

**Mathematics Subject Matter Knowledge (SMK) for Teaching at Primary Level: How much is enough?**

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**Introduction**

A mathematics lecturer responding to the NCCA (2006: 29) consultation document declared that ‘...students need their best teachers at a young age. Teachers who really know what they are doing and really understand the simplicity of what they are doing’. Therefore one must ask the question: what is it that primary teachers should ‘know’ and ‘understand’ in order to teach mathematics effectively? It is suggested that in order to be able to teach any subject effectively teachers require three categories of subject knowledge: **content knowledge** (*know the maths*), **subject specific pedagogical content knowledge** (*know how to teach*) and **curriculum knowledge** (*Primary School Mathematics Curriculum*) (O’Meara, 2010)

For the purposes of this article, the focus is on the ‘mathematics content knowledge’ (**‘mathematics subject matter knowledge’ (smk)**) required for effective teaching in the primary school classroom (Rowland *et al*, 2005). Given that primary teachers are generalist teachers, consensus exists that it is an almost impossible challenge to have expert knowledge in every subject. In spite of this, one would expect that a solid understanding of the material to be taught is a pre-requisite for good mathematics teaching. However, that is not to say that ‘good mathematics’ alone is enough.

**Mathematics Subject Matter Knowledge (SMK): Important?**

In the past there was a perception that mathematics smk is something ‘...that secondary teachers already have it and elementary teachers need very little of it’ (Rowland *et al*, 2005: 256). It is only in the past two decades that primary teachers’ mathematics smk has become a source of interest and concern. The main reason for this was that many nations especially the US and UK, following poor performance in international comparative studies e.g. TIMSS, sought to identify the ‘causes’ of this unsatisfactory scenario (Wall, 2001). Over time however internationally there is general

agreement that a teachers’ smk of the mathematics to be taught is central to effective teaching. Supporting this belief Rowland *et al* (2005) propose that teachers’ knowledge of mathematics is central in the **selection and use of appropriate analogies, illustrations, explanations and demonstrations** i.e. ‘transformation’. Subject matter knowledge is also perceived to affect **decision making regarding ‘connections’ i.e. sequencing between and within lessons** (Rowland *et al*, 2005). Mathematics smk also influences a teacher’s ability to **‘think on ones feet’** when responding appropriately to children’s questions, unexpected answers and misconceptions i.e. ‘contingency’. In short, a teacher’s mathematics smk is believed to affect his/her ability to make apt decisions regarding the most appropriate instructional materials, presentation, emphasis, and sequence of instruction (Ball *et al*, 2005).

In the Irish context, even among teachers themselves, the complexity of mathematics smk deemed necessary for effective teaching is generally underestimated. While one would assume that you knew a lot about mathematics if you announced that you were an engineer, no assumption of mathematical prowess would be assumed if you reported that you were a primary mathematics teacher (Delaney, 2008 a.; b.).

**How much Mathematics SMK is enough?**

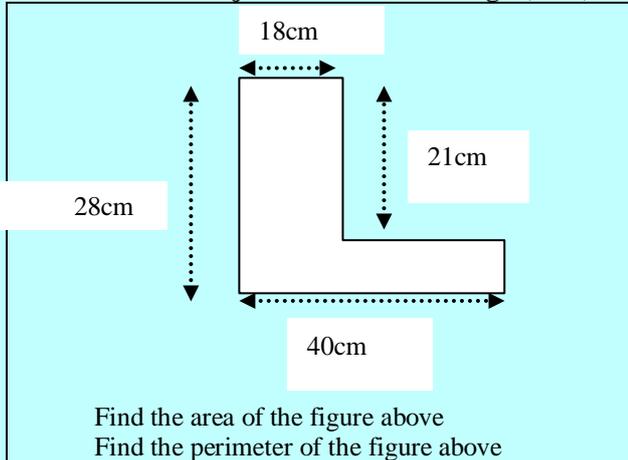
Research challenges the assumptions that mathematical concepts and procedures addressed at primary level are easy (Hourigan, 2009). While consensus exists that practicing primary teachers require ‘deep’ and ‘rich’ mathematics smk, the question remains as to *the* nature of the mathematics primary teachers require.

It is now recognised internationally that smk beyond a certain ‘threshold’ is not associated with greater pupil achievement i.e. primary teachers do not need to study mathematics to degree level (Department of Education and Science (DES), 2002). However, this finding does not suggest that a teacher’s knowledge of mathematics is irrelevant to the quality of mathematics teaching and learning.

If one were to adopt a ‘minimalist view’, this would suggest that *‘He who knows mathematics, knows how to teach it’* i.e. that any

well-educated adult possesses the subject matter knowledge required to teach at primary level (Hourigan, 2009). While teachers need to be able to ‘do’ mathematics (**‘common’ knowledge**) (see Figure 1), procedural knowledge alone is insufficient (Ball *et al*, 2005).

**Figure 1: Items requiring ‘common’ mathematics subject matter knowledge (smk)**

 <p>Find the area of the figure above Find the perimeter of the figure above</p>
Evaluate: $5 \times 8 + 15 - 10 \div 2$
Holly spent $\frac{1}{4}$ of her weekly wages on rent and $\frac{2}{3}$ of the remainder on living expenses. She saved what was left over. What fraction of her money did Holly save?
Place the following numbers in order with the smallest first: 56%      0.06 $\frac{5}{8}$ 0.056 $\frac{3}{5}$

A certain kind of mathematics smk is needed to teach the subject effectively at primary level, additional to that required by those pursuing other mathematically intensive careers e.g. accountants i.e. they must “...*know mathematics differently*”. [16: 104]. Ball *et al* (2005) refer to this knowledge as **‘specialised’**. To be able to meet the demands of a job which includes explaining terms and concepts, interpreting pupils’ statements and solutions, selecting appropriate materials and examples (Hourigan, 2009), a teacher must possess **conceptual understanding** of the various mathematical concepts and procedures as well as recognising and understanding the **interconnections between them** (see Figure 2) (Ball *et al*, 2005; Delaney, 2008 a.; b.).

**Figure 2: Items requiring ‘specialised’ mathematics subject matter knowledge (smk)**

Solve and explain your actions $3\frac{3}{4} \times 5$ Can you think of another (equally correct) way of solving this calculation?
‘A square is a special type of rectangle’ True or false? Explain your answer.

Consider the various possible approaches to solving the subtraction calculation 43-29 (mental maths, paper and pencil, estimation, concrete materials, link to addition)  
Detail the errors of the following two children:

A: 43	B: 502
$\begin{array}{r} 43 \\ - 29 \\ \hline 26 \end{array}$	$\begin{array}{r} 502 \\ - 6 \\ \hline 406 \end{array}$

A prize of €625 is divided between two sisters Ann and Brenda in the ratio 3: 2. How much did Ann get?  
Explain your procedure (i.e. why)

A student used the following procedure to multiply  $0.25 \times 5.25$

	0.25
2nd line	x <u>5.25</u>
3rd line	125
4th line	500
5th line	<u>12500</u>
6th line	1.3125

Is the answer correct?

Explain lines 4-6

Explain the position of the decimal point

The heights of 5 students are 1.4m, 1.6m, 1.5m, 1.55m and 1.45m. Find the mean (average) height of the students.

Explain your method (i.e. why): \_\_\_\_\_

### The Reality of the Situation:

#### The Nature of Primary Teachers’ Mathematics SMK

Internationally, there is overall consensus supported by reports that *‘all is not well’*. Characteristics of elementary teachers who were deemed to have ‘substandard’ mathematics smk (e.g. UK, US, Hong Kong) include a dependence on rule-bound knowledge. Shortcomings in both procedural and conceptual understanding of concepts are also reported. Other common characteristics include ignorance of connections between concepts and gaps in knowledge (Ball *et al*, 2005; Rowland *et al*, 2005). Such dissatisfaction has resulted in the introduction of ‘standards’ in the US and UK (e.g. Qualified Teacher Standards (QTS, UK)) which are rigorously tested at various levels (e.g. numeracy skills test for licensing purposes) (Wall, 2001; Rowland *et al*, 2005).

It is understandable that one may be under the impression that there is little concern regarding the mathematics smk of Irish primary teachers, given the sparse amount of publicity given to the phenomenon (Wall, 2001; Corcoran, 2005 a.). Although this issue has received little attention until the recent past, research has been carried out by a number of individual researchers

within the various Colleges of Education (e.g. Wall, 2001; Oldham, 2005; Corcoran, 2005 a.; b.; Leavy and O' Loughlin, 2006; Delaney, 2008 a.; b.; Hourigan, 2009). Such findings reflect the characteristics of their international peers. However, unlike the UK and US systems, the mathematics smk required' by Irish primary teachers is quite limited (Wall, 2001; Corcoran, 2005 b.). Once prospective teachers achieve the minimum entry requirement i.e. D3 at Ordinary/Higher Level Leaving Certificate Mathematics, they are not obliged to provide any further evidence of their mathematics smk (Corcoran, 2005 a.; b.).

### **Consequences of Inadequate Mathematics SMK**

'Weak' mathematics smk is associated with less competent mathematics teaching (DES, 2002). It is suggested that teachers cannot promote mathematical learning outside the limits of their own understanding. The coping strategies used by such teachers include

- avoiding topics altogether
- overdependence on the text
- limited interaction
- a focus on rules and procedures as isolated facts (DES, 2002).

In such contexts pupils must depend on memorization rather than understanding which in turn leads to the '*...failure to lay the groundwork for future development of student understanding*' (Leavy and O' Loughlin, 2006: 54).

### **Analysis of Past Weaknesses while Looking to the Future**

While the amount of attention given to the mathematics smk is limited, concern regarding substandard mathematical skills evident among Leaving Certificate students generally has been escalating for some time. Consensus is now widespread that, regardless of level of study or achievement, pre-tertiary mathematics education in its present form is short-changing those who wish to pursue further education (NCCA, 2006). Agreement exists that the nature of predominant classroom practices, especially at the senior cycle of second level education i.e. exam-led, teacher-led didactic approach focusing on rules and procedures which are likely to be examined, is not conducive to the development of conceptual knowledge among students (NCCA, 2006; Hourigan, 2009). Into the future, efforts to '*address the problem where it arises*' have begun.

'Project Maths' seeks to promote conceptual understanding and problem solving within realistic contexts as well as smooth transitions within and between mathematics courses at the respective levels. The success of this initiative provides hope that into the future student teachers will enter third level with appropriate conceptual knowledge. For this to become a reality sustained support is essential at all levels (Oldham, 2005; NCCA, 2006).

From a third level perspective, in the past, the sole form of preparation for teaching mathematics in many of the Colleges of Education was the mathematics pedagogy course. As these courses are expected, within limited time constraints, to provide student-teachers with the necessary knowledge to teach mathematics at all primary class levels, it is not surprising that finding the time to explicitly address student-teachers' mathematics smk proves problematic (Wall, 2001; DES, 2002; Corcoran, 2005 a.; b.; Leavy and O' Loughlin, 2006). Consequently in the past within many of these courses it was often taken for granted that the mathematical smk relating to the various concepts and procedures was addressed 'somewhere else' e.g. pre-tertiary mathematics. In such contexts no distinction was generally made between knowledge of content and knowledge of how to teach it (Wall, 2001; Ball *et al*, 2005).

In more recent times as a result of an increased awareness of the fact that the mathematics smk of some entrants to primary teacher education required enhancement, Irish Colleges of Education have made various efforts to address the issue of student teachers' mathematics smk. While some courses strive to include more explicit focus on the subject matter of the primary curriculum within pedagogy courses, other Colleges have chosen to provide some extra support in the form of 'Professional Mathematics' or peer tutoring (Oldham, 2005; Corcoran, 2005 b.; Hourigan, 2009).

### **Reflection**

It is intended that this article would act as a device to '*surface and challenge*' teachers' awareness of and desire to develop existing levels of smk (Goulding *et al*, 2002: 2).

The items (see Figures 1 & 2) provide a sample of the nature of mathematics smk required for effective teaching. These items can act as a 'self-audit' for teachers to facilitate the identification of strengths and weaknesses in both

their ‘common’ and ‘specialised’ mathematics smk.

### What Can Teachers Do?

Many teachers, on becoming aware of the types of knowledge required, may be in a position to address weaknesses through further personal study of particular concepts. At a local level support may be available from staff members with a particular strength in mathematics. Structured support may be sought through the various professional development avenues available to Irish primary school teachers. Such support may come in the guise of the Primary Professional Development Service (PPDS) or similar support bodies, Education Centres and/or summer courses. Alternatively teachers within individual schools or areas may seek to set up a ‘community of practice’ which would facilitate the sharing and resolution of subject matter knowledge issues.

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