a kind of geometry. (Wittgenstein 1998, p.131)

In *Philosophical Grammar* Wittgenstein comes to the realisation that his criticism of other logicians in the *Tractatus* is now equally relevant to his own work. As such in the following Wittgenstein acknowledges that the *Tractatus* was deeply enmeshed within a language game of which he was then explicitly unaware and which in his pursuit of the ideal, failed to question the very process of logical analysis which he was participating in:

How strange if logic were concerned with an 'ideal' language and not with *ours*. For what would this ideal language express? Logical analysis is the analysis of something we have, not of something we don't have. Therefore it is the analysis of propositions as they stand. (It would

be odd if the human race had been speaking all this time without ever putting together a genuine proposition)(Wittgenstein 2005, p .52)

In denying any significance to an ideal logical language which in his early work was intrinsically part of creating a closed logical model, he now recognises “ordinary language” as the language which must be investigated. He immediately follows this remark with his new pedagogical methodology which signals a mammoth repositioning of the status of logical analysis itself and its relation to pedagogical practice which is intended to both explicate the process of logical analysis – from the move to primary and secondary language use – and to reflect the pedagogical process inherent in the language game of aspect seeing which allows for continuous revision and new ways of seeing language:

When a child learns ‘Blue is a colour’, red is a colour, green, yellow – all are colours’, it learns nothing new about the colours, but the meaning of a variable in such propositions as: ‘There are beautiful colours in that picture’ etc. The first proposition tells him the values of a variable.(Wittgenstein 2005, p.53)

This central conceptual insight would prove to be one of the most critical right through to the Investigations and signals an understanding of the process of logical analysis itself and the secondary use of language, as being rooted in the humble pedagogical process of language acquisition:

The arbitrariness of linguistic expressions: might we say: A child must of course learn to speak a particular language, but doesn’t have to learn to think, i.e. it would think spontaneously, even without learning any language?

I mean there is no preliminary stage in which a child already uses a language, so to speak uses it for communication, but does not yet think in it. (Wittgenstein 2005, p.54)

Aligning logical analysis and secondary language use with the pedagogical process is intended to remove the idea of mystery surrounding non referential language use – which Wittgenstein was himself guilty of in the Tractatus. One process in this sense throws light on the other both as language games of comparison and also in the sense that logical analysis is itself pedagogical in terms of its self reflexivity. This point is inherent in the very concept of aspect seeing which allows for the logician or analyst to learn through new ways of seeing, thereby allowing the language game of linguistic aspect seeing to evolve by means of new logical models.

In the following a decisive and unambiguous movement from what were previously classified as nonsense propositions – in so far as they had no objective isomorphic corollary in the world – to a consideration of the logical role of nonsense as illustrated in the non referential use of language is set in motion:

It cannot be proved that it is nonsense to say of a colour that it is a semitone higher than another. I can only say ‘If anyone uses words with the meanings that I do, then he can connect no sense with this combination. If it makes sense to him, he must understand something different by these words from what I do’.(Wittgenstein 2005, p. 53)

Here Wittgenstein’s presentation of nonsense highlights the sharp reorientation of the new free moving logical bolt of aspect seeing wherein secondary language use

and meaning are no longer limited and prohibited. In focusing on what would previously have been considered nonsense propositions Wittgenstein now locates therein a key logical insight which amounts to an acceptance that sense and meaning do not always operate along clearly defined systematic rule following processes. Retrospectively considered these remarks validate the open logical model of Russell which he so severely criticised. Here the logical focus on the particular in tandem with the idea of rule creation or invention overturns the position of nonsense in the *Tractatus*:

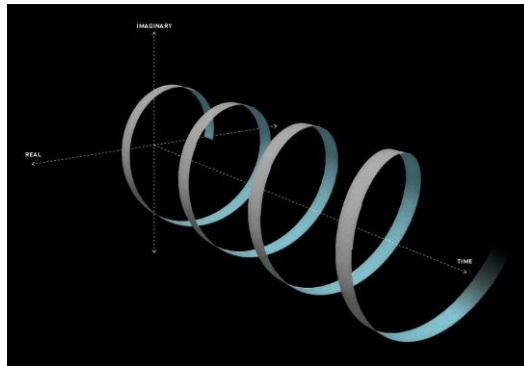
Can anyone believe it makes sense to say “That’s not a noise, it’s a colour”?

On the other hand you can of course say ‘It’s not the noise but the colour that makes me nervous’, and here it might look as if a variable assumed a colour and a noise as values. (Sounds and colours can be used as vehicles of communication.) It is clear that this proposition is of the same kind as ‘If you hear a shot or see me wave run.’ For this is the kind of co ordination on the basis of which a heard or seen language functions.(Wittgenstein 2005, p.55)

In establishing this critical point Wittgenstein once again focuses on the work of Hilbert:

I can play with chessmen according to certain rules. But I can also *invent a game* in which I play with the rules themselves. The pieces in my game are now the rules of chess, and the rules of the game are, say, the laws of logic. *In that case I have yet another game and not a metagame.* What Hilbert does is mathematics and not metamathematics. It’s another calculus just like any other. (Wittgenstein 2005, p.319)

Despite his overt hostility the distinction between a game and a meta game is blurred such that regardless of whether we call it a meta game or a game the possibility of a new logical model is accepted. Moreover his later presentation of aspect seeing essentially operates as a meta-analysis therefore casting these remarks of his middle period in a transitory light. He thus notes that “our normal mode of expression carries the seeds of confusion right into its foundations, because it uses the word “series” both in the sense of “extension” and in the sense of “law”.” (Wittgenstein 2005, p.430) The relationship between the word “series” in the sense of extension and in the sense of law “can be illustrated by a machine for making coiled springs, in which a coil is pushed through a helically shaped passage to make as many coils as are desired.”(Wittgenstein 2005, p. 430)



What is called an infinite helix need not be anything like a finite piece of wire, or something that that approaches the longer it becomes; it is the *law of the helix*, as it is embodied in the short passage. Hence the expression “infinite helix” or “infinite series” is misleading.

So we can always write out the recursive proof as a limited series with “and so on” without losing any of its rigour....The recursive definition is a rule for constructing replacement rules, or else the general term of a series of definitions. It is a signpost that shows the *same way* to all expressions of a certain form. (Wittgenstein 2005, p.430)

In the context of the language game of logical analysis as presented in the Investigations, the idea of an infinite series applies not to the language game as a totality, but to the logical law or principle of the open logical model of aspect seeing. The idea that nothing can be said about this logical process is now rejected, and is consequently reflected in the Investigations:

120 In giving explanations I already have to use language full-blown (not some sort of preparatory provisional one); this is enough to show that I can only come up with externalities about language. Yes – but how can these observations satisfy us? – Well, your very questions were framed in this language; they had to be expressed in this language if there was anything to ask!

And your scruples are misunderstandings. (Wittgenstein 2009, p. 54)

Here we see the end of the logical position held in the Tractatus that language cannot be used to refer to itself on pain of introducing Russell's paradox. Instead we have a logical model which is open ended in structure and which is entirely devoid of the immutable logical bolt of elementary propositions. As illustrated in the above remark his "scruples" regarding the logical models operative in mathematics and in Russell's type theory retrospectively reflect Wittgenstein's own "misunderstandings" regarding both the nature of the process of logical analysis itself and the idea of an immutable fixed terminus of elementary propositions to close this infinite language game.

## *Chapter Six*

### *The second Stage of Evolution in Wittgenstein's Analysis of Russell's paradox and the problem of self reference*

#### *6.1 Resolving the defective surroundings of paradox through the concept of aspect seeing*

Having considered Wittgenstein's revision of the role of the infinite as a logical concept which is now central to both his open logical model of aspect seeing and to the process of logical analysis itself, I now turn to consider his revision of the generated problem of Russell's paradox and self reference. In the Tractatus I argued that Russell's paradox was rejected and vanished within that closed logical model on foot of Wittgenstein's rejection of the infinite. Within the later Wittgenstein's open logical model of aspect seeing, the characteristic of self reference and self reflexivity which pervades Russell's paradox, assumes a logical centrality. Just like Russell's paradox, the example of the duck rabbit illustration can be seen now one way now another but not both ways simultaneously. Its sense – or lack of – ultimately stems from that fact that self reference and reflexivity within the model or set of the duck rabbit is entirely dependent on two mutually independent entities – a duck and a rabbit. It is simply that both concepts are bound together in this peculiar way in the visual image that the paradox emerges. Similar to Russell's paradox both pictures point at each other, generating our fascination with the picture either in its linguistic and logical or purely visual context. As Hofstadter remarks on the issue of self reference and reflexivity in relation to Russell's paradox:

Taken together, these sentences have the same effect as the original Epimenides paradox; yet separately they are harmless and even potentially useful sentences. The "blame" for this strange loop cant be pinned on either sentence – only on the way they "point" at each other...each local region is quite legitimate; it is only the way they are globally put together

that creates an impossibility.  
(Hofstadter 1979, p.21)

That the idea of a paradox or contradiction being inherent in set theory and therefore in the foundations of mathematics signalled a disastrous logical impasse, is I contend used by the later Wittgenstein to restructure the failed logic of the Tractatus. This second stage of the evolution of Wittgenstein's position, on the inter-related problems of paradox, self reference and contradiction which as shown stemmed from the problem of the infinite, is expressed in material from the middle period of his work:

Say we often arrived at the results of our calculations through a hidden contradiction. Does that make them illegitimate? – But suppose that we now absolutely refuse to accept such results, but still are afraid that some might slip through. Well then, in that case we have an idea which might serve as a model for a new calculus. As one can have the idea of a new game. (Wittgenstein 2001, p.369)

For Wittgenstein the idea that a contradiction somehow signals the demise of all logical certainty is entirely misplaced, and is now considered alternatively, and in stark contrast to the Tractatus, as “an idea which might serve as a model for a new calculus.” (Wittgenstein 2001, p. 369) The critical exegetical point in this remark is that Wittgenstein directly links the idea of a hidden contradiction serving as a logical model to language games, signalling a much more multifaceted approach to language than appears in the Tractatus. Ultimately the idea that a contradiction or paradox might serve as a model for a new game, anticipates the idea of aspect seeing wherein language can be seen in multiple and therefore contradictory and paradoxical ways. As he comments further:



But you can't allow a contradiction to stand! – Why not? We do sometimes use this form in our talk, of course not often – but one could imagine a technique of language in which it was a regular instrument. (Wittgenstein 2001, p.371)

Wittgenstein's own change of linguistic aspect perception or secondary use of language is also revealed in these telling remarks, as he begins to logically re-orientate the core problem which the Tractatus had claimed to make disappear. Rather than making it disappear it now seems that the problem of paradox and its characteristic of self reference is essential to the new logical model under construction:

The various half joking guises of logical paradox and only of interest in so far as they remind anyone of the fact that a serious form of the paradox is indispensable if we are to understand its function properly. The question is: what part can such a logical mistake play in a language game? (Wittgenstein 2001, p.397)

This parallel reconsideration of the role of the infinite, in tandem with the problem of paradox and self reference proves fundamental in allowing the new logical bolt of the infinite as operative in non referential language use to operate in a logically cohesive manner. In the Tractatus it was essential that the paradox should disappear. Wittgenstein believed this had been accomplished by excluding as untenable the non referential use of language in the process of logical analysis, specifically as evidenced when addressing the concept of the infinite. This had allowed him to exclude the concept of the infinite in both a logical and visual contexts. As Russell's paradox emerges from a logical consideration of the infinite, its emergence within the closed logical model of the Tractatus was therefore an impossibility as the non

referential use of language in relation to logical concept was rejected. In rejecting these logical concepts Wittgenstein also rejected the process by means of which they were generated – i.e. non referential language use – and in so doing he also condemned the activity of logical analysis itself to eternal silence. However in his middle period and later work the role of paradox, self reference and reflexivity, becomes essential to the language game of logical analysis itself, in so far as it serves to further explicate the idea of his distinction between infinite reality and infinite possibility, the latter of which is inherent in the process of logical analysis.

For Russell unlike Wittgenstein the visual field is infinitely complex and displays at all points a decidedly open ended infinite structure. Russell's consideration of the visual field is most explicit in *The Problems of Philosophy* (1912), which is an extension of the 1905 work *On Denoting*. Russell's example of the experience of looking at a table in the following echoes Wittgenstein's later work on aspect seeing revealing a much more dynamic model of the visual field than presented in the one dimensional account of the *Tractatus*:

To the eye it is oblong brown and shiny, to the touch it is cool and smooth and hard. Although I believe the table is 'really' of the same colour all over, the parts that reflect the light look much brighter than other parts, and some parts look white because of reflected light...It follows that if several people are looking at the table no two will see exactly the same distribution of colours, because no two can see it from exactly the same point of view. (Russell 1912, p. 3)

Russell's consideration of the visual field continues as above in increasing complexity such that the appearance of anything in the visual field possesses the logical possibility of being seen in innumerable ways. Similar to Wittgenstein's later work on aspect seeing, contradictions in terms of what is visually perceived

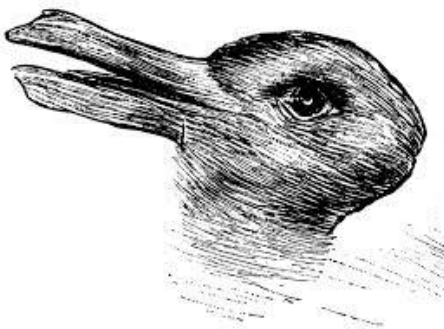
by different subjects is inevitable as “no two can see it from exactly the same point.”(Russell 1912, p.3)

A similar view is expressed in the following where Russell’s reference to different appearances of the visual field in the language of set theory reflects Wittgenstein’s remarks on different aspects:

A chair presents at each moment a number of different appearances. All the appearances that it is presenting at a given moment make up a certain class. If I take a chair and smash it, it will present a whole set of different appearances from what it did before, and without going as far as that, it will always be changing as the light changes and so on. (Russell 2007, p.275)

Inherent in Russell’s paradox the issue of aspect seeing emerges, as we seem to be able to see the particular class of concern, under two different aspects as both being and not being a member of itself, which in turn reflects the contradictory and self referential nature of aspect perception. If we consider Russell’s paradox from the later Wittgenstein’s concept of aspect seeing, the difficulty can be expressed in an alternative form which situates the problem of self reference and contradiction in a new light. The problem of paradox, self reference and contradiction is acutely pronounced in instances of aspect seeing and retrospectively considered we can consider Russell’s paradox as illustrative of the aspect switch which is central to the experience of aspect seeing. In the case of the duck rabbit for instance we define the set as a duck which is entirely valid if ones sees it as a duck. However this can be contradicted by the ability of the pictorial content to change via its internal self reference whereby the set becomes most definitely a rabbit which contradicts the first statement. Both seem to be valid yet the image cannot be seen as both *simultaneously*, just as in Russell’s paradox a class can’t simultaneously both be and not be a member of itself. Despite this restriction we yet can easily

revert from one aspect of both the theoretical and visual paradox to the other, fascinated by the aspect switch. There is of course no such animal as a duck-rabbit which has the characteristics of both animals which allows for the set called duck-rabbit to be considered as both being and not being a member of itself. Its sense therefore depends entirely on how we visually perceive the content.



Anonymous, Duck Rabbit Illusion (1892)

What Russell's paradox reveals retrospectively rather than a logical oddity, is the ability of language to display a dual aspect wherein the concept of meaning changes dramatically. This dual aspect is embedded in the distinction between referential and non referential uses of language. In considering Russell's paradox as an exemplar of linguistic aspect seeing, it reveals that in the language game of logical analysis, other possibilities are always present and that the language game of logical analysis itself only becomes problematic if we try to limit it – as Wittgenstein did in the *Tractatus*.

In his later rejection of the closed logical model of the *Tractatus*, Wittgenstein specifically singles out the one dimensional model of visual and linguistic representation which it champions. He remarks that the idea of logical form providing an internal closed isomorphic logical necessity between thought and reality is incorrect. Firstly it was based on “comparing the method of projection with projection lines which go from one figure to another” (Kenny 1994, p.43). In

the context of the Tractatus the method of projection began with logical form and terminated with atomic propositions:

This comparison conceals the fact that the picture plus the projection lines leaves open various methods of application. (Kenny 1994, p.43)

He reflects, that in the closed logical model of the Tractatus he imagined

the difference between proposition and reality is ironed out by the lines of projection belonging to the picture, and no further room is left for a method of application, but only for agreement and disagreement.(Kenny 1994, p. 43)

In reflecting that the closed logical model of the Tractatus is untenable because it allows “no further room for a method of application” the premises that language cannot refer to itself, and that the non referential use of language as operative in logical analysis is nonsense, must be reconsidered. The fact that Wittgenstein now acknowledges that his earlier position had concealed the fact that “various methods of application” are open to language, retrospectively legitimates the open logical model of Russell which he had earlier rejected.

This position is rooted in the revision of the problem of paradox which occurs in his middle period. He thus observes:

The various half joking guises of logical paradox are only of interest in so far as they remind anyone of the fact that a serious form of the paradox is indispensable if we are to understand its function properly. The question is: what part can such a

logical mistake play in a language game? (Wittgenstein 2001, p.397)

Wittgenstein's remark that a "serious form of the paradox is indispensable" in our understanding of language games rests on a reassessment of the nature of rule following and rule invention both of which parallel the distinction between infinite reality and infinite possibility. In terms of his later work these terms correspond to and parallel his later formal expression regarding primary and secondary uses of language and meaning where we see its inversion reflect the dramatic change between his early and later work. It is however in his middle period where we see the beginnings of how the grammar of the infinite, is now seen as central in correctly understanding the role of paradox and self reference:

If you accept the rule you must do this" – This may mean: the rule doesn't leave two paths open to you here. (a mathematical proposition) (Wittgenstein 2001, p.406)

He continues his exposition observing that in this instance the "rule conducts you like a gangway with rigid walls."(Wittgenstein 2001, p.406) However this rigid conception of rule following must take account of seeing the proposition in a different way. Thus "against this one can surely object that the rule could be interpreted in all sorts of ways. – Here is the rule, like an order!" (Wittgenstein 2001, p.406):

Something surprising, a paradox, is a paradox only in a particular, as it were, *defective surrounding*. One needs to complete this surrounding in such a way that what looked like a paradox no longer seems one. (Wittgenstein 2001, p. 410)

When situated in the context of aspect seeing this is precisely what happens. He thus remarks in one of his later works *Remarks on the Philosophy of Psychology*:

In the language game of a tribe there might be a pronoun, such as we do not possess and which we have no practical use, which 'refers' to the propositional sign in which it occurs. I will write it like this: I'. The Proposition "I' am ten centimetres long" will then be tested for truth by measuring the written sign. The proposition "I' contain four words" for example is true; and so is "I' do not contain four words". "I' am false" corresponds to the paradox of the Cretan Liar. – The question is What do people use this pronoun for? Well the proposition "I' am ten centimetres long" might serve as a ruler, the proposition "I' am beautifully written" as a paradigm of beautiful script. (Wittgenstein 1980, p. 15)

In considering Russell's paradox as a clash of two opposing rules, the paradox can be seen as a logical indicator of the language game of linguistic aspect seeing or the secondary use and meaning of language. In this sense it mirrors the infinite possibility of language to move beyond itself and generate new meaning and new logical models – all of which cannot be proven to follow a singular consistent rule, but rather follow the rule of rule deviation. Wittgenstein thus remarks that:

A proposition like "I' contain four words" might be used as a paradigm for the number four and in another sense so might the proposition "I' do not contain four words." (Wittgenstein 1980, p.15)

If we treat the paradox as such an indicator, the paradox only becomes problematic if we try to quantify over all rules as a totality – akin to the sense of trying to

quantify over the infinite and all cardinal numbers. In the context of aspect seeing, the idea of a defined set or totality of rules becomes paradox generating, as the principle of aspect seeing is the infinite possibility of generating new rules via the secondary use of language and meaning – therefore it is impossible to define linguistic aspect seeing as a totality:

It is the business of philosophy, not to resolve a contradiction by means of a mathematical or logico-mathematical discovery, but to make it possible for us to get a clear view of the state of mathematics that troubles us: the state of affairs before the contradiction is resolved.

The fundamental fact here is that we lay down rules, a technique, for a game, and that when we follow the rules, things do not turn out as we had assumed. That we are therefore as it were entangled in our own rules. (Kenny 1994, p. 285)

In our efforts to understand this entanglement the paradox or contradiction:

Throws light on our concept of meaning something. For in those cases things turn out otherwise than we had meant, foreseen. That is just what we say when for example a contradiction appears: “I didn’t mean it like that”

The civil status of a contradiction or its status in civil life: there is the philosophical problem. (Kenny 1984, p.284)

This logical point is thus similarly illustrated, in the Necker cube, the duck rabbit, and in Russell’s paradox. In all cases of paradox and self reference within both visual perception and linguistic expression, two opposing rules occupy the same



logical and visual space, allowing us in the case of the duck rabbit to see it according to the rule of a duck or according to the rule of a rabbit – but not both simultaneously. What such instances of paradox reveal retrospectively is not a limit of language, but rather the characteristic of the language game of logical analysis itself which is the ability to use language in a dual manner via referential and non referential uses. Such non referential use of language now consists in the ability of language to break off from its primary use and construct a secondary use and meaning in a self reflexive manner, thereby allowing language to construct infinite new logical models within the open logical model of the language game of aspect seeing. It is in this sense in which Russell's paradox becomes central to Wittgenstein's work on aspect seeing. Not only is it central but Russell's response to the paradox also mirrors Wittgenstein's. Just as Russell's response to the problem of the infinite and the generated problem of paradox demanded the open logical model of type theory, so too Wittgenstein's exposition of linguistic aspect seeing and the secondary use and meaning of language epitomises a similar model.

It is in this sense that language games becomes “objects of comparison”(Kenny 1994, p.284) In the following the problem of paradox, contradiction and self reference, is considered in the context of a language game where the process of logical analysis is likened to playing a game of chess. Using the analogy with a chess game, the pieces of which are objective contents which we can see, in conjunction with mathematical axioms which we can understand, the game can be considered as either “the rules according to which you play” or “the opening position of the game”. (Wittgenstein 1998, p.319) Wittgenstein thus observes:

The rules are in a certain sense statements. They say: you may do this or this, but not that. Two rules can be inconsistent. Suppose, e.g. that in chess one rule ran: under such and such circumstances the piece concerned must be taken. But another rule said: a knight may never be taken. If now the piece concerned happens to be a knight, the rules

contradict one another: I don't know what I'm supposed to do. What do we do in such a case? Nothing easier: *we introduce a new rule and the conflict is resolved.*

My point then is: if consistencies were to arise between the rules of the game of mathematics, it would be the easiest thing in the world to remedy. All we have to do is to make a new stipulation to cover the case in which the rules conflict, and the matter's resolved. (Wittgenstein 1998, p. 319)

For Wittgenstein aspect seeing provides the necessary rule or stipulation, allowing the apparent contradiction to be resolved. In essence what Wittgenstein is doing is applying an open logical model, which is structurally similar to Russell's type theory. In the *Tractatus* Wittgenstein's efforts to eliminate paradox by considering language as one universal isomorphic system failed, not because he had not discovered the correct logical rules but because – as he now realises – there are no correct logical rules. Moreover the logician doesn't discover such logical rules but creates them such that by introducing “a new rule the conflict is resolved.” (Wittgenstein 1998, p. 319) In this context aspect seeing as expressed in Russell's paradox, can be considered as a rule generator with infinite possibility, a position which reveals the mammoth divergence between the closed logical model of the *Tractatus* and the open logical model of aspect seeing.

Previously paradox had been seen as being the logical indicator which suggested that mathematics itself was inconsistent, as the infinite could not be quantified over within a closed consistent logical model. While it had been shown in the logicist efforts of Russell and Frege that delivering consistent logical foundations could not be offered within a closed logical model, Russell still maintained – unlike Wittgenstein - that the open logical model was structurally necessary:

A contradiction is only a contradiction when it arises. People have the idea that there might at the outset be a contradiction hidden away in the axioms which no one has seen, like tuberculosis: a man doesn't suspect anything and then one day he's dead. That's how people think of this case too: one day the hidden contradiction might break out, and then the catastrophe would be upon us. (Wittgenstein 1998, p. 319)

In the following remarks this point is addressed from a new perspective and clearly illustrates how the concept of language games originally evolved in response to the crisis of mathematics and the problem of the infinite and paradox which had first sparked Wittgenstein's interest in logic leading ultimately to his magnum opus the *Tractatus*:

I've been reading a work by Hilbert on consistency. It strikes me that this whole question has been put wrongly. I should like to ask: Can mathematics be inconsistent at all? I should like to ask these people: Look, what are you really up to? Do you really believe there are contradictions hidden in mathematics? (Wittgenstein 1998, p. 318)

This insight can be considered in the light of Berto's paraconsistent argument:

There is no metasytem in which one establishes that the Gödel sentence is true; there are no metasytems. Consequently one cannot 'get out of' a system and solve it in its metasytem, problem which were meaningfully expressible but undecidable within the system. (Berto 2009, p.206)

In the context of aspect seeing as an open logical model and as reflective of the process involved in logical analysis itself, statements about a particular logical model, and statements about the process of linguistic aspect seeing itself can't be used to prove the particular logical model consistent or not. Thus Wittgenstein's criticisms of Hilbert assuming to speak from the perspective of a meta game, is problematic at this point because the language game of aspect seeing cannot be defined as a totality and therefore must be inconsistency tolerant. In considering aspect seeing as a meta-analysis on the process of logical analysis Wittgenstein is careful not to overstep this mark and as such no definition of aspect seeing is provided. Rather it is the creation of a new rule and a new logical model which allows us to see things differently, rather than to confer absolute truth or falsity on the logical model itself. This applicative potential of a given logical model – particularly within the sphere of logical analysis within any given discipline e.g. theoretical physics – is of significant value.

The crucial logical insight of aspect seeing is that any one logical model, does not exhaust the possibilities of secondary meaning or linguistic seeing as relative to a given concept. Thus while we can't define the language game of logical analysis, this is only problematic if we consider it within "defective surroundings" such as those of the Tractatus. If we take any individual logical model – or any logical concept - it of necessity occupies a paradoxical position in the language game of linguistic aspect seeing. Why? Ultimately all logical models simultaneously do and do not represent the language game of linguistic aspect seeing, in so far as they do they serve as exemplars, but in so far as they do not they cannot define or limit the game itself, which the Tractatus had tried to do. In short the language game of secondary language use and meaning cannot be defined - in the same way as cardinal numbers or the infinite cannot - as it is logically part of an open logical model. Russell's remarks on the necessity of an open logical model now have a clear applicative sense to Wittgenstein's logical model of aspect seeing:

Every account of structure is relative to certain units which are, for the time being, treated as if they were devoid of structure, but it must never be assumed that these units, will not in another context, have a structure which it is important to recognise.(Russell 2007, p.223)

If we try to define the language game of aspect seeing along the lines of set membership, we will easily end up in the mire of Russell's paradox. This is so in two senses. Firstly the class of all instances of aspect seeing is not the logical concept of aspect seeing but rather the instances of aspect seeing, therefore aspect seeing as a concept can be considered as not being a member of itself. Therefore if we try as Russell did to ask if all such classes including aspect seeing make up a set we will repeat the procedure as operative in Russell's paradox. In the second sense we can't include all instances of aspect seeing – both visual and linguistic- in the totality of the language game - defined as a total set, as another logical model will always serve to enlarge the set thereby falsifying any totality - therefore we cannot quantify over the set. Thus any efforts to define the set will ultimately be contradicted by the fact that a new member will have to be included. We therefore cannot define aspect seeing, as the open logical model of the language game cannot have a totality so it therefore cannot be a closed set or closed logical model, as Russell's type theory illustrates. We can appreciate the impossibility if we imagine the alphabet analogously to the number line and efforts to define all referential and non referential uses of language through combinations of letters, then words, then sentences, then logical models, then meta models etc. This implicit impossibility is reflected in Russell's theory of types wherein the structural complexity of an open ended logical model prohibits the establishment of final boundaries:

The correspondence of propositions and fact grows increasingly complicated as we pass to more

complicated types of propositions: existence propositions, general propositions, disjunctive and hypothetical propositions, and so on. The subject is important and capable, I believe, of throwing much new light on logic. (Russell 2007, p.319)

Here we can appreciate the logical similarity between Russell's model wherein logical analysis sees language as different types and Wittgenstein's model wherein we see language in the context of different aspects. It is this reality of the language game which allows different logical models operating with different rules for the same concept to be established and co exist one alongside the other:

Now if the case arose that a formula counted as having been proved on the basis of one method, but as refuted on the basis of another, then that wouldn't in the least imply we now have a contradiction and are hopelessly lost; on the contrary we can say: *the formula simply means different things. It belongs to two different calculi.* In the one calculus its proved, in the other refuted. And so we really have two different formulae in front of us which by mere accident have their signs in common.(Wittgenstein 1998, p.344)

Logical models are now considered in their new surroundings as part of a language game which allows for "two different calculi" or logical models to generate two opposing systems which "simply mean different things".(Wittgenstein 1998, p.344) Here the idea of meaning as use is understood not by following rules which we discover but by inventing rules and as such accounts for the open ended structure of the language game itself. In this sense it is not logical experience which is at issue and which the Tractatus had rejected but logical language use in

the process of logical analysis. In the new open logical model of aspect seeing non referential language use thus constitutes a linguistic experience:

649 For is what is linguistic not an experience?(Wittgenstein 2009, p.174)

As Wittgenstein remarks in the following Cantors discovery of the infinite simply amounts to a new logical model:

Cantor defines a *difference of higher order logic*..

For I can of course form the expression: “class of all classes which are equinumerous with the class “infinite series” (as also “class of all angels that can get onto a needlepoint”) but this expression is empty so long as there is no employment for it. Such an employment is not: yet to be discovered, but: still to be *invented*. (Wittgenstein 2001, p.135,336)

Ironically the employment of the infinite series as operative in the process of secondary use and meaning within logical analysis, can be retrospectively identified in Wittgenstein’s own invention of both a closed and open logical model, in the logical model of Russell’s type theory, and in Mandelbrot’s work on fractals where the complex spatial objects of the Tractatus find a logical home. Here we see real exemplars – all vastly different yet related - of how linguistic aspect seeing operates. Understood in this sense the concept of the infinite is transformed to indicate the logical impossibility of limiting the language game of logical analysis:

To say that a technique is unlimited does not mean that it goes on without ever stopping – that it increases immeasurably; but that it lacks the institution of the end, that it is not

finished off. As one may say of a sentence that it is not finished off if it has no period. Or of a playing field *that is unlimited when the rules of the game do not prescribe any boundaries* – say by means of a line.(Wittgenstein 2001, p.138)

Rather the process of logical analysis now allows for new techniques, new pictures, and new ways of seeing to evolve and develop:

For the point of a new technique of calculation is to supply us with a new picture, a new form of expression. (Wittgenstein 2001, p.138)

The process of logical analysis in the creation of all new logical models is thus inevitably involved in a change of aspect:

It is true enough that I changed the *aspect* of the logical calculation by introducing the concept of the number of negations: “I never looked at it like that” – one might say. But this alteration only becomes important when it connects with the application of the sign. (Wittgenstein 2001, p.181)

The crucial logical point is that in perceiving a change of aspect, we create a new secondary use of language itself derived from the grammar of the infinite as operative in the language game of linguistic aspect seeing:

Suppose it were said “By calculating with numbers we get acquainted with the properties of numbers” But do the properties of numbers exist outside the calculating?(Wittgenstein 2001, p.189)



Just as the properties of numbers do not exist outside the practice and experience of using numbers so too the realm of the infinite does not exist outside of the language game which creates it. There is no final system no final language game. In the efforts to solve paradox and self reference Wittgenstein refers to Russell's theory of types, observing that "while it "could it be said, e.g. that while Russell's Theory of Types avoids the contradiction, still Russell's calculus is not *THE* universal logical calculus but perhaps an artificially restricted mutilated one" (Wittgenstein 2001, p.217) While Wittgenstein almost reluctantly accedes that the model of type theory is logically purposeful in avoiding the paradox or contradiction, his innate hostility to both Russell's type theory and Cantors set theory is still as ferocious as it had been in the Tractatus albeit, the process of evolution in his own logical analysis is ironically entirely dependent on them:

Like the enigma of time for Augustine, the enigma of the continuum arises because language misleads us into applying to it a picture that doesn't fit. (Wittgenstein 2005, p.471)

The following remarks from Remarks on the Foundations of Mathematics decisively links the problem of the infinite, and the generated problems of paradox, self reference and contradiction with their ultimate solution - aspect seeing. Firstly the secondary nature of non referential language use in set theory is identified:

What sort of proposition is: "The class of lions is not a lion, but the class of classes is a class"? How is it verified? How could it be used? – So far as I can see, only as a grammatical proposition. To draw someone's attention to the fact that the word "lion" is used in a fundamentally different way from the name of a lion; whereas the class word class is used like the designation of one of the

classes, say the class lion.  
(Wittgenstein 2001, p.403)

The logical problem of treating logical concepts on a par with concepts which have clear public ostensive counterparts results in erroneously considering the proposition ‘the class of lions’ as being logically comparable with ‘class of classes’. Here we have a clear instance of a rule of reference and a rule of invention. Paradox and self reference only arise in so far as the logician fails to recognise that two different uses of language – primary and secondary uses – are at issue in both respective language games. In such cases “we draw someone’s attention” to two very different uses of language. Through identification of the language game of aspect seeing it becomes clear that concepts such as lion and class are not comparable in the sense imagined by the early Wittgenstein. Just because the concept of a lion refers to a reality outside of itself, it does not logically follow that this is also the case with the concept class. That this is not the case with the concept of class, does not make the use of the concept redundant, but rather highlight the different sense of non referential language use:

*The sudden change of aspect in the picture of a cube and the impossibility of seeing ‘lion’ and ‘class’ as comparable concepts.*

One can examine an animal to see if it is a cat. But at any rate the concept cat cannot be examined in this way.  
(Wittgenstein 2001, p.403)

In his later work in the Investigations this self reflective process of analysis would be more explicitly understood as indicating that what we do have to examine is the logical model in which such classification is used. In Zettel he thus observes the experience of aspect seeing as one in which language points beyond itself in a self reflexive and self referential manner in so far as it relates to the entire field of language:

Doesn't the theme point to anything outside itself? Yes it does! But that means – it makes an impression on me which is connected with things in its surroundings – e.g. with our language and its intonations; and hence with the whole field of our language games. (Wittgenstein 1970, p.31)

As Wittgenstein observes in the Investigations, just as the words “Now I am seeing this as an apex” cannot so far mean anything to a learner who has only just met the concepts of apex, base and so on.” (Wittgenstein 2009, p.178), so too, logical concepts can only mean something to someone who is familiar with playing the language game of logical analysis:

“Now he's seeing it like this”, “now like that” would only be said of someone capable of making certain applications of the figure quite freely. The substratum of this experience is mastery of a technique. (Wittgenstein 2009, p.178)

The problem of the infinite, paradox and self reference are now unproblematic in their new surroundings, within the language game of linguistic aspect seeing, the foundations of which are the “mastery of a technique” of primary language use.

Aspect seeing as inherent in paradox and self reference such that we see a class as both a member and not as a member of itself, amounts in the language game of aspect seeing, to now seeing it in this way and now in that, eliminating its logical mystery. It reveals a conflict of rule following which when situated within the context of aspect seeing is unproblematic. This same point is explicitly reiterated in the following reference to “paradigms” or logical models, which is decisive that the language game of logical analysis is one of linguistic invention and not metaphysical discovery:

When I said that a proof introduces a new concept, I meant something like the proof puts a new paradigm among the paradigms of the language;

One would like to say: the proof changes the grammar of our language, changes our concepts. It makes new connections, and it creates the concept of these connections. (*It does not establish that they are there, they do not exist until it makes them*). (Wittgenstein 2001, p. 166)

It is therefore through the invention or creation of new logical models that our concepts and ways of seeing logic in a generic sense are changed and challenged:

What is the invention of the decimal system really? The invention of a system of abbreviations – but what is the system of the abbreviations? Is it simply the system of the new signs or is it also a system of applying them for the purpose of abbreviation? And if it is the latter, *then it is a new way of looking at the old systems of signs*. (Wittgenstein 2001, p.152)

The idea of “a new way of looking at an old system of signs” (Wittgenstein 2001, p.152) was also a logical insight which proved critical in Russell’s open logical model of type theory, and retrospectively reflects Wittgenstein’s evolving consideration of logical models:

I think any valid kind of interpretation may give new meaning to fundamental ideas. In practice this means that structure must be preserved. And a test of this is that all the propositions of a science should remain, though new

meanings may be found for their terms. (Russell 2007, p.340)

The importance of the concept of aspect seeing is further presented by Wittgenstein as being the catalyst for continuation of this open logical model. In this sense both the concept of aspect seeing, and Russell's conception of the unlimited nature of logical analysis, carry a mammoth intellectual importance and value in so far as the logical models of both logicians present the process of logical analysis as the process of new creative and inventive ways of looking at the world which, over time become embedded in and part of our language game and culture. In the following sequence of remarks Wittgenstein describes how the process of aspect seeing might take hold in a case of linguistic seeing as. He imagines someone who has been shown a proof indicating that there is an endless series of prime numbers:

He will say that he has drawn conclusions from what he has seen. – Not however as one does from an experiment. Could he say “What I have seen was very impressive? I have drawn a conclusion from it. In future I shall...” (E.g. in future I shall always calculate like *this*) He tells us: “I saw that it must be like that”

*I decide to see things like this.* And so to act in such and such a way. It must be so means that this outcome has been defined to be essential to this process. This *must* shows that he has adopted a concept. (Wittgenstein 2001, p. 309)

The logical ‘must’ is here considered as a new way of seeing in so far as we adopt new concepts. The process as described in the above makes concrete the complex process of seeing something as, and of then inventing a new logical model via the

secondary use of language and meaning. Linguistic seeing as thus acts as a catalyst for the possibilities of the language game of aspect seeing:

I should like to say: we should perhaps originally never have thought of the possibility of such a sequence and we have now introduced a new concept into our calculation. *Here is a new 'aspect'*.(Wittgenstein 2001, p.178)

In seeing a new aspect within a given logical model our conception of a given concept has changed precisely because new linguistic possibilities have been seen and experienced linguistically:

The different conceptions must correspond to different applications. For there is indeed a distinction between these two things: being surprised that the figures on the paper seem to behave like this; and being surprised that this is what comes out as the result. In each case however, *I see the calculation in a different context*. (Wittgenstein 2001, p.362)

When we want to avoid paradox or a contradiction, what the logician needs to do is invent a new logical model, while realising simultaneously that the emergence of paradox and contradiction is inherent in the language game being played, illustrating the necessity of the self reflexive nature of language in creating and in resolving logical conflict. In this sense our primary use and acquisition of language always has the ability to reflect back on itself when we encounter new concepts and to thereby create new combinations of meaning and sense which generate in turn new logical models. This allows us to truly understand the grammar of the

infinite as operative in the language game of aspect seeing and retrospectively invalidates Wittgenstein's criticism of Russell's type theory:

When for some practical purpose, you want to avoid a contradiction mechanically, as your calculus so far cannot do...*What is done here is not to improve bad mathematics, but to create a new bit of mathematics.* (Wittgenstein 2001, p.371)

In the sense in which Wittgenstein's work on aspect seeing can be considered as a meta-analysis of logical analysis itself – explored through both the infinite and paradox - the position expressed by Berto assumes parallel significance. Berto's argument that Wittgenstein's suggestion that Gödel's system can itself be seen as another variation of self reference and paradox, which is identified in the work of Priest and Routley. (Berto 2009, p.205) In considering the paraconsistent relevance of Wittgenstein's work stemming from this observation Berto observes that “by turning Godels proof into a paradox, it places inconsistencies at the very core of (the theory which supposedly captures) our mathematical practice. (Berto 2009, p.207). Its impression that we are at the end or limit of possibility, is for Wittgenstein a view generated by a failure to understand the role of the infinite and paradox- namely at the interface between primary and secondary language wherein the grammar of the infinite is generated. In asserting that we can't prove a system consistent, Gödel can be seen in the context of Wittgenstein open logical model of aspect seeing, to illustrate the secondary use and meaning of language through generating a new logical model. In contrast to Godels conclusion, for the later Wittgenstein the point lost on the world of mathematicians was that no fixed system of language no matter how complicated can represent the complexity of the language game of aspect seeing which is inherent in the process of logical analysis, and therefore the language game they are playing. What they certainly cannot do is limit it – a point retrospectively validated by the invention of paraconsistent logic and mathematics.

Changing the rules of the game in order to avoid or resolve paradox and self reference in a particular way, is precisely what Gödel achieved and what Wittgenstein attempted in the Tractatus. However the process of changing the rules is now for Wittgenstein understood as operating within the complex language game of aspect seeing and as such a final and definitive logical model which answers all questions and ends the language game, is precluded as a logical impossibility. This crucial logical principle of the language game of aspect seeing is reflected in the following where mathematical use is presented as no different in kind to language use. Retrospectively considered these remarks on the nature of mathematics stand in sharp contrast to the austere declarations of the Tractatus in turn reflecting Wittgenstein's transition from a closed to an open logical model:

Mathematics I want to say – teaches you, not just the answer to a question, but a whole language game with questions and answers.

Mathematics is then a family: but that is not to say that we shall not mind what is incorporated into it.

The introduction of a new rule of inference can be conceived as a transition to a new language game. (Wittgenstein 2001, pp.381, 399, 425)

This new position is illustrated in the following where Wittgenstein presents us with a visual and linguistic illustration of aspect seeing with a “surface which is divided into a number of strips and is observed by several people. The colours of the strips change every minute, all at the same time.”(Wittgenstein 2001, p.427) The colours are red, green, blue, white, black, blue. Various representations of what is seen emerge which Wittgenstein states “are material implications in Russell's sense.”(Wittgenstein 2001, p.427) Unlike the Tractarian position where a proposition was either true or false depending on its isomorphic relation to the world, now infinitely many different ways of seeing the visual field and



representing it are possible. His earlier view expressed in the closed logical model of the Tractatus, that something cannot simultaneously be two opposing things is no longer problematic within the grammar of the infinite, as something can now simultaneously be three opposing things and infinitely many more, thereby accommodating paradox self reference and self reflexivity. In highlighting the role of paradox, self reference and self reflexivity within language and the visual field, Wittgenstein points out the aspect with which an observer may be occupied may proceed as follows:

*And might not someone be preoccupied with the aspect red.blue ]black. ].white? If for example he has been taught to forget everything else, and only to look at the surface from this point of view. (Wittgenstein 2001, p.427)*

However given that any multiplicity of combinations can inform the aspect from which the coloured strip is seen, *how* it is meant to be seen cannot be prescribed by rules in all cases. Thus in the case of three opposing representations of aspect seeing Wittgenstein observes that “If these are three observations then it must also be possible for the third observation not to agree with the logical conclusion from the first two. (Wittgenstein 2001, p.427) In this sense different contradictory propositions are allowed to co-exist side by side. While paradoxical that the same content can be represented in many different ways, the problem of paradox in this context simply reveals the nature of the language game being played within which self reference and reflexivity is central.

Retrospectively considered Russell’s paradox, the Necker cube and the colour exclusion problem, which had been the catalyst in the demise of the Tractatus, assume a logical centrality in terms of the self reflexive self reference which they

all display in different ways. This self reflexive self reference ultimately serves as the logical bolt of the later Wittgenstein's conception of language games:

But how many different kinds of sentence are there? – There are countless kinds. And this diversity is not something fixed, given once for all; but new types of language, new language games, as we may say, come into existence, and others becomes obsolete and forgotten. (We can get a rough picture of this from the changes in mathematics) (Wittgenstein, 2009, p.15)

In moving from a position which demanded uniformity by removing the infinite, and by extension Russell's paradox and self reference, Wittgenstein now situates infinite diversity at the heart of his new logical model where the generation of new language games emerges as a result of the ability of language to look at itself differently in a self reflexive manner. Thus in his later work the ability of language to generate new language games and logical models is considered as being dependent on the self reflexive ability of language to overlap and criss-cross in an infinite variety of ways:

The upshot of these considerations is: we see a complicated network of similarities overlapping and criss-crossing: similarities in the large and the small. (Wittgenstein 2009, p.36)

It is this self reflexive ability of language to self reference itself or to look at itself in other contexts or surroundings, which allows language to point beyond itself:

Doesn't the theme point to anything beyond itself? Yes it does! But that

means – it makes an impression on me which is connected with things in its surroundings – e.g. with our language and its intonations; and hence with the whole field of our language games. (Wittgenstein 1970, p.31)

The ability of language to self reflexively look at itself within other surroundings or other language games, allows it to see or point beyond itself in an inventive generative and secondary manner:

How did I arrive at the concept ‘sentence’ or ‘language’? Surely only through the languages that I have learnt. – But they seem to me to have led beyond themselves, for I am now able to construct new languages, e.g. to invent words. (Wittgenstein 1970, p.60)

It is of critical significance that in the Investigations Wittgenstein presents the ability of language to self reference itself through criss-crossing and overlapping with other language games in infinite ways, in the context of mathematics. In order to highlight the infinite and open as opposed to finite and closed nature of self reflexive self reference within language Wittgenstein notes the following:

“Right; so in your view the concept of number is explained as the logical sum of those individual interrelated concepts: cardinal numbers, rational numbers, real numbers and so forth; and in the same way, the concept of a game as the logical sum of corresponding sub-concepts.” – This need not be so. For I can give the concept of number rigid boundaries, that is use the word “number” for a rigidly bound concept; but I can also use it so that the extension of the

concept is not closed by a boundary.  
And this is how we use the word  
“game”. (Wittgenstein 2009, p.37)

Highlighting the vast difference in approach featured in his earlier closed logical model, Wittgenstein’s later view of language is one of diversity wherein the self reflexive engagement between primary and secondary language use, or referential and non referential language use is essential, allowing language to “overlap” in infinite ways:

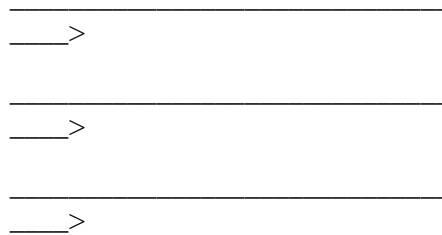
And we extend our concept of number, as in spinning a thread we twist fibre on fibre. And the strength of the thread resides not in the fact that some one fibre runs through its whole length, but in the overlapping of many fibres. (Wittgenstein 2009, p.36)

Within the language game of linguistic aspect seeing - or logical analysis - how we see language determines the logical models we invent, and of course all such endeavours necessitate self reference and reflexivity between primary and secondary uses of language. What this illustrates is how Russell’s paradox and the problem of self reference have been transformed and reconsidered to now accommodate the infinite language game of aspect seeing which crucially is also an inconsistency tolerant one.

***6.2 Bounded and Unbounded Language Games considered as a reflection of Wittgenstein's accommodation of the infinite, Russell's paradox and self reference through his pedagogical method***

In The Blue and Brown Books Wittgenstein introduces the logical models of bounded and unbounded language games which signal the final evolution in his logical analysis of the concept of the infinite and the related problems of paradox self reference and contradiction. Having allowed the infinite to reside within both the visual field and within language use considered as the grammatical rule of infinite possibility, Wittgenstein now applies these new logical positions to the active use of language in the context of language games. Here the use of this distinction between bounded and unbounded language games also serves to parallel the divergence between his early closed logical model and that of Russell's, while methodologically elaborating the structural similarity of an open logical model as expressed in Russell's type theory. Wittgenstein opens the discussion on bounded and non bounded language games as follows:

Consider this example: We introduce different ways of reading tables. Each table consists of two columns of words and pictures as above. In some cases they are to be read horizontally from left to right, i.e. according to the scheme:



In others according to such schemes as:

≠ ±

Schemes of this kind can be adjoined to our tables, as rules for reading them. Could not these rules again be explained by further rules? Certainly. On the other hand is a rule completely explained if no rule for its usage has been given? (Wittgenstein 1965, p.91)

These illustrations of rule following in the process of logic analysis or the language game of aspect seeing introduce the view that rules can always be “explained by further rules”. Not only does this illustrate the divergence between the logical models of Russell and the early Wittgenstein, but also signal his move to accept the open logical model of Russell as superior. The following pedagogical illustration serves as an essential aid in understanding the logic underpinning the secondary use of language as it appears in *Philosophical Investigations*:

We introduce into our language games the *endless series of numerals*. But how is this done? Obviously the analogy between this process and that of introducing a series of twenty numerals is not the same as that between introducing a series of twenty numerals and introducing a series of ten numerals....The difference between it and 2) would not be just that more numerals were used.(Wittgenstein 1965, p.91)

His reference to language game 2) is critical. The logic presented here underpins the self reference between the primary secondary language use distinction and its relation to aspect seeing which is essential in considering aspect seeing as operative in the process of logical analysis. In language game one Wittgenstein had introduced ostensive language use as a means of looking at “the extension of language” (pg.79) From here he considers the acquisition of numerical skills noting:

Learning the numerals by heart will be one of the essential features of learning this language. The use of the numerals will again be taught demonstratively. But now the same word “three” will be taught by pointing either to slabs or to bricks or to columns etc.(Wittgenstein 1965, p.80)

Here the logical form of language is presented as the first principle of language acquisition and not something metaphysical which is hidden within language, highlighting the central role of pedagogical method in his new open logical model. In this regard he refers to Russell remarking:

Compare Russell’s idea of the ‘individual’. He talks of individuals as the ultimate constituents of reality, but says that it is difficult to say which things are individuals. The idea is that further analysis has to reveal this. We on the other hand introduced the idea of a proper name in a language in which it was applied to what in ordinary life we call “objects”, “things”, (“building stones”).(Wittgenstein 1965, p.8)

Of course Wittgenstein’s criticisms of Russell are equally and even more so relevant to the Tractatus. In the following Wittgenstein fleshes out the concept of the secondary use of language as it appears in the Philosophical Investigations. Here the mathematical concept of the infinite is aligned with that of an unbounded – and as such infinite – language game, and the generated problem of paradox and self reference is now critical in the operation of the language game, illustrating the evolution of these concepts from his early closed model of logic. In asking what the difference between the two language games outlined in the previous page consists in Wittgenstein observes:

The difference between the finite and infinite game does not seem to lie in the material tools of the game; for we should be inclined to say that infinity can't be expressed in them, that is, that we can only conceive of it in our thoughts, and hence that it is in these thoughts that the finite and infinite game must be distinguished. (It is queer though that these thoughts should be capable of being expressed in signs.) (Wittgenstein 1965, p.91)

These considerations stand in stark opposition to the Tractatus where the infinite was rejected in order to limit the bounds of legitimate linguistic expression - and by extension the process of logical analysis as expressed within the open logical model of type theory which attempted to accommodate the logical principle of the infinite. This same concept now serves as the logical foundation of the open logical model of aspect seeing, wherein the secondary use and meaning of language represent the unbounded and infinite language game of logical analysis. The nature of this secondary use of language is reflected in the following:

One game is played with a fixed number of such cards, say 32. In the other game we are under certain circumstances allowed to increase the number of cards to as many as we like, by cutting pieces of paper and writing numbers on them. We will call the first of these games bounded, the second unbounded. (Wittgenstein 1965, p. 92)

He asks us to imagine “a hand of the second game” being played where “the number of cards actually used was 32.” (Wittgenstein 1965, p.92)The question now is “what is the difference in this case between playing a hand a) of the unbounded game and



playing a hand b) of the bounded game?”(Wittgenstein 1965, p.92) For Wittgenstein the difference does not lie “between a hand of a bounded game with 32 cards and a hand of a bounded game with a greater number of cards” as the number of cards used was in this case the same. The difference is of another kind such that “the bounded game is played with a normal pack of cards, the unbounded game with a large supply of blank cards and pencils.” (Wittgenstein 1965, p.92) The unbounded game is opened with the question “How high shall we go?” If the players look up the rules of this game in a book of rules, they will find the phrase “and soon ad infin., at the end of a certain series of rules” just as in the case of the concept of infinity in set theory and in Russell’s hierarchical type theory. The difference between the two games rather “lies in the tools we use though admittedly not in the cards they are played with. (Wittgenstein 1965, p.92) Crucially the process involved demands moving from the position of the bounded game – representing language acquisition and the referential use of language – to the unbounded language game representing non referential language use wherein invention is essential. Here aspect seeing allows language to self reference itself in the creation of new rules. The tools in the unbounded game reveal in a concrete sense the grammar of the infinite, which admit of the invention of new rules and not of following rules as in the case of the bounded game. In such cases we encounter a secondary use of language which allows for the creation of new symbols and new meanings.

In the bounded game the crucial logical principle is that the meaning of the cards is fixed analogously to referential ostensive definition in the public language game. If when we extend language from its primary use to its secondary use with the assumption that the picture of referential ostensive reference is also extended intractable problems arise. Central to all of these observations is the identification of pattern formation and linguistic creation, and in turn their relation to the creation of new rules and new systems within the language game of aspect seeing. For Wittgenstein the logical importance of this ability is not the untenable assumption that a new metaphysical entity has been discovered but rather that language itself in its secondary use within the language game of aspect seeing allows us to see

things – logical concepts, the visual field - in different ways. In this regard such secondary use of language which the early Wittgenstein rejected as nonsense becomes the new logical bolt of his open logical model of aspect seeing. Crucially this ability of *seeing as* is rooted not in a metaphysical or meta logical world but within the matrix of language acquisition and ostensive referential language use, relating to the public world which we all see and learn to see initially in the same way with the acquisition of a public language.

This new position retrospectively reveals Wittgenstein's own misguided view – representing a certain form of aspect blindness in the Tractatus - which fails to perceive the process of logical analysis as an unbounded language game - wherein logical concepts cannot logically have a final fixed form or definition. The crucial point is that this realisation is no longer prohibitive in speaking of the logic of our language or in the process of logical analysis itself – as both were clearly retrospectively operative in the Tractatus itself:

One might also ask: Does a man who regards the sign III..... as a sign for the concept of number (in contrast with III to denote 3) see the first group of lines differently from the second? Even if he does see it differently does he *see* there anything like the *essence* of the concept number?

The example is the point of departure for further calculation. (Wittgenstein 2005, pp. 272 273)

In concluding that the example is a point of departure for “further calculation” the use of aspect seeing constituted by the secondary use and meaning of language, alongside a secondary way of seeing is made explicit through illustration. In this sense aspect seeing as Wittgenstein presents it becomes a meta-analysis of the language game of logical analysis. The fact that there are multiple and opposing ways of seeing exclude the possibility of an ideal ‘essence’ of the concept number

– and retrospectively of logical form etc. - indicating an open rather than a closed logical model. To the following different ways of *seeing as* correspond different logical models of representation of what is seen with the conclusion for Wittgenstein that there is no final one correct way of seeing just as there is no one final correct system of logical analysis.

In this context he remarks that “to know the prime numbers only up to 7 and thus to have a finite system” when contrasted with “what we call the discovery that there are infinitely many primes” is in truth “the discovery of a system with no greater rights than the other”.(Wittgenstein 2005, p.322) As such the discovery of the prime numbers – of infinity – can only be generated by means of one’s primary use of numbers. As such an essential self reference between primary and secondary language, resides in the movement from following a rule to creating a new rule, from a primary way of seeing to a secondary way of seeing, and from a primary use of language to a secondary use:

Imagine a calculating machine that calculates not with beads but with colours on a strip of paper.....If it’s a question of different colours, you can imagine a way of thinking in which you don’t say that here we have two colours but here we have a distinction between colours; a style of thought which does not see 3 at all in red, green and yellow; which does indeed recognise as a series, a series like: red; blue; green; yellow; black; white; etc. but doesn’t connect it with the series I; II; III etc., or not in such a way as to correlate I with the term red.(Wittgenstein 2005, p. 322)

This meta-analysis of the process of logical analysis reveals the complexity of how linguistic aspect seeing is expressed within different logical models. The ability to see *something as* is now entirely dependent on the possibility of self reference being inherent in the language game which allows ones primary experience of a concept –

numbers or colours – to be extended to generate a secondary way of seeing the primary use. This is therefore achieved by the ability of language to reflect back upon itself - to essentially look at itself in the process of logical analysis - under the guise of a new aspect:

We can describe the way a rectangle is divided by saying: it is divided into five parts, or 4 parts have been cut off it, or; its division schema is ABCD, or; you can reach every part by crossing four boundaries or; the rectangle is divided (i.e. into 2 parts) one part is divided again, and both parts of this part divided, etc. I want to show that *there isn't only one method of describing the way it is divided.* (Wittgenstein 2005, p.326 328)

While this means that logic has to sacrifice a general universal consistency as expressed in the Tractatus, this sacrifice is rather essential to the value of the language game itself. In this sense the concept of pictorial representation so central to the Tractatus takes on a very new significance where logic is now longer a sterile cold system of prohibitive laws, but is understood as being an inherently inventive process:

To say that a proposition is a picture gives prominence to certain features of the grammar of the word "proposition".

Thinking is quite comparable to the drawing of pictures.(Wittgenstein 2005, p.163)

In the language game of logical analysis, we thus show language in its secondary use rather than revealing some metaphysical realm. Rather the metaphysical is constituted and as such created by language and in this sense removing the

metaphysical as understood previously, deepens the possibilities of the language game rather than removing them. Wittgenstein's development of the concept of aspect seeing and its relation to bounded and unbounded language games - in the following referred to as closed and open respectively – thus accommodates the problem of the infinite and its generated problem of paradox, self reference and contradiction within the structure of an open logical model.

He presents a tribe with two systems for counting. People learn to count “with the alphabet from A to Z and also with the decimal system.” The first represents “the closed way”, the second “the open way”, and they are taught “that the arithmetic of their language is not a finite one, that their series of numbers has no end.”(Wittgenstein 1965, p.95) For Wittgenstein this corresponds to the problem of the infinite, of paradox and self reference, such that “when numerals are constructed ‘indefinitely’ we say that people *have* the infinite series of numbers”. Introduced by way of his example he points out that there is now “nothing mysterious about the word “open” (Wittgenstein 1965, p.95):

This word correspond to our “infinite” and the games we play where the latter differ only by being vastly more complicated. In other words our use of the word “infinite” is just *as straightforward* as that of “open” in 31) and our idea that it's meaning is ‘transcendent’ rests on a misunderstanding. (Wittgenstein 1965, p.95)

He concludes that all such unlimited cases of language use “are characterised by this: that they are not played with a definite supply of numerals, but instead with *a system* for constructing numerals (indefinitely). (Wittgenstein 1965, p.95) Of course the system or model in question is one the logician creates or invents within the language game of aspect seeing as opposed to discovers. It is only when we try to formalise the language game of aspect seeing – the unbounded, open or infinite

game – as Wittgenstein tried to do in the Tractatus that a false picture will be generated:

Reflections such as the preceding will show us the infinite variety of the functions of words in propositions, and it is curious to compare what we see in our examples with the simple and rigid rules which logicians give for the construction of propositions. (Wittgenstein 1965, p.83)

In effecting a logical transition from the domain of visual seeing to linguistic seeing, it becomes clear that within every unbounded or open system of secondary language use in the language game of aspect seeing, the concept of new rule formation is central, but can only be generated by reference to one's primary language use or to other systems of secondary use– not to some ideal hidden logical language. Most significantly the concept of pattern and rule formation which deviates and breaks off from a primary use, reflects the fractal nature of language itself such that just in the same way in which different fractal curves exhibit a unique or specific fractal pattern, within a given logical model – allowing the transformation of the continuum from an equation to a curve - so too language itself exhibits this fractal dimension:

7)B has a table in which written signs are placed opposite to pictures of objects (say a table, a chair, a tea cup)...The pictures in 7) and other instruments of language which have a similar function I shall call patterns. (This explanation as others which we have given is vague and meant to be vague) We may say that words and patterns have different kinds of functions. (Wittgenstein 1965, p.84)

In language when we “make use of a pattern we compare something with it e.g. a chair with the picture of a chair.” (Wittgenstein 1965, p.84) However when an ostensive correlate is absent the extension of the concept of pattern as seen in the process of logical analysis, does not presuppose a metaphysical correlate. In highlighting this point Wittgenstein’s notes that “the distinction ‘word/pattern’” is “not set up as a final logical duality.” Rather it is to highlight the open ended nature of the language game of aspect seeing whereby the words “ “one” “two” “three”, etc.” can be substituted for a pattern such as ““ \_\_\_ ” “ \_\_\_ \_\_\_ ” “ \_\_\_ \_\_\_ \_\_\_ ”” (Wittgenstein 1965, p.84)

Suppose in a language the numerals were “one” “one one” “one one one” etc. should we call “one” a word or a pattern. *The same element may in one place be used as a word and in another as a pattern.* A circle might be the name for an ellipse or on the other hand a pattern with which the ellipse is to be compared by a particular method of projection.(Wittgenstein 1965, p.84)

Such pattern formation had already been linked in Remarks on the Foundations of Mathematics to the invention of systems as seen in higher order logic:

Dedekind gives a general pattern of expression; so to speak a logical form of reasoning. (Wittgenstein 2001, p.294)

The following distinction between copying and deriving is highlighted showing how language in its secondary use operates in the invention of new linguistic systems again illustrating the nature of the process of logical analysis. The remark

opens with reference to a pupil who has begun to derive a new system instead of simply copying by rote that which he sees:

He didn't stick to this way of transcribing. In fact he changed it, but according to a simple rule: After having transcribed "A" into "n" he transcribes the next "A" into "o", and the next "A" into "p", and so on. But where is the sharp line between this procedure and that of producing a transcription without any system at all? Now you might object to this by saying in the case 71) you obviously assumed that he *understood the table differently*; that he didn't understand it in the normal way" But what do we call "understanding the table in a particular way?" (Wittgenstein 1965, p. 125)

The logical point here is twofold. Firstly there is no inner process – for e.g., thinking, reading, deriving - occurring within the individual independently of the linguistic process – which crucially is an outward process. What is occurring is rather the process of understanding the table differently or seeing it differently within the language game of aspect seeing. Within the language game of logical analysis we are likewise involved in the inventive and infinite process of secondary language use and meaning. This is not a mysterious inner process, nor is it a process which points to a metaphysical object which the system is somehow referring to – such as infinity or in Wittgenstein's case a hidden primary language of fixed elementary propositions. Rather the invention of the system constitutes the concept or the logical model created and the logicians own experience of secondary language use or linguistic seeing as:

It was not the function of our examples to show us the essence of 'deriving', 'reading' and so forth through a veil of inessential features;



the examples were not descriptions of an outside letting us guess at an inside which for some reason or other could not be shown in its nakedness. (Wittgenstein 1965, p.125)

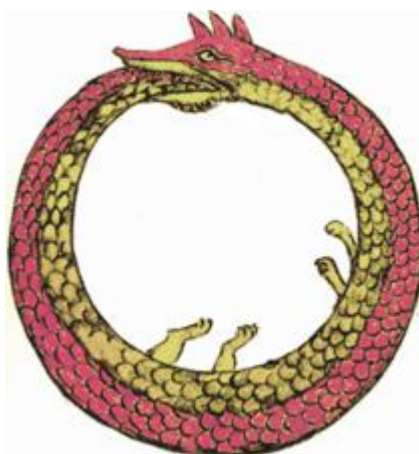
All of the aforementioned concepts – reading deriving etc- are linguistic activities central to what today we refer to as critical and creative thinking skills, reflecting what Wittgenstein regards as the secondary use and meaning of language within the language game of aspect seeing. In tracing the logical evolution of Wittgenstein's work this chapter has shown how the concept of the infinite, paradox and self reference ultimately led to his later conception of language games which is presented in the Blue and Brown Books as bounded and unbounded games. That two inherently contradictory processes as presented in this section– referential and non referential language use – operate in tandem via self reflexivity and self reference is not paradoxical when considered from the perspective of aspect seeing which accommodates the dual aspect of language as expressed in Russell's paradox and the duck rabbit illustration. Nor is the concept of the infinite problematic when understood as reflecting the infinite possibilities of the unbounded language game. What is also revealed is the sustained influence of Russell's open logical model which is implicit in Wittgenstein's logical evolution. In conclusion the result of Wittgenstein's revision of the logical position of the infinite and associated problem of paradox and self reference not only reveal his own logical evolution allowing us to see how language in its secondary use and meaning operates, but also signals the applicative potential of his work, which as I will highlight in the conclusion is one of Wittgenstein's greatest legacies.

## *Chapter Seven*

### *The third stage of evolution in Wittgenstein's analysis of the infinite and paradox and self reference*

#### *Aspect Seeing – A Retrospective consideration of Language looking at Language*

##### *7.1 Pedagogical Method, Self Reference and Reflexivity as illustrated in the contrast between the logical models of the early and later Wittgenstein*



Pelecanos, T. Ouroboros (1478)

Retrospectively considered, Wittgenstein's later work on aspect seeing reveals that the infinite has its roots not in any metaphysical realm but within the acquisition of language and the infinite possibilities of seeing as, which are inherent in both the visual field and language. In this sense Wittgenstein's earlier closed logical model can be seen as an exemplar of the secondary use and meaning of language which is now being 'looked' at and 'seen as' through the secondary use and meaning of language as represented in aspect seeing. This reveals the inherent self referential and self reflexive characteristic in the dynamic between his early and later work, whereby his analysis of aspect seeing functions as a meta-analysis not only of his earlier work, but of the status of the ladder of non referential use of language in the process of logical analysis itself. As the image of self reference above suggests, the

closed logical model of the Tractatus is effectively swallowed and accommodated within his later open logical model of aspect seeing becoming part of the language game of aspect seeing, allowing us to retrospectively see the Tractatus as an exemplar of aspect seeing or aspect blindness. In this sense Wittgenstein looking at Wittgenstein, illustrates the logical principle of the language game of aspect seeing, wherein the self referential and self reflexive ability of language allowing language to look at itself and transform itself is revealed. The seeing of aspects on the continuum, within the ambiguous images such as the duck rabbit, and within Russell's paradox itself thus retrospectively reflects Wittgenstein's journey from the closed logical model of the Tractatus to the open logical model of aspect seeing. The use of aspect seeing in logical models in the field of mathematics represents just one illustration of aspect seeing. It is of course equally present in other 'systems' and logical models, generated via the secondary use and meaning of language – in theoretical physics, the great literary and artistic works, and in the games children play.

While in the Investigations much of the groundwork which features in his middle period regarding the infinite, paradox and self reference is no longer explicit in his writings on aspect seeing, its implicit presence is evident. There are some significant reminders of the role which both the infinite and the problem of paradox assumed in arriving at the open logical model of aspect seeing which signal the final evolution of the concept of the infinite, Russell's paradox and self reference.

Firstly we encounter the problem of paradox as it features in the writing of Saul Kripke, which was addressed in the literary review, where the paradox of rule following is shown to be inherent in the language game of aspect seeing. Secondly we see Wittgenstein's intellectual nod to the role which the infinite played in transforming the closed logical model of the Tractatus. One of the most significant remarks regarding the infinite features in part two of the investigations, where

Wittgenstein is illustrating the impossibility of defining aspect seeing. He first remarks:

163 You can think now of *this*, now of *this*, as you look at it, can regard it now as *this*, now as *this*, and then you will see it now *this* way, now *this*" – What way? There is after all no further qualification (Wittgenstein 2009, p.211)

He then relates the process and activity of logical analysis – seeing something now this way now that – to the imaginary numbers of the infinite on the continuum, where their status is now that of an infinite possibility as opposed to an infinite reality as it was in his early work:

164 (It is in this sense that there is no room for imaginary numbers in the continuum of real numbers. And this surely means: the application of the concept of imaginary numbers is less like that of real numbers than is revealed by the look of the calculations. It is necessary to descend to the application, and then the concept finds a different place – one which, so to speak, one never dreamed of) (Wittgenstein 2009, p.211)

Significantly the imaginary numbers referenced here - also known as complex numbers - are used in the construction of fractal curves, illustrating how the complex spatial objects of the Tractatus which were originally used to reject the concept of the infinite, have come full circle. Here the sense of the fractal connection self references his earlier position and in turn reflects the very process which has led Wittgenstein to this insight. Of course the sense of the infinite is

now embedded in the language game being played, where the tension and self reference between the concepts of closed and open, finite and infinite, and primary and secondary uses and meaning of language find an applicable context in the “unlimited application of a rule”:

147 So you mean you know the application of the rule of the series quite apart from remembering actual applications to particular numbers. And you'll perhaps say “Of course! For the series is infinite, and the bit of it that I could develop finite.”(147 PI)

218 Whence the idea that the beginning of a series is a visible section of rails invisibly laid to infinity? Well we might imagine rails instead of a rule. And infinitely long rails correspond to the unlimited application of a rule. (Wittgenstein 2009, pp.64,91)

This unlimited application of a rule is now in turn reflected in the pedagogical process:

208 Teaching which is not meant to apply to anything but the examples given is different from that which ‘points beyond’ them. (Wittgenstein 2009, p.89)

Such pointing beyond can now be considered as the beginning of the language game of aspect seeing or logical analysis, wherein secondary use and meaning of language assume a creative and inventive role in the process of generating new logical models. This position standing in opposition to the views set forth in the Tractatus where invention was prohibited and denied any logical validity:

492 To invent a language could mean to invent a device for a particular

purpose on the basis of the laws of nature (or consistently with them), but it also has the other sense, analogous to that in which we speak of the invention of a game.

Here I am saying something about the grammar of the word 'language' by connecting it with the grammar of the word 'invent'. (Wittgenstein 2009, p.145)

These remarks considered together, reveal the complexity involved in the logical journey from closed to open logical model. Wittgenstein's earlier position that there was only one possible method of projection – that of isomorphic projection - concealed “the fact that the picture *plus* the projection lines leaves open *various methods of application*.”. (Kenny 1994, p.43) In order to reach this new logical position the non referential use of language had itself to be granted a logical experiential status which the Tractatus had ruled out:

So if a man has not learned a language, he is unable to have certain memories? Of course – he cannot have verbal memories, verbal wishes or fears, and so on. And memories etc. in language are not threadbare representations of the real experience; for is what is linguistic not an experience?

This radically new approach to the picture theory of meaning allowed the paraconsistent rule of “various methods of application” to replace the isomorphic principles laid down in the Tractatus, revealing in turn the self referencing between language as presented in the closed logical model of the Tractatus and in his later open logical model of aspect seeing. In the Tractatus logical form functioned as the logical background, assuming a mysterious and paradoxical position – which shows itself but cannot in turn be spoken of. Instead of this logical background -

retrospectively reflecting Wittgenstein's earlier secondary use of language and aspect seeing/blindness - which had silenced forever the process and activity of logical analysis itself, the pedagogical process of the mastery of a primary language now assumes this essential background role. In this sense the pedagogical and the logical become two sides of the same coin:

Language games are the forms of language with which a child begins to make use of words. The study of language games is the study of primitive forms of language. When we look at such simple forms of language the mental mist which seems to enshroud our ordinary use of language seems to disappear. We see that we can build up the complicated ones from the primitive ones by gradually adding new forms.(Wittgenstein 1965, p.17)

Moreover retrospectively considered the paradoxical duality inherent in the concept of logical form can be seen as an aspect switch – seeing in one way, now another – revealing Wittgenstein's own experience of aspect seeing. Aspect seeing, considered as a meta-analysis of the process of non referential secondary language use and meaning, and retrospectively evident in the Tractatus, demands looking at language in an entirely different manner:

12 It is like looking into the cabin of a locomotive. There are handles there, all looking more or less alike. But one is the handle of a crank, which can be moved continuously (it regulates the opening of a valve); another is the handle of a switch, which has only two operative positions...(Wittgenstein 2009, p.10)

Wittgenstein's new logical position on the infinite and the role of self reference, ultimately led to the inversion of the primary secondary language distinction, allowing the concept of meaning in logical analysis and the secondary use and meaning of language to assume a new dynamic character:

When language games change then there is a change in concepts and with the concepts the meanings of words change.(Wittgenstein 1975, p.10)

The concepts of primary and secondary language use as operative in the process of logical analysis, thus parallel early language use where in the first instance language is taught by means of ostensive reference, and where in the second case the primary use of language evolves through the language game of aspect seeing to more complex language games. Here self reference functions as an essential logical part of secondary language use where, it is exhibited in the interface between the transitions from primary to secondary uses of language. By extension we can appreciate the self reference between logical models wherein this primary secondary distinction itself becomes extended to deal with the secondary use of meaning of language. When asked what we mean by the rules of fat and lean when referring to Tuesday and Wednesday Wittgenstein observes that:

I could only explain the meanings in the usual way. I could not point to the examples of Tuesday and Wednesday. Here one might speak of a primary and secondary sense of a word. It is only if the word has the primary sense for you that you can use the secondary one. (Wittgenstein 2009, p.227)

The purpose of the secondary use of language within the language game of aspect seeing as illustrated above is thus to account for the non referential use of language



in the process of logical analysis, where there is no public object to serve as an external reference. Wittgenstein's comment in the Blue and Brown Books that logical "discussions constantly compare language with a calculus proceeding according to strict rules." a problem found in both "the sciences and mathematics" (Wittgenstein 1965, p.24) can retrospectively be seen as applicable to the closed logical model of the Tractatus:

The man who is philosophically puzzled sees a law in the way a word is used, and trying to apply this law consistently comes up against cases where it leads to paradoxical results. (Wittgenstein 1965, p. 27)

While the non referential use of language in the language game of logical analysis is complex, the context and logical foundation is now entirely unproblematic, unlike in the Tractatus where the logical foundation of logical form lead to impossible, paradoxical and baffling difficulties:

Would a child understand what it means to see the table 'as a table'? It learns this is a table, that's a bench etc., and it completely masters a language without any hint of there being an aspect involved in the business.

"Yes it's just that the child doesn't analyse what it does"

Once more: what is in question here is not an analysis of what happens. Only an analysis – and this word is very misleading - of our concepts. And our concepts are more complicated than those of the child; in so far as our words have a more complicated employment than its words do.(Wittgenstein 1980, pp.81 82)

Unlike the child the logician who plays the game of logical analysis, is looking at and seeing concepts in different ways and in particular is seeing them through the aspect or prism of rule formation and rule following. When logicians “talk about investigating, analysing the meaning of words” Wittgenstein cautions that we must not forget that “a word hasn’t got a meaning given to it as it were, by a power independent of us, so that there could be a kind of scientific investigations into what the word really means.” In short “a word has a meaning someone has given to it.”(Wittgenstein 1965, p.27):

There are words with several clearly defined meanings. It is easy to tabulate these meanings. And there are words of which one might say: They are used in a thousand different ways which gradually merge into one another. No wonder that we can’t tabulate strict rules for their use. (Wittgenstein 1965, p. 27)

The essential difference between the language game of primary language use and secondary language use or aspect seeing thus amounts to the fact that in the latter language game the role of an objective public content to which we point is redundant. Meaning Wittgenstein retrospectively observes is a “primitive concept” wherein “the word means this belongs to it” (Wittgenstein 1982, p. 46) Here:

The explanation of a meaning is by pointing. This works well in certain circumstances and with certain words. But as soon as the concept is expanded to include other words difficulties arise. (Wittgenstein 1982, p. 46)

Referential and non referential uses of language now serve to highlight the fundamental difference which the concept of meaning has in relation to both

language games. In rejecting the Augustinian referential picture of language he thus remarks:

For a large class of cases – though not for all – in which we employ the word ‘meaning’ it can be defined thus: the meaning of a word is its use in the language. Any the meaning of a name is sometimes explained by pointing to its bearer. (Wittgenstein 2009, p.18)

This difference in meaning as operative in cases of seeing and seeing as is emphasised in the following:

A difference ‘trying to see something’ and ‘trying to form an image of something’. In the first case one says ‘Look just over there!’ in the second ‘Shut your eyes’. (Wittgenstein 1980, p. 14)

The logical foundations of logical analysis are no longer something which is shown but paradoxically cannot be spoken of, but the original acquisition of language. Wittgenstein thus makes the following remark regarding aspect seeing urging us to avoid attempting “to make fine distinctions” as he had done in the *Tractatus*:

161 The everyday language game is to be *accepted*, and *false* accounts of it characterised as false. The primitive language game which children are instructed in needs no justification. (Wittgenstein 2009, p.210)

We thus begin with the acquisition of the public language – primary language use – *from which* the experience of seeing as or secondary language use evolves through self reference to one's mastery of its primary use:

The language game “What is that?” – “A chair” – is not the same as – “What do you take that for?” – It might be a chair.

To begin by teaching someone “That looks red” makes no sense. For he must say that spontaneously once he had learned what ‘red’ means, i.e. has learned the technique of using the word.

(Any explanation has its foundation in training. (Educators ought to remember this)

“It looks red to me” – “And what is red like?” – “Like this” Here the right paradigm must be *pointed to*.

Why doesn't one teach a child “It looks red to me” from the first? Because it is not yet able to understand the rather fine distinction between seeming and being?

The red visual impression is a new concept. (Wittgenstein 1970, p.75)

Crucially in this context the secondary employment of language cannot get into conflict with the original primary use and as such the logical foundations of language acquisition remain necessarily unchanged by secondary language use. Thus in the creation of logical models there is no final definitive one, thereby allowing the entire field logical analysis to remain open, as any secondary use of language cannot limit the field of possibilities of secondary use and meaning, which may arise from primary language use:

Suppose I had agreed on a code with someone; 'tower' means bank. I tell him 'Now go to the tower' – he understands me and acts accordingly, but he feels the word 'tower' to be strange in this use, it has not yet 'taken on' the meaning.

When I pronounce this word while reading with expression it is completely filled with its meaning...The figurative employment of the word can't get into conflict with the original. (Kenny 1994, p.186)

Considered from the perspective of aspect seeing, when logicians talk about infinity, infinite sets, type theory, paradox and self reference they are looking at and seeing language in different ways rather than using language to refer to something in any metaphysical sense. They are essentially creating complex linguistic models analogous to the code in the previous remark, which serve as new revelations of aspects within the language game of logical analysis.

The experience of finding a logical model inadequate or the rules within the model inadequate, has for Wittgenstein the same sense as someone who says "I find this handwriting unattractive"(Wittgenstein 1980, p.102) As Wittgenstein points out this experience is only possible in the following context:

If someone has just learned to read and write, can he find a handwriting unattractive? – it may perhaps in some sense put him off. It makes sense to say that someone finds handwriting unattractive only if he is capable of forming all sorts of thoughts about handwriting."(Wittgenstein 1980, p. 544)

Retrospectively this point applies to Wittgenstein's experience and perception of the open logical model as presented in Russell's type theory. In now locating his early idea of the logical form of language as constituted within the acquisition of the public language game, Wittgenstein successfully provides a consistent foundation for the ladder of non referential language use and crucially one which can accommodate the infinite and self reference, both within language and within the visual field. The difference between referential and non referential language use is thus reflected in the language game of linguistic aspect seeing – the process of logical analysis - where we are looking at language from an entirely different perspective:

Language can be observed from various points of view. And they are reflected in the respective concepts of meaning. (Wittgenstein 1970, p.104 )

As he reminds us in his later work:

Remember that you have to teach a child the concept. Therefore you have to teach it the game of evidence. You learned the *concept* 'pain' when you learned a language. (Wittgenstein 1980, p.81)

While Wittgenstein's growing realisation that aspect seeing was indeed the logical panacea to all of the logical difficulties which he had encountered himself in the Tractatus,:

Seeing aspects in built up on the basis of other games. (Wittgenstein 1980, p.96)

unlike the Wittgenstein of the Tractatus, the process of logical analysis is now a limitless process, and therefore an open logical model just as type theory:

The formation of a concept has, for example the character of limitlessness, where experience provides no sharp boundary lines” (Wittgenstein 1980, p.109)

The long self reflexive journey from a closed to open model of logic through two radically different considerations of the role of the infinite and paradox is reflected in the following retrospective remark on the infinite in the Investigations. Remarking that while the concept of number just as the concept of a game can be “considered as the logical sum of corresponding sub-concepts” which reflects his early closed logical model, this is not a logically tenable limit:

68 For I can give the concept number rigid boundaries in this way, that is use the word ‘number’ for a rigidly bound concept; but I can also use it so that the extension of the concept is not closed by a boundary. And this is how we do use the word game. (Wittgenstein 2009, p.38)

Wittgenstein’s early efforts in the Tractatus can thus be seen as impossible efforts to define concepts which are part of a language game which cannot by definition be defined, as concepts used within the language game of aspect seeing do not have fixed boundaries. As he remarks:

76 If someone were to draw a sharp boundary, I couldn’t acknowledge it as the one that I too always wanted to draw, or had drawn in my mind. For I

didn't want to draw one at all.  
(Wittgenstein 2009, p.40)

Rather the purpose of all logical models as exemplars of secondary language use and meaning is to reveal “changes which would have a particular effect on someone who looked at it.”(Wittgenstein 1982, p.83) In the following example of an arbitrary cipher we can imagine a concept such as logical form or type theory assuming the place of the cipher:

I can imagine some arbitrary cipher – this for instance dz to be a strictly correct letter of some foreign alphabet. Or again, as a faultily written one; and faulty in one or more of several ways; For example, it might be a slap dash, or t typical childish awkwardness, or like the flourishes in a legal document. It could deviate from the correctly written letter in a variety of ways. – And I can see it in various aspects according to the fiction I surround it with. – And here there is the close kinship with experiencing the meaning of an isolated word. (Wittgenstein 1982, p.90)

Seeing such aspects is further linked to seeing and recognising linguistic or musical style, allowing us to appreciate how both Wittgenstein's and Russell's experience of language and respective expressions of their individual secondary use of meaning of language is immediately recognisable - even in the context of an entirely different logical model:

I hear a melody completely differently after I have become familiar with its composers style. Previously I might have described it as happy, for



example, but now I sense that it is the expression of great suffering. Now I describe it completely differently, group it with different things.

The name, the picture of its bearer.

“I feel as if I knew the city lay over there” – “I feel as if the name Schubert fitted Schubert’s works and his face.”(Wittgenstein 1982, p.101)

In using the concept of aspect seeing as a meta- analysis on the process of logical analysis itself, he remarks that the concept of seeing in its ordinary context “makes a tangled impression” (Wittgenstein 1980, p.82). Wittgenstein notes that in all such cases:

There is not one genuine proper case of such description – the rest being just vague, something which awaits clarification, or which must be swept aside as rubbish (Wittgenstein 1980, p.82).

The danger he cautions against, is the logicians’ compulsion to “make fine distinctions” just as he was in the Tractatus:

Here we are in enormous danger of wanting to make fine distinctions. It is the same when one tries to define the concept of a material object in terms of what is really seen. – What we have to do is to accept the everyday language game, and to note false accounts of the matter as false. The primitive language game which children are taught needs no justification; attempts at justification need to be rejected. (Wittgenstein 1980, p.453)

However the practice of the language game of aspect seeing within logical analysis - the playing of the game itself - demands not final fixed rules by means of which we can finally understand logic, but demands the absence of final definitive rules or definitions – a logical position which he had reached in the Blue and Brown Books:

We are unable to clearly circumscribe the concepts we use; not because we don't know their real definition but because there *is* no real definition to them. To suppose there must be would be like supposing that whenever children play with a ball they play a game according to strict rules.(Wittgenstein 1965, p.25)

Like the early Wittgenstein, in trying to tabulate strict rules for concepts which cannot by their very nature be defined we become entangled not in the rules we have discovered but in the rules we have invented, and as such we see and experience aspect seeing – the dual possibility of referential and non referential language use - as paradoxical and logically problematic. While this was experienced by both Russell and Wittgenstein, as an entanglement in their own rules, it now indicates for Wittgenstein as it implicitly did for Russell also, the essential dimension of secondary language use and meaning in the process of logical analysis:

For in those cases when things turn out otherwise than we had meant, foreseen. That is just what we say when for example a contradiction appears: “I didn't mean it like that”(Wittgenstein 2009, p.43)

In this sense the idea of Wittgenstein's later open logical model of aspect seeing being generated as a result of the secondary use and meaning of language

reflecting back on itself and swallowing itself within the new model to reveal a new aspect, is applicable to all instances of self reference between primary and secondary language use, and between different models of secondary language use. Just as one side of Russell's paradox consumes itself to reveal another, so too the entire process of secondary language use in the language game of logical analysis operates in the same manner. The non referential or secondary use of language, when considered within the framework of aspect seeing, is now understandable and demystified, without any need to limit the game of logical analysis as was the case in the Tractatus. Considering both the infinite and the problem of paradox as indicators of the experience of aspect seeing, and of the infinite possibility in the creation of new logical models, allows Wittgenstein to move away from his early conception of a closed logical model wherein concepts were artificially forced to fit in a manner which ultimately proved flawed:

The criteria which we accept for 'fitting', 'being able to', 'understanding' are much more complicated than might appear at first. That is the game with these words, their employment in the linguistic intercourse that is carried on by their means, is more involved – the role of these words in our language is other than we are tempted to think. (*This role is what we need to understand in order to resolve philosophical paradoxes.* And hence definitions usually fail to resolve them; and so a fortiori does the assertion that the word is 'indefinable') (Wittgenstein 2009, p.63)

The crucial point now is that it is logically essential to the language game that such concepts cannot be defined in a final manner, and as such the problem is not that the concept is indefinable. Rather the language game of linguistic aspect seeing

inherent in the process of logical analysis, operates in a manner no different than the following children's game:

How does one play the game it could be this too? But one could play that game for instance with a child. Together we look at a shape; or at a random object (a piece of furniture) – and then it is said: “That is now supposed to be a house”. And now it is reported, talked about and treated as if this were a house, and it is altogether interpreted as this. Then when the same thing is made to stand for something else, a different fabric will be woven around it. (Wittgenstein 1980, p.95)

Here the pedagogical sheds light on the process of logical analysis in the following sense. In both contexts of playing the language game of aspect seeing “a different fabric” is “woven around” either the object or the concept in question amounting to different ways of seeing either object or concept. The rule now is that there is no final definitive rule – the rule is you can proceed like this ad infinitum where “like this” refers not to a rule but to the invention of rules in the secondary use of language and as such to the infinite grammar and possibilities of the language game. Wittgenstein thus retrospectively remarks that the experience of linguistic aspect seeing as presented in Russell's paradox and indeed in the *Tractatus* is a technique which flows along underground:

What is showing its face here is the invisible application. We are not aware of the particular technique, for it flows along underground, as it were, without our noticing it; and not until it openly contradicts our false imagination do we suddenly become aware of it. (Wittgenstein 1980, p.417)

It is as Wittgenstein points out only when “it openly contradicts our false imagination” as it did for him in relation to the colour exclusion problem “do we suddenly become aware of it”. (Wittgenstein 1980, p.417) If at this point we ask what is aspect seeing? it should be logically apparent that such a question is itself already on the wrong logical track if a definition is sought. Aspect seeing cannot be defined but rather its logical principle can be identified, wherein meaning breaks off in a fractal like manner allowing the secondary use of language which is the life of the sign in logical analysis to emerge in the creation of new logical models. In this sense aspect seeing is innately rooted in the logical structure of our visual world, our perceptual abilities and within the logical structure of our language, only developing once one has acquired a language.

The language game of aspect seeing whether played in relation to the world of visual perception – as in the case above – or in the case of linguistic perception within the process of logical analysis, gives the language game and as such our method of representation a “new joint” which unlike the immovable elementary propositions of the Tractatus is entirely free from constraint:

You give the language game a new joint. Which does not mean however that now it is always used. The language game “What is that?” – “A chair” – is not the same as : “What do you take that for?” – “It might be a chair”.

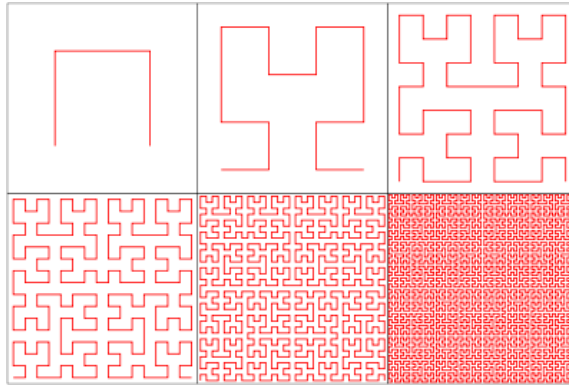
In the beginning we do not teach the child “It’s probably a chair” but “That’s a chair”. Don’t fancy for a moment that “probably” is left out because it is still too difficult for the child to understand, that things are simplified for the child.(Wittgenstein 1980, p.316,317)

The fundamental difference between aspect seeing as expressed within the games of children and within the logicians' game of logical analysis is the different surroundings in both cases:

The glance a word in a certain context casts at us. Of course the way in which it looks at us depends on the surroundings in which it is located. (Wittgenstein 1982, p. 50)

The glance which a word or concept such as logical form retrospectively casts on us when considered from the later open logical model is now reminiscent of the aspect switch expressed in the duck rabbit illustration or Russell's paradox aspect switch. We see it one way now another, logical form *as* something which is shown logical form *as* something which cannot be spoken of. How can *that* be? How can language assert something which simultaneously is and is not? What sort of logical limit or logical object is this? Retrospectively considered its function was to keep the infinite out but really it simply exposed the concept of infinite possibility as the technique flowing underground, inherent in the aspect switch which secondary use and meaning of language allows to emerge. The aspect switch and puzzlement logical form generates can now be considered as reflecting Wittgenstein's own entanglement in a set of rules which cannot be applied to the language game of logical analysis he was playing as there is no final atomic, or elementary proposition. In this way also we can appreciate how the fractal connection as an explanatory device and as a conceptual device implicit in the work of Wittgenstein reveals the constant recursive self referencing (language looking back on itself) and iterative (language moving beyond itself to create new pictures and meaning) movement between primary and secondary meanings of language. This phenomenon of the language game of logical analysis is duly presented in the logical tension between open and closed logical models, via the self referencing between primary and secondary language use and meaning within referential and non referential contexts, and as such primary and secondary ways of seeing

linguistically and visually. We can now consider the new validity of the complex spatial objects of the Notebooks as illustrative of this dynamic:



Hilbert D., Hilbert Curve (1891)

In the Tractatus logical analysis had been considered as an activity which produced nonsense propositions, as a direct result of making clear and identifiable errors in language use and proposition formation. Moreover such errors were seen by Wittgenstein as being resolvable and most importantly as logical errors over which one had a choice of either making or not. This Tractarian position carried with it the implicit assertion that there is only one way to do philosophy, strictly only one correct way to see the world – which is to draw limits to it.

While in the language game of aspect seeing as operative in logical analysis, the logical models we invent are obviously more complex than those of children the logical principle underpinning such games is no different in kind:

Well our operations are not more remote from that object than is, say, dividing in the decimal system from sharing out nuts. (Wittgenstein 2001, p.368)

That the pedagogical process – considered as a process of looking at language – sheds light on the process of logical analysis looking at language and revealing itself under new aspects is indeed critical, illustrating both that the process can never be finalised that the logician must always be looking for new aspects just as the student who is learning:

I would never have thought of laying the two pictures one on the other like *that*, of comparing them in *that* way. For they suggest a different method of comparison. The ← picture hasn't even the slightest resemblance to the → picture, one would like to say although they are congruent.

One needs to remember that seeing-as may have an effect like that of an alteration of what is seen, e.g. by putting between brackets, or underlining, or making a connection of one kind or another etc. and that in this way again there is a similarity between seeing-as and imagining. (Wittgenstein 1980, pp.155, 173)

Retrospectively considered the criteria within the Tractatus for resolving nonsense or meaningless propositions was achieved by comparing “them with reality”. The question which Wittgenstein had not asked in the Tractatus is how did he arrive at a position wherein he himself could construct a logical model which had no isomorphic relation to reality?, a system which was by his own admission both nonsense in a significant manner at the time it was written, and retrospectively nonsense in a logically flawed and untenable manner. Retrospectively considered Wittgenstein's logical analysis was itself constituted by playing the language game of linguistic aspect seeing rather than decreeing the activity itself as nonsense – as he believed he had achieved by suavely rejecting the Tractatus itself as nonsense. Rather he was playing the language game of linguistic aspect seeing where the



secondary use and meaning of language served to invent as opposed to discover the atomic logic of showing which lay concealed by ordinary secondary language. Considered from the perspective of his later work, the Tractatus is therefore entirely dependent on the open logical model of aspect seeing and the infinite possibilities of language both of which the Tractatus rejected. As such the Tractatus and Russell's type theory can be retrospectively seen as respective exemplars of aspect blindness and aspect seeing. While the early Wittgenstein had rejected the non referential use of language in the process of logical analysis, he now realises its absolute necessity in that same process – albeit in an entirely different way:

For understanding a sentence we say points to a reality outside the sentence. Whereas one might say "Understanding a sentence means getting hold of its content; and the content of the sentence is in the sentence. (Wittgenstein 1965, p. 167)

The content of the sentence and by extension of the logical model is rooted in the experience of meaning which is the central characteristic of aspect seeing, where "The case of 'meaning experienced' is related to that of seeing a figure as this or that. (Wittgenstein 1980, p.186) Here the concepts of aspect seeing and linguistic experiences of meaning are presented via an entirely subjective illustration which is devoid of any objective referential counterpart which could justify the proposition on isomorphic grounds:

Someone might see a boulder and exclaim: "A man!", and then he might point out to someone else how he sees the man in the boulder – where the face is, the feet are etc. (Someone else might see a man in the same shape but in a different way.) It will be said that it takes imagination to see that, but

that no imagination is required to recognise a true to life picture of a dog as a dog.

“He’s comparing the boulder to a human shape.” “He sees a human shape in it” – but it’s not in the same sense that we say: He’s comparing the picture with a dog, or this passport photograph with a face.(Wittgenstein 1980, p.93)

Seeing the man *in* the boulder is of course no different in kind than seeing fractal curves *in* the straight line or on the continuum. With this insight the idea that there is any one correct way of seeing the boulder or the continuum is rejected, and as such any attempt to limit the use and meaning of language as operative in the process of logical analysis within a closed logical model like the Tractatus, is the real nonsense which must be exposed. If there is no one correct representation of reality, then there is consequently no necessary isomorphic relation between proposition and reality.

Thus for the later Wittgenstein “a metaphysical question is always in appearance a factual one, although the problem is a conceptual one.”(Wittgenstein 1980, p.168) Such conceptual investigations constitute the activity of logical analysis allowing what Russell referred to as new structural possibilities to emerge when seen under new aspects:

What is it however that a conceptual investigation does? Does it belong in the natural history of human concepts? – Well natural history we say describes plants and beasts. But might it not be that plants had been described in full detail, and then for the first time someone realised the analogies in their structure, analogies which had never been seen before? And so that he establishes a new order among these descriptions. He says eg

“compare this part, not with this one, but rather with that” (Goethe wanted to do something of the sort) and in so doing he is not necessarily speaking of derivation; nonetheless the new arrangement *might* also give a new direction to scientific investigation. He is saying “Look at it like *this*” (Wittgenstein 1980, p.168)

In this sense all invention through the process of logical analysis illustrates how looking at something under a new aspect “might also give a new direction to scientific investigation.” Without the ability to see something as there would only be one way of taking things, one way of representation, and one way of expression. In such unbounded games, even in the case of children’s games – which may seem far removed from the world of logical analysis – depth grammar becomes a crucial logical feature of aspect seeing whether in its linguistic or visual context:

In the use of words one might distinguish ‘surface grammar’ from ‘depth grammar’. What immediately impresses itself upon us about the use of a word is the way it is used in the construction of the sentence, the part of its use – one might say – that can be taken in by the ear. And now compare the depth grammar, say of the word to mean, with what its surface grammar would lead us to suspect. No wonder we find it difficult to know our way about. (Wittgenstein 2009, p.142)

While the early Wittgenstein can be seen to have applied a logical model wherein surface grammar dominated, his open logical model of aspect seeing demands that depth grammar as evidenced in the language game of aspect seeing assumes logical dominance allowing logical diversity to supersede logical uniformity. The use of imagination, creativity and invention in creating rules, a central feature of

the language game of aspect seeing, is no less pronounced in the games of children than it is in the process of logical analysis:

I learn the concept seeing along with the description of what I see. I learn to observe and describe what I observe. I learn the concept to have an image in an entirely different context....The concept of imagining is rather like than of doing than of receiving. Imagining might be called a creative act (And is of course so called) (Wittgenstein 1980, p.2)

The ladder of non referential language use which the Tractatus discarded, now leads us not to some final destination but to the open logical model of aspect seeing. Here the ladder of non referential language use reveals not a singular ladder or way of representing, but rather a fractal ladder, where the invention of rules holds precedence.

Accepting Wittgenstein's logic of aspect seeing therefore necessitates accepting both the infinite and self reference as positive and progressive logical forces inherent in language and visual experience where it extends to all parts of life and academic enquiry:

If there were no change of aspect then there would only be a way of taking, and no such thing as seeing this or that. (Wittgenstein 1980, p.80)

Such a language which is described in the Tractatus would logically be incapable of accommodating the reality of logical analysis as presented in all disciplines from literary and poetic expression, to theoretical physics. Consistent foundations are thus not to be discovered through the identification of an ideal language within a 'hidden' logical matrix concealed by ordinary language but rather in ordinary language itself, where we recognise that from the acquisition of a language the

problem of self reference and paradox evolve within the language game of aspect seeing in the visual field and in the linguistic field, reflected in our secondary ways of seeing and using language. Here aspect seeing in all its guises allows for infinite possibilities of expression. There is of course no conceptual vantage point which allows us to define aspect seeing, precisely because any such effort will simply produce another system which cannot of logical necessity encompass all present systems or all possible future systems. The indeterminacy of seeing something as something and its inherent paradoxical logic can be identified in the most simple of examples:

Take as an example the aspects of a triangle. This triangle can be seen as a triangular whole, as a solid, as a geometrical drawing;; as standing on its base, as hanging from its apex; as a mountain as a wedge, as an arrow or pointer, as an overturned object which is meant for example, to stand on the shorter side of the right angle, as a half parallelogram, and as various other things. (Wittgenstein 2009, p.211)

While the complexity of linguistic uses of aspect seeing changes, the fundamental logic underpinning the open logical model of the language game does not, as Wittgenstein illustrates in the following where the ability of meaning to be infinitely diverse is placed centre stage:

How does one play the game: “It could also be this?” (This which the figure could also be, which is what it can be seen as - is not simply another figure. Someone who said I see (triangle) as  $\_>$  might still mean very different things.)(Wittgenstein 2009, p. 217)

Just as the child quite forgets in playing a game “that it is a chest” transforming it through the experience of aspect seeing so that “for him it actually is a house” thereby “giving expression to the lighting up of an aspect” (Wittgenstein 2009, p. 217), so too in the language game of logical analysis, regardless of discipline, the experience of seeing something new through the invention of a new logical model – the lighting up of an aspect – is no less pronounced or fascinating.

In the classical visual instances of aspect seeing the Necker cube and the duck rabbit are seen according to which ‘rule’ we follow – either the duck or the rabbit – and while seeing it as both at the same time is logically precluded the possibility of seeing it in one way or another is inherent in the internal self reference between both rules. While such visual instances of aspect seeing are fixed expressions of the self reference of rule following, the linguistic instances of aspect seeing through the secondary use of language are not. For Wittgenstein all such efforts of logic which attempt to define the infinite considered as operative in logical analysis – and by extension to define final and definitive rules of the game endeavour to do so as a result of aspect blindness to game being played – a point applicable to the early Wittgenstein. The task is logically impossible. Just as the concept of infinity, and indeed all logical concepts – are logically unbounded – any system or theory, which attempts to produce final rules relating to them is in effect trying to define and limit the language game of logical analysis or linguistic aspect seeing itself, and is a futile effort.

Unlike the closed logical model of the Tractatus there now cannot be any final fixed limits to the process of logical analysis and by extension to the secondary use and meaning of language in that process:

‘Heap of sand’ is a concept without sharp boundaries – but why isn’t one with sharp boundaries used instead of it? – Is the reason to be found in the nature of the heaps? What phenomenon is it whose nature determines our concepts?  
(Wittgenstein 1980, p.107)

The concept 'heap of sand' by analogy illustrates the concept of aspect seeing, which operates against a background of the complexity operative in the invention and creation of logical models, which is of course the hallmark of all logical endeavour as understood by Wittgenstein in his later work:

The formation of a concept has, for example, the character of limitlessness, where experience provides no sharp boundary lines. (Approximation without a limit).(Wittgenstein 1980, p.109)

The experience which provides no sharp boundary is the secondary use and meaning of language in the process of logical analysis, and is as such a linguistic experience. It is precisely the experience of linguistic aspect seeing which stipulates against any sharp boundary lines. In enabling us to see the possibility of the infinite and self reference in a new context, the logic of aspect seeing affords these logical problems a positive dimension within both linguistic and visual perception. It is only when this is appreciated and accepted as a logical principle that the complexity of seeing – in the case of ostensive definition – and the further complexity of *seeing as* in language game of aspect seeing, can logical analysis truly begin to understand itself and extend itself in new, creative and innovative ways.

## *7.2 Inside the Self Reflexive Process of Aspect Seeing*

### *Wittgenstein reflecting Russell reflecting Wittgenstein*

While Wittgenstein's later open logical model of aspect seeing, signals a decisive move away from the logic of the *Tractatus*, he still maintains that at the centre of all logical problems is a certain "puzzlement" generated by language. As such he still holds the view that language itself is the penultimate object of philosophical analysis – albeit the role of the infinite, paradox and self reference are now essential to both understanding such puzzlement and generating new logical models, to respond to the glance which language casts upon us. While his early distinction between a primary and secondary language in the *Tractatus*, proved its undoing regarding the position of the independence of atomic or elementary propositions, the distinction now presented in a radically different and inverted form, proves *essential* in his efforts to rework that same logic:

My notion in the *Tractatus Logico-Philosophicus* was wrong: 1) because I wasn't clear about the sense of the words 'a logical product is hidden in a sentence' (and suchlike), 2) because I too thought that logical analysis had to bring to light what was hidden (as chemical and physical analysis does.)  
(Kenny 1994, p.41)

Retrospectively considered Wittgenstein describes the *Tractatus* as a "dogmatic account" of logic. In addition to dogmatism he also identifies something "much more dangerous" which "pervades my whole book (the *Tractatus*)". This was his view that:



There are questions the answers to which will be found at a later date. Thus I used to believe, for example, that it is the task of logical analysis to discover the elementary propositions. ..I did think the elementary propositions could be specified at a later date. (Pears 2003, p. 204)

From his later work it is clear that Wittgenstein's reflection on the Tractarian logic signified an experience wherein he was unknowingly held captive by a certain way of seeing language and logic:

A picture held us captive. And we could not get outside it; for it lay in our language and language seemed to repeat to us inexorably.(Wittgenstein 2009, p.53)

The concept of aspect seeing was thus presented from the outset as a framework which could capture the logical significance of the particular, reflecting in turn the non uniform nature of rule invention, as expressed in different logical models which capture the linguistic experience of aspect seeing as operative in logical analysis:

Two uses of the word "see".

The one: "What do you see there?" – "I see *this*" (and then a description, a drawing, a copy).

The other: "I see a likeness in these two faces" – let the man to whom I tell this be seeing the faces as clearly as I do myself. What is important is the categorical difference between the two objects of sight.

The one man might make an accurate drawing of the two faces, and the other notice in the drawing the likeness which the former did not see.

I observe a face and then suddenly notice its likeness to another. I see that it has not changed; and yet I see it differently. I call this experience “noticing an aspect”. (Wittgenstein 2009, p.203)

It is therefore not the similarity of each instance of aspect seeing, as expressed in a given logical model, which is significant but the difference of each instance and as such the impossibility of defining the language game itself. As Berto points, Wittgenstein’s criticisms of Gödel are not criticisms of mathematics per se but rather of the assumption – what Wittgenstein refers to as the prose – which is attached which demands similarly to the Tractatus an ideal language:

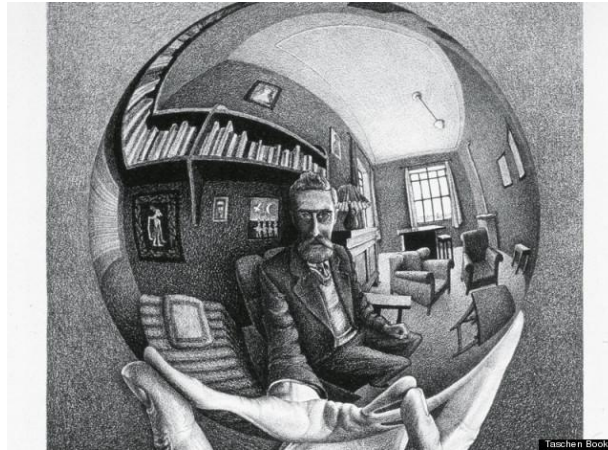
When Wittgenstein made such claims he was not questioning formalisation itself, but the overwhelming importance attached to it by philosophers and logicians looking for the ‘ideal language’. On the contrary we are now assuming precisely that formalisation is nothing but the translation of vague ordinary prose. (Berto 2009, p. 202)

While Berto’s comments are pertinent, it is also abundantly clear that the criticism is equally applicable to the Tractatus in a retrospective manner. Thus when the logician plays the language game of aspect seeing and tries to determine the rules of the game – as Wittgenstein did in the Tractatus – he will of logical necessity fail. While the Philosophical Investigations extends the concept of aspect seeing beyond the mathematical world it is only in his mathematical work that the origins of aspect seeing can be fully appreciated.

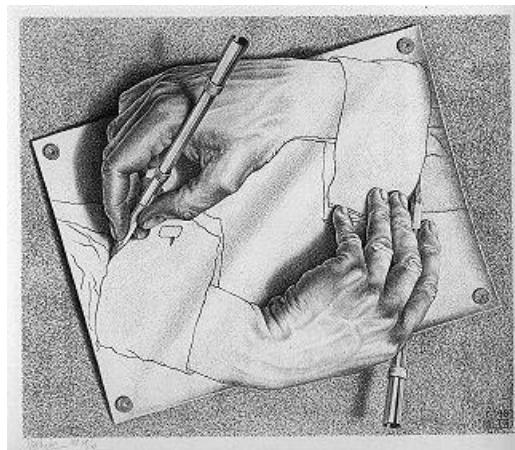
In progressing from the idea of an ideal closed logical model which he so desperately sought in the Tractatus, Wittgenstein retrospectively remarks:

We want to say there can't be any vagueness in logic. The idea now absorbs us that the ideal '*must*' occur in reality. At the same time, one doesn't yet see how it occurs there, and doesn't understand the nature of this "must". We think the ideal must be in reality; for we think we already *see* it there. (Wittgenstein 2009, p.50)

The position of the Tractatus had barred the infinite and by extension Russell's paradox and self reference, and demanded in its place a singular way of seeing language and the visual field – a structure which he maintained would be reflected in the atomic or elementary proposition – ultimately proved its fatal flaw, but retrospectively delivered a clear exemplar of the fine line between aspect seeing and aspect blindness. It was only in revisiting this flawed assumption that Wittgenstein would begin to move towards the concept of aspect seeing. While Wittgenstein instigates the concept of aspect seeing in Philosophical Remarks deep within the mire of the mathematical landscape following the crisis in mathematics, by grappling with the concepts of infinity, paradox, self reference and consistency, the Philosophical Investigations are devoid of this detailed analysis. Without this background detail the concept of aspect seeing as presented in the Investigations can seem quite a baffling and strange concept and unrelated to his earlier work. It is only in looking into the inner workings of Wittgenstein's own secondary use and meaning of language in both the Notebooks and the works of his middle period do we find a consistent link between both periods. This requires Wittgenstein's later work to self reference his earlier logical model such that one logical model is self reflexively looking at the other, just as the following drawings of Escher reveal in a visual sense:



Escher, M.C. Hand With Reflecting Sphere (1935)



Escher, M.C. Drawing Hands (1948)

In Philosophical Grammar Wittgenstein had imagined a man who claimed to be able to copy shades of red into green. In such an instance of aspect seeing the ambiguity of the rule of translation resided in the fact that we don't understand what is meant, but a clarification such as "this shade of colour is a copy of this note on the violin."(Wittgenstein 2005, p.92) could act as new aspect allowing us to understand the new meaning. In all such examples of playing the language game of aspect seeing or non referential language use – specifically the linguistic experience of aspect seeing - accounts for how the original rules and meaning of a

primary use of language break off to create new rules and meaning within a secondary use of language. Such creation of meaning thereby allows for the generation of a new system. In the new system the self reference of language between its primary and secondary employment is generated through “the logic of association” and not through isomorphic copying as had been the case in the Tractatus:

The difference between the meanings of “associate” and “copy” shows itself in the fact that it doesn’t make sense to speak of a projection method (rule of translation) for association. We say “you haven’t copied correctly” but not “you haven’t associated correctly”. (Wittgenstein 2005, p.92)

What is significant in a retrospective context is that in abandoning the referential model of language and logic, Wittgenstein is now freely talking about and referring to language itself in an unproblematic manner. Russell’s reflections on the tension between speaking about a logical model and speaking within a logical model reflect the dramatic change evident in the later Wittgenstein’s work allowing us to consider the entire project of aspect seeing as a meta-analysis of the process of logical analysis, as expressed in the secondary use and meaning of language. Russell retrospectively recalls that Wittgenstein “had maintained that the form of a sentence can only be shown and not stated”, and furthermore that “the apprehension of form was something that was ineffable in the strict sense, and only possible in virtue of some kind of metaphysical insight” (Russell 2007, p.370) He further reflects on the impossibility of this situation regarding the activity of logical analysis itself and his belief in the opposing open logical model format:

It appeared that given any language, it must have a certain incompleteness, in the sense that there are things to be said *about* the language which cannot be said *in* the language. This is

connected with the paradoxes – the liar, the class of classes that are not member of themselves, etc. These paradoxes had appeared to me to demand a hierarchy of logical types for their solution and the doctrine of a hierarchy of languages belongs to the same order of ideas. For example if I say ‘all sentences in the language L are either true or false’ this is not itself a sentence in the language L. It is possible as Carnap has shown, to construct a language in which many things about the language can be said, but never all the things that might be said: some of them will always belong to the metalanguage.(Russell 2007, p. 371)

Russell further reflects that an open logical model which allows language the freedom to operate in a non referential manner is essential to logic. He had criticised the closed logical model of the Tractatus for its failure to accommodate the infinite remarking that within the limitation of the Tractatus “the theory of the infinite collapses” (Russell 2007, p.123) It is therefore as Russell recognised only possible within an open logical model to say what the Tractatus had forbidden:

For example there is mathematics, but however mathematics may be defined there will be statements about mathematics which will belong to metamathematics...A new set of puzzles has resulted from the work of Gödel in which he proved that in any formal system it is possible to construct sentences of which the truth or falsehood cannot be decided within the system. Here again we are faced with the essential necessity of a hierarchy, extending upwards ad infinitum, and logically incapable of completion. (Russell 2007, p. 371)

In this sense not only does Wittgenstein come to utilise the very principle of an open logical model in aspect seeing, but constructs a meta-analysis which refers to the nature of logical analysis itself via the concept of aspect seeing. Moreover Wittgenstein's entire methodological approach in his later work can be seen as a clear reflection of Russell's:

The way to clarify controversial questions is by a more careful scrutiny of the premises that are apt to be employed unconsciously, and a more prolonged attention to fundamentals. After that a philosophical argument can only take the form of saying, 'Look can't you see what I see?' A philosophical advance consists in suddenly seeing a new way of looking at something. (Russell 2007, p.266)

Rather than exactness and crystalline purity logical analysis is now conceived in a radically different manner. In his own self reflexive analysis of the secondary use of language operative in the Tractatus, he had thus asserted in Philosophical Remarks that logical "vagueness isn't something provisional, to be eliminated later on by more precise knowledge" but rather that it is "a characteristic logical peculiarity" (Wittgenstein 1998, p.263) Such vagueness is evident "the moment we try to apply exact concepts of measurement to immediate experience".(Wittgenstein 1998, p.263) As in his earlier work he further connects the logical problems of definition and rule formation as originating within the visual field – albeit now in a radically different manner -observing that "This is all connected with the problem 'How many grains of sand make a heap?'" (Wittgenstein 1998, p.263) We can try to formulate strict rules such as "any group with more than a hundred grains is a heap and less than ten grains do not make a heap". (Wittgenstein 1998, .263), however "this has to be taken in such a way that ten and a hundred are not regarded as limits which could be essential to the concept

‘heap’.” (Wittgenstein 1998, p.263) He had thus asked the following question of mathematics which can also be considered as a self reflexive question:

Why do you always want to look at mathematics under the *aspect* of finding and not doing? (Wittgenstein 2005, p. 362)

In the opening of Philosophical Grammar Wittgenstein had addressed this core issue in an explicit manner highlighting the central logical error he had made in the Tractatus:

I do not now have phenomenological language or ‘*primary language*’ as I used to call it, in mind as my goal. I no longer hold it to be necessary. All that is possible and necessary is to separate what is essential from what is inessential in *our* language.

That is if we so to speak describe the class of languages which serve their purpose, then in so doing we have shown what is essential to them.

Each time I say that instead of such and such a representation, you could also use this other one, we take a further step towards the goal of grasping the essence of what is represented. (Wittgenstein 2005, p.51)

These critical remarks reveal the key logical pillars which proved essential in considering logical analysis itself as an instance of the language game of aspect seeing. Firstly his Tractarian idea of a ‘primary language’ hidden within ordinary language is abandoned. Secondly and more importantly he now observes that in “describing the class of languages which serve their purpose” the logical configuration must take account of the fact that in all instances of representation where a system lays down a set of logical rules the rules of this system can be



challenged by the fact that “you could also use this other one”. (Wittgenstein 2005, p.51)

In the Philosophical Investigations aspect seeing is thus presented as an experience wherein the infinite, paradox, self reference and contradiction are essential components, remarking that “the expression of a change of aspect is an expression of a new perception, and at the same time, an expression of an unchanged perception. (Wittgenstein 2009, p.206). Wittgenstein likens the experience to the sudden solution of a puzzle picture, reflecting the original aspect change expressed in Russell’s paradox and in the transformation of equations to fractal curves on the continuum:

I suddenly see the solution of a puzzle picture. Where there were previously branches there is now a human figure. My visual impression has changed, and now I recognise that it has not only shape and colour but a quite particular ‘organisation’.  
(Wittgenstein 2009, p. 206)

Finding solutions to puzzle pictures is qualitatively no different than Russell’s effort to find a solution to his infamous paradox, which crucially resulted in the hierarchical open ended model of type theory. In the case of visual instances of aspect seeing the illustration of the change of aspect is quite immediately accessible. However the critical logical application of aspect seeing is Wittgenstein’s extension from the visual field to language itself. This is the most significant dimension of aspect seeing as it brings into sharp relief his later acceptance of the non referential use in logical analysis as essential to that process. Within secondary language use and meaning, we see the crucial element of non referential language use being allocated what I suggest is the most essential role, ultimately becoming the logical bolt analogous to how the atomic proposition had functioned in the Tractatus. We therefore see that the inversion of the primary

secondary distinction is rooted in a radical reconsideration of how meaning operates in the new open logical model of aspect seeing:

The importance of this concept lies in the connection between the concepts of seeing an aspect and of experiencing the meaning of a word. For we want to ask “What would one be missing if he did not experience the meaning of a word?”(Wittgenstein 2009, p.225)

In the context of a comparison between referential and non referential uses of language the concept of meaning now proves critical in granting the complex process of logical analysis the highest logical status when retrospectively considered against its nonsense status within the Tractatus:

The question remains why in connection with this game of experiencing the meaning of a word, we also speak of ‘the meaning’ and of ‘meaning it’. – This is a different kind of question. – It is a characteristic feature of this language game that in this situation we use the expression ‘We pronounced the word with this meaning’ and take this expression over from that other language game. Call it a dream. It does not change anything.(Wittgenstein 2009, p.227)

In the Investigations contextualisation of the phenomenon of linguistic aspect seeing, was ultimately offered by way of the acute and unusual experience of synaesthesia, wherein the secondary use and meaning of language in the construction of new logical models is most pronounced:

Given the two concepts ‘fat’ and ‘lean’ would you be inclined to say

that Wednesday was fat and Tuesday lean, or the other way round?

Asked “What do you really mean here by ‘fat’ and ‘lean’?” I could only explain the meanings in the usual way. I could not point them out by using Tuesday and Wednesday as examples.

Here one might speak of a primary and secondary meaning of a word. Only someone for whom the word has the former meaning uses it in the latter.

The secondary meaning is not a ‘metaphorical’ meaning. If I say “For me the vowel e is yellow”, I do not mean ‘yellow’ in a metaphorical meaning – for I could not express what I want to say in any other way than by the means of the concept yellow.(Wittgenstein 2009, pp.227-228)

The logical operation of linguistic aspect seeing considered as a meta critique of the process of logical analysis itself is certainly not as immediately accessible as the purely visual instances such as the duck rabbit. In both cases there is recognition of a certain pattern and also a certain logical model either visual or linguistic. More significantly there is an internal self reflexivity and self reference between primary and a potential secondary meaning, as we encounter in for instance Russell’s type theory or Wittgenstein’s understanding of logical form in the Tractatus. In this sense all logical models function as exemplars of the tension between aspect seeing and aspect blindness, which is itself mirrored in the structure of the logical interface and self reference between primary and secondary language, and between open and closed logical models. Crucially Wittgenstein’s new open logical model of aspect seeing retrospectively mirrors the logical principle of Russell’s type theory, in defence of which Russell had argued the

following, illustrating the inherent connectedness of Wittgenstein's work to that of Russell's:

It is perfectly possible to suppose that complex things are capable of analysis ad infinitum. (Russell 2007, p. 222)

Both Russell and Wittgenstein's open logical models reflect the sentiments of Hofstadter who highlights his view that the strange loops of paradox and self reference are essential to the modern phenomenon of AI:

The flexibility of intelligence comes from the enormous number of different rules and levels of rules. In some situations, there are stereotyped solutions which require just "plain rules". Some situations cannot be classified thus there must exist rules for inventing new rules....and on and on. Without doubt, Strange loops involving rules that change themselves, directly or indirectly, are at the core of intelligence. (Hofstadter 1979, p.35)

While Wittgenstein did not get the chance to complete the Investigations in the manner which he had originally intended, there is a responsibility on the part of the researcher to extend his work in diverse and purposeful directions. This work has traced the development of Wittgenstein's work from the zero method of logical analysis in the closed logical model of the Tractatus, to the infinite method of logical analysis presented in the open logical model of aspect seeing through the concepts of the infinite, Russell's paradox, self reference and self reflexivity. I have sought to illustrate how Russell's open logical model of type theory and Russell's position on the nature of logical analysis as an open ended activity, is essentially

the same position which the later Wittgenstein adopts allowing Russell's early diagnosis of the problems inherent in the Tractatus to be seen in a new light:

That every language has as Mr. Wittgenstein says, a structure concerning which, in the language, nothing can be said, but that there may be another language dealing with the structure of the first language, and having itself a new structure, and that to this hierarchy of languages there may be no limit. (Wittgenstein and Ogden 1999, p.23)

Retrospectively considered we can understand how both Russell's and Wittgenstein's logical models –as exemplars of aspect seeing - through the process of linguistic self reflexivity and self reference allowed for the creation of the later Wittgenstein's new logical model of aspect seeing, which can be considered as a new extension to language game of logical analysis. As Wittgenstein's later work on aspect seeing stands as an open logical model and in complete contrast to the closed logical model of the Tractatus, I conclude by inverting the final line of the Tractatus to reflect the inversion of the primary secondary language distinction, and its role in creating the open logical model of aspect seeing which has been considered as a meta-analysis on the process of logical analysis itself. While the closed logical model of the Tractatus ended with the instruction - whereof one cannot speak therefore one must be silent - an apt ending here is its inverted form, reflecting Russell's logical position regarding the open ended nature of language, where we end with the instruction; whereof one sees an aspect thereof one must not remain silent – as it could perhaps be the beginning of a revolutionary logical model.

## *Conclusion*

Monk observes Wittgenstein always regarded his work on mathematics as central to the Investigations:

Despite the fact that he never returned to his work on mathematics Wittgenstein continued to regard his remarks on mathematics as belonging to his Philosophical Investigations. Thus the preface to the book written in 1945 still lists 'the foundations of mathematics' as one of the subjects about which the book is concerned. (Monk 1991, p.467)

In his later work the mathematical was not abandoned but rather transformed and expanded upon, both in terms of analysis itself and in terms of the applicative potential of aspect seeing. While in the Investigations the world of mathematics may seem far removed, I have shown that it was the set theoretical mathematical concept of the infinite which proved critical in the logical evolution of Wittgenstein's work. The transition from the closed logical model of the Tractatus, to the open logical model of aspect seeing reveals how Wittgenstein realised that the infinite, paradox and self reference rather than being logical problems to be removed were in fact central to the language game of aspect seeing of which logical analysis is a part. Here the movement from a primary use of language to its infinite potential application in its secondary use demands a constant operation of self reflexivity and self reference between primary and secondary uses and meanings of language, as we invent un-limitable new ways of representing the world through the creation of new and diverse logical models.

This thesis has endeavoured to illustrate the central role which Russell occupies in Wittgenstein's logical journey from the closed logical model of the Tractatus to his

open logical model of aspect seeing. Reflecting the evolving significance of the infinite, paradox and self reference in the parallel status of non referential language use and the process of logical analysis, I identified Wittgenstein's insistence on the independence of atomic or elementary propositions as the logical bolt of the closed logical model of the Tractatus, and his later insistence on the self reflexive and self referential nature of secondary language use and meaning as the logical bolt of the open logical model of aspect seeing. I have shown that this leads to a self reflexive conclusion where the closed logical model of the early Wittgenstein can be considered as an exemplar of aspect seeing which proves essential in the creation of his later open logical model of aspect seeing. In this sense both logical models reflect the process of logical analysis itself, which for the later Wittgenstein is constituted by non referential language use or the secondary use and meaning of language, which had been rejected in the Tractatus, and criticised specifically in Russell's open logical model of type theory. From a retrospective position I have illustrated that Wittgenstein's efforts to try to contain the infinite within a closed logical model was a logical impossibility.

Retrospectively considered from Wittgenstein's later work on aspect seeing, I have illustrated how the problem of paradox represents an exaggerated instance of linguistic and logical aspect seeing, where similar to the purely visual instance of the duck rabbit image for example, the set in question is inherently plagued by the problem of self reference and contradiction. Just like the duck rabbit, Russell's paradox suffers from the logical difficulty that two contradictory statements can accurately describe the given set. In both cases the contradiction emerges because of the intractable issue of self reflexivity and self reference within either the logical or visual set in question. I have shown that Wittgenstein identified in Russell's paradox and the concept of the infinite acute instances of what he regarded as an untenable use of non referential language which informed his early position on the role and status of primary and secondary language use in the closed logical model of the Tractatus. Most importantly I have shown how these same concepts were utilised in a retrospective and self reflexive manner to generate an inversion of the

primary secondary language distinction and in turn the open logic model of aspect seeing. While in his early work Wittgenstein was adamant that such contradictions must be removed from logic, I have shown that in his later work, self reflexivity, self reference and contradiction are no longer considered problems or logical errors which must be eliminated, but rather indicate the inherent self reflexive and infinite nature of secondary language use and meaning which is intrinsic to the very process of logical analysis itself – a realisation lost on the early Wittgenstein.

Ironically then Wittgenstein ultimately moves from a closed model of logic in the *Tractatus* where being blind to the issue of aspect perception he attempts to eliminate it by banishing the infinite, to an open model of logic in his work on aspect seeing where the infinite and its generated problem of paradox and self reference, prove essential to the logical model which bookends his life's work. This observation allows us to present the later Wittgenstein's work on aspect seeing as a process involved in the self reflexive act of language looking at itself under different aspects as operative in the process of logical analysis, which within his open logical model of aspect seeing constitutes an infinite and unquantifiable language game. As aspect seeing encompasses all logical models as exemplars of aspect seeing, we can consider how the later open logical model swallows or encompasses the early closed logical model of the *Tractatus*, which can retrospectively be utilised to demonstrate the tension between the conceptual terms aspect seeing and aspect blindness, primary and secondary language use and meaning, referential and non referential uses of language, and between closed and open logical models.

My analysis has shown that the resolute reading of Wittgenstein which claims that there is no difference in logical position between his early and later work is untenable. Rather it is essential, and the means by which the depth of Russell's influence can be revealed. This is crucial in bringing to light the progressive and dynamic nature of Russell's work which has been very much overshadowed by Wittgenstein. My analysis also supports the view that while aspect seeing is



implicit in the work of the early Wittgenstein it is not operative as an explicit working concept as Floyd argues.

My analysis has also revealed current academic analysis of Russell's paradox and its role in the Tractatus, to be limited. It is as I have argued only by tracing the complex links and aspects between his early, middle and later periods of work, that the concept of the infinite and its relation to Russell's paradox and self reference can be seen as having a fundamental role in the entire evolution of Wittgenstein's work. Not only is this central in the context of Wittgensteinian scholarship but also in the context of Russellian scholarship, opening up new and dynamic ways in which his work can be considered.

My analysis of the Necker cube compliments and adds significantly to the work of Ter hark on that specific issue. Similar to the concept of the infinite and Russell's paradox, the Necker cube and other ambiguous images, assume significance in Wittgenstein's later work as logical indicators of the language game of logical analysis, highlighting how language functions in a self reflexive and self referential manner when used in a non referential context. The work of O'Sullivan regarding a possible Tractarian theory of perception is addressed in a comprehensive manner through the consideration of how the infinite, Russell's paradox and self reference are utilised in both the visual field and the linguistic field in his early, middle and later works as a means of moving from a closed to an open model of logic. Again in identifying Wittgenstein's move from a one dimensional and limitative account of visual perception and logic in the Tractatus, to a much more logically dynamic one in his later work, the influence of Russell becomes pivotal.

In my view the most important conclusion is the applicative potential of aspect seeing which I have endeavoured to illustrate in this particular work by considering aspect seeing as a meta-analysis of the activity and process of logical analysis. I have also identified such applicative potential in the work of Berto whose consideration of Wittgenstein's work as paraconsistent logic reflects the central tenets of the work presented here. Not only has this presentation an applicative

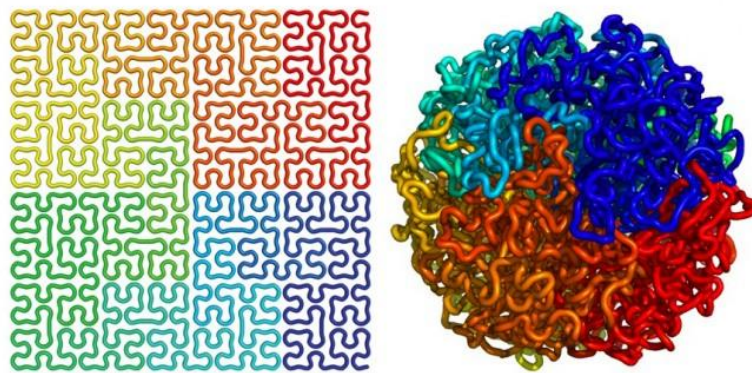
significance in an academic context, but also in a pedagogical context opening up a new way of presenting Wittgenstein's work to undergraduate students in a manner which can bring a new sense to his complex and exegetically challenging body of work. Finally, I believe that the fractal connection identified herein is highly significant not only in playing a critical role in reaching a comprehensive understanding of the logical evolution within Wittgenstein's work, or in offering an original and new way of considering the traditional picture theory of meaning as presented in Wittgenstein's earlier work. Rather its potential as presented in the work of Wittgenstein, has a multi disciplinary applicative potential and in this sense confers on traditional philosophical and logical analysis a very relevant role in the modern world.

Given Mandelbrot's discovery of fractal geometry in the latter half of the twentieth century it is fair to say that Wittgenstein was operating without the necessary language and understanding which we can retrospectively apply to his work. Just as Wittgenstein saw that within the fractal curves generated by paradox and self reference, are illustrations of the concept of aspect seeing in so far as they allow us to see the continuum in a new way, so too Mandelbrot saw in this gallery of monsters, something unique which allows us to see the world in a new way. Today there is nothing mysterious about fractals – they are used in mobile phone technology, in medical devices, in how we measure coastlines, observe the universe, and in how we understand the physiology of the human body. We also simply enjoy looking at these fractal images for their beauty and complexity – much like we enjoy and value looking at and reading the work of Joyce, Beckett, Plato or Wittgenstein.

Just as Mandelbrot succeeded in showing that fractals rather than being a gallery of monsters are in fact implicit in the structure of the world around us – in coastlines, clouds, tress, rivers etc. – recent studies have discovered that fractals are not just inherent in the world outside of us but also inherent within our DNA in the very building blocks of life. In a paper entitled *Comprehensive Mapping of long Range*

*Interactions Reveals Folding Principles of the Human Genome*, published in Science in 2009 science researchers discovered that the human genome is in fact a fractal:

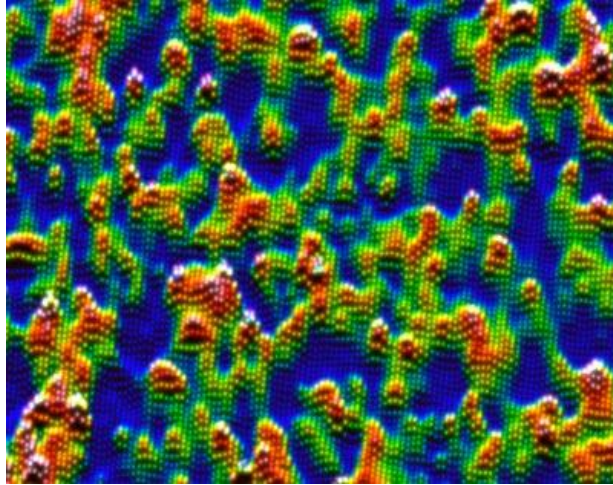
In mathematical terms, the pieces of the genome are folded into something similar to a Hilbert curve, one of a family of shapes that can fill a two dimensional space without ever overlapping – and then do the same trick in three dimensions.(2009 Science)



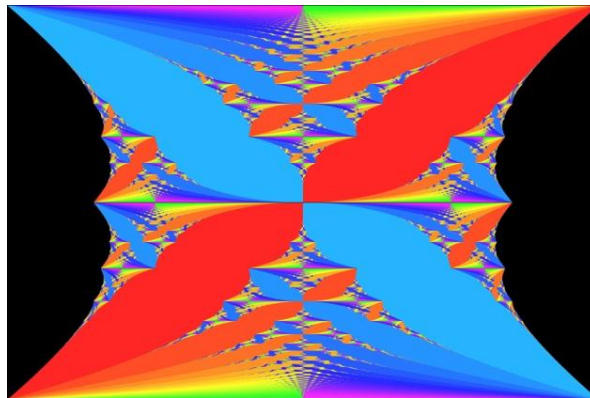
This fractal revelation was achieved “by breaking the human genome into millions of pieces are reverse engineering their arrangement to produce the highest resolution picture ever of the genomes three dimensional structure.” (Science, 2009) The picture revealed

Is one of mind blowing fractal glory and the technique could help scientists investigate how the very shape of the genome, not just its DNA content, affects human development and disease.(2009 Science)

A 2010 publication *Fractals Spotted in the Subatomic Realm* reveals that even at the subatomic level the fractal dimension is at play:



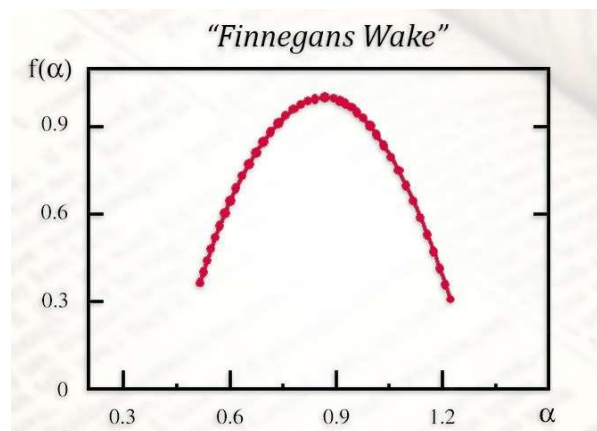
Again a 2013 publication *Physics net fractal butterfly* details the fractal dimension at play in a decades old search for a recursive pattern that describes electron behaviour, first discovered by Hofstadter.



The research which is most fascinating in the context of Wittgenstein's and Russell's work, is a publication from the Institute of Nuclear Physics and the Polish Academy of Sciences, (2016) entitled Quantifying origin and character of long range correlations in narrative texts. In this paper it is revealed that the world's greatest literary works possess an inherent fractal dimension - a dimension

Wittgenstein recognised many years earlier in the secondary use and meaning of language, and Russell recognised in his insistence that within the process of logical analysis, language always has the potential to make further structure explicit in an unlimited manner:

Regardless of the language they were working in, some of the world's greatest writers appear to be, in some respects, constructing fractals. Statistical analysis carried out at the Institute of Nuclear Physics revealed something even more intriguing. The composition of works from within a particular genre was characterised by the exceptional dynamics of a cascading (avalanche) narrative structure. This type of narrative turns out to be a multifractal. That is, fractals of fractals are created. (Drozd 2016)



As Professor Drozd observes, "The absolute record in terms of mutifractality turned out to be Finnegans Wake by James Joyce. The results of our analysis of

this text are virtually indistinguishable from ideal, purely mathematical multifractals.” (Drozd 2016) This result would I imagine have been unsurprising to Wittgenstein or Russell as they had already identified – without knowing anything about the formal categorisation of fractals – that such a dimension was operative in the logic of our language, within the language game of aspect seeing and indeed within the process of logical analysis itself. The fractal connection thus offers a new way to enrich and add to all other secondary literature referenced, which opens new points of interest in all areas of research both old and new.

One such logical application is found in the computer programming language Perl created by Larry Wall. In a speech delivered called *Perl The First Postmodern Computer Language* Wall details why Perl stands apart from all other computer programming languages. Its methodology as outlined in the following has a clear parallel with Wittgenstein’s open logical model of aspect seeing, where the onus in problem solving is on the ability of a person to use language to see things under new aspects. While Wall claims that “in many ways, it’s still the only language to do that.” Wittgenstein’s work has shown that this dimension is inherent in the structure of language itself:

How does Perl put the focus onto the creativity of the programmer? Very simple. Perl is humble. It doesn't try to tell the programmer how to program. It lets the programmer decide what rules today, and what sucks. It doesn't have any theoretical axes to grind. And where it has theoretical axes, it doesn't grind them. Perl doesn't have any agenda at all, other than to be maximally useful to the maximal number of people. To be the duct tape of the Internet, and of everything else. (www.Pperl.com)

Like the later Wittgenstein Wall contends that “things that are different should look different.” Echoing Wittgenstein’s work on secondary language use, Wall highlights the inventive dimension of his computer language where rules are not final and fixed, allowing the programmer or language user to change rules as and when necessary. Just as in traditional logical models, some computer language systems will be more useful, more elegant and more internally consistent than others:

If the burden of decision making is on the programmer, then it's possible for the programmer to make a mess of things. It's possible for Perl programmers to write messy programs. (In case you hadn't noticed.) It's also possible for Perl programmers to write extremely clean, concise, and beautiful programs.(www.Pperl.com)

Just as within the secondary use and meaning of language it is possible to invent and create new rules, new meanings and ultimately new logical models –such as type theory or the Tractatus- which are counterproductive when the nature of the language game is unclear to the language user, this same principle applies to Perl:

The very fact that it's possible to write messy programs in Perl is also what makes it possible to write programs that are cleaner in Perl than they could ever be in a language that attempts to enforce cleanliness. (www.Pperl.com)

Wall’s observation that the potential for clarity and internal consistency can only be delivered in a system which does not enforce a rule “for cleanliness” parallels the central position Wittgenstein affords to the infinite, paradox and self reference within the language game of aspect seeing and its paraconsistent ‘rule’ of infinite grammar. As logic and pedagogy are presented as two sides of the same coin in his later work on aspect seeing, I suggest a further fundamental and important area of

application is within the domain of educational and pedagogical theory and practice. The work of James Guetti is a case in point who in the *Grammar of Literary Experience* establishes the groundwork for such a potential application. In the following Guetti refers to the linguistic sense of aspect seeing as applied in a literary context:

We see through and past words in one way and this seems like seeing through them in another. But what we see through to, when we look at it, is of course words again. And when the differing process is sustained, when sequences simply continue until they stop and leave us, as it were 'hanging'...then we are propelled toward other words outside them..In this sort of case ones attention to language is always very quickly extended past or through that language in the expectation of continuing applications that are supposed to be of more interest than language itself. One looks at language here only to look beyond it. (Guetti 1993,.181)

As the later Wittgenstein retrospectively observes:

How did I arrive at the concept 'sentence' or language? Surely only through the languages that I have learnt. – But they seem to me in a certain sense to have led beyond themselves, for I am now able to construct new languages, e.g. to invent words – So such construction also belongs to the concept of language. But only because that is how I want to fix the concept. The concept of a living being has the same indeterminacy as that of a language. (Wittgenstein 1970, p.60)



How language leads beyond itself is for the later Wittgenstein through the self reflexive and self referential ability of language to look at itself, which constitutes our experience of linguistic aspect seeing specifically within the field of logical analysis. It is a skill we continuously renew and refine wherein the pedagogical significance of language looking at itself is crucial. Wittgenstein and Russell have in this sense provided the open logical model, its application is open for us to apply it as we choose. When retrospectively considered, Wittgenstein's conclusions regarding the status of the process of logical analysis are entirely positive, reflecting those of Russell's and providing language infinite possibility. The real potential of Wittgenstein's work is therefore in its applicative uses. Language it seems always offers us a new way of considering old problems, and once we recognise the dynamics of the infinite, paradox and self reference inherent in the self reflexive process of secondary language use, its potential is further enhanced and refined. This constant – and consistent - logical fractal self reflexivity and self reference between primary and secondary language use, between referential and non referential language use, and between open and closed logical models, ensures that the language game of aspect seeing is kept precisely as it should be – both infinite and fascinating.

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