

## **WORKING TOWARDS ADDRESSING THE MATHEMATICS SUBJECT MATTER KNOWLEDGE NEEDS OF PROSPECTIVE TEACHERS**

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*This study is the result of a perceived need for action within one Irish College of Education. During professional interactions with prospective teachers in both mathematics pedagogy sessions and teaching practice, the author (MH) considered that the number of incidents where prospective teachers demonstrated weaknesses in their mathematics subject matter knowledge was excessive. In response, the first step taken was to seek further insight into the phenomenon both nationally and internationally (Hourigan and O' Donoghue, 2007 a).*

### **Theoretical Framework**

International research highlights the importance of adequate mathematics subject matter knowledge among elementary teachers (Shulman, 1986). Consensus exists that rather than achieve extra qualifications in mathematics e.g. study to degree level, elementary teachers of mathematics require a certain type of subject matter knowledge additional to the 'common' subject matter knowledge needed for other numerate professions. It is proposed that 'specialised' knowledge of mathematics is required given the need to 'transform' his/her personal mathematical knowledge as well 'think on ones feet' in order to respond to pupil answers, queries and misconceptions (Department of Education and Science, 2002; Corcoran (2005 b.; Hill et al, 2005). Therefore conceptual understanding of the various mathematical concepts and procedures as well as an understanding of the interconnections between them is essential (Conference Board of the Mathematical Sciences (CBMS), 2001; Ball et al, 2005). There is also agreement in relation to the reality of mathematics subject matter knowledge demonstrated by both prospective and qualified elementary teachers internationally. In many cases, weaknesses are apparent in teachers' conceptual understanding of the relevant concepts and skills, and a tendency to depend on rule-bound knowledge. Shortcomings in procedural knowledge are also reported as well as gaps in knowledge and ignorance of the connections between concepts (Rowland et al, 2005; Ball et al, 2005). Within the Irish context, although this issue has received little attention until the recent past, the findings of the relevant studies reflect many of the international issues (Corcoran, 2005 a, b; Leavy and O' Loughlin, 2006; Delaney 2008 a). This is no surprise, given the highly publicised discontent regarding the mathematical abilities apparent among Irish students entering Third level numerate courses among the relevant parties (Murphy, 2002; Oldham, 2005; NCCA, 2006). The nature of the 'typical' pre-tertiary mathematics experience; that is a teacher-led didactic approach focusing on the rules and procedures likely to be examined; has been found wanting in its ability to develop conceptual understanding among students (Murphy, 2002; NCCA, 2006; Hourigan and O' Donoghue, 2007 b.) Into the future, efforts to 'address the problem where it arises' have begun. In September, 2008, following review and consultation, the phased implementation of the syllabus 'Project Maths' was initiated within a group of pilot schools. This programme seeks to promote conceptual understanding and problem solving within realistic contexts as well as smooth transitions within and between mathematics courses at the

respective levels. Undoubtedly the success of this initiative requires ongoing support for teachers and schools in the form of resources and professional development as well as changes to the terminal examination if the intended and implemented curricula are to coincide (Oldham, 2005; NCCA, 2006; EGFSN, 2008). Until the envisaged positive changes associated with the nationwide implementation of such a 'reform' approach to mathematics education at post-primary level become a reality, the predominant pre-tertiary mathematics experience is 'short-changing' entrants to Third level courses, pre-service education included. The author (MH) felt an onus to address the issue as inadequate subject matter knowledge among graduates would have negative implications for the experiences and knowledge of the pupils they teach (CBMS, 2001). Internationally responses developed by initial teacher education to address the specific needs of the population in question consists of one or more of the approaches ranging from the modification of existing courses to the promotion of self study, the development of a peer assisted learning teaching structure or the provision of an 'extra' specialised maths course (CBMS, 2001; Starkings, 2005; Corcoran, 2005 b).

### **The Study's Context**

Despite the importance associated with the development of mathematics subject matter knowledge within initial teacher education courses, the reality of time limitations within the College of Education in question meant that the main-stream mathematics 'pedagogy' classes; which are the sole form of preparation for teaching mathematics; could not begin to explicitly address prospective teachers' mathematics subject matter knowledge (DES, 2002; Corcoran, 2005; Leavy and O'Loughlin, 2006). Therefore it was deemed necessary for some 'extra' provision to be made available to prospective teachers to facilitate the 'audit and remediation' of their mathematics subject matter knowledge. A purposive sample was utilised which consisted of the cohort of second year prospective teachers (Cohen et al, 2000; Mertens, 2005). It was perceived that this cohort would be optimally motivated to partake as their subsequent teaching practice placement was in the senior classes.

### **The Methodology**

The author took a 'pragmatic' approach to the study, believing that approaches should be selected on the basis of 'fitness-for-purpose' (Cohen et al, 2000; Mertens, 2005). As the author (MH) sought to attain further insight into the perceived problem, prior to addressing the issues which became apparent, action research was selected as the most appropriate research methodology (Opie, 2004; Mertens, 2005). The study consists of two cycles of action research (i.e. Cycle 1 or Preliminary study and Cycle 2 or Main study). While the first cycle began in February 2006 and concluded in May 2006 with the selected cohort of prospective teachers, the second cycle, which built upon the learning of the preliminary phase, commenced in February 2007 with the subsequent cohort of prospective teachers. Throughout the necessary ethical obligations were fulfilled (Cohen et al, 2000). Within the preliminary study, the initial idea was 'to address the issue of substandard mathematics subject matter knowledge among prospective teachers'. The reconnaissance stage facilitated the collection of data which shed further light on the nature of mathematics subject matter knowledge apparent among the participating prospective teachers i.e. the characteristics and needs of the population i.e. through the development and administration of a paper-based

assessment. The findings reflect both national and international reports, in that a proportion of participating prospective teachers demonstrated mere procedural knowledge without limited awareness of connections. Evidence also existed that a number of students lacked conceptual knowledge of the various concepts in question, relying on memory and procedural knowledge (Corcoran, 2005; Ball et al, 2005) (For details and findings see Hourigan and O’ Donoghue, 2007 a.). Subsequently the findings were instrumental in the design of the ‘general plan’ i.e. the development of a suitable intervention.

### **Cycle 1: The Intervention**

On consideration of the potential intervention approaches, the ‘Extra Support’ model of intervention was selected as most appropriate for the particular context in light of considerations such as the potential benefits as well as the reality of financial and time constraints (CBMS, 2001; DES, 2002; Murphy, 2002; Ball et al, 2005).

The ‘Professional Mathematics’ course developed within the College of Education sought to provide support to all interested prospective teachers within the cohort who demonstrated or perceived possessing inadequate mathematics subject matter knowledge. It also strove to develop a deep and connected understanding of the fundamental mathematics concepts. The sessions provided participants with opportunities to experience the ‘reform’ mathematics for themselves, to challenge and make sense of their previous experiences, perceptions and misconceptions e.g. through sharing ideas or the use of structural materials/other representations (CBMS, 2001; DES, 2002; Oldham, 2005). In terms of the course content, a strong focus was placed on the development of deep understanding of the fundamental principles that underlie the ‘very mathematics they are charged with teaching’ particularly within the ‘weak’ areas as gauged by analysis of pre-test findings and student-feedback i.e. number (CBMS, 2001).

Regardless of involvement in the testing phase of the initiative, all prospective teachers within the cohort were invited to attend one of the three weekly alternative Professional mathematics session available to the cohort (Wednesday: 12 noon, 1p.m., 3p.m.). Weekly reminders facilitated prospective teachers to attend sessions they perceived to best meet their needs. In all, 7 Professional mathematics sessions were provided to the cohort by two members of staff (MH and a colleague).

### **Cycle 1: The Evaluation Stage**

Evaluation is defined as ‘periodic assessment of the relevance, performance, efficiency and impact (both expected and unexpected) of the project in relation to the stated objectives’ (Fort et al, 2001 cited in Mertens, 2005: 47). The author was committed to the evaluation process, open to change and the desire to do things better i.e. ‘developmental evaluation’. The findings serve a formative purpose i.e. facilitate the author in sustaining the strong features of the initiative into the future and making appropriate adjustments to the apparent shortcomings in the future (Murchan et al, 2005; Huntley, 2005). In line with Cromptons’ (1999) proposal, the author intended that the evaluation would focus on factors such as effectiveness, efficiency as well as satisfaction among the users of the initiative.

### **Conceptual Framework**

Mullan and Travers (2007) report that an evaluation should focus on contexts, mechanisms and outcomes. Reflecting this belief, Shapiro's (1987: 290) framework for the analysis of the evaluation process and findings was adopted. This states that the criteria by which the intervention should be judged include: Treatment effectiveness; Treatment integrity; Social Validity and Treatment Acceptability.

The criterion 'treatment effectiveness' requires insight into the amount of change or improvement evident among the participant group, ideally in comparison with the control group who have not experienced the intervention. The author interpreted change/improvement to refer to both mathematics subject matter knowledge as well as the beliefs/attitudes of all concerned (Mullan and Travers, 2007). The criterion of 'treatment integrity' appraises the extent to which the intervention is implemented as intended across all presentations. Social validity describes the 'effectiveness of the programme' as perceived by the consumers or participants. 'Treatment acceptability' of the intervention determines whether or not the potential participants 'like' the intervention procedure implemented. Shapiro suggests that treatment acceptability is an important criterion because even highly effective interventions fail if judged as unacceptable by the potential 'consumers'. The unintended side effects of the intervention require attention when examining the acceptability.

While this evaluation framework was developed to evaluate the intervention stage only, the author believed that these criteria would also shed light on the effectiveness of the testing phase of the initiative.

### **Multi-Method Approach**

Aware that no one methodological instrument guarantees a holistic grasp of the 'truth' and the wide range of research questions and hypotheses, a multi-method approach was utilised within this evaluation process (Murchan et al, 2005; Crompton, 1999). The sources of data included usage statistics, reflective journal, as well as the administration of a post-test and post-survey. The author believed that narrative information could add meaning to numeric data and vice versa thus increasing the validity and reliability of findings (Murchan et al, 2005).

Throughout the project, the author (MH) systematically recorded all of the relevant events, informal conversations and feedback as well as reflections and interpretations of the situation within a reflective diary (Elliot, 1991). Usage statistics were collected through a weekly log of student participation at the sessions. This data provided insight into the number of student-teachers who partook in the initiative as well as the number of return visits.

Two further instruments were administered after the initiative. As it was necessary to attain feedback from the large cohort of prospective teachers (both initiative participants and non-participants) regarding the accessibility, management and perceived effectiveness of the initiative, the survey was considered most appropriate instrument given its efficiency and the facilitation of anonymity of respondents (Opie, 2004). Although the Likert scale was used for the majority of the survey items within this study, each subsection contained open-ended items which provided respondents with an opportunity to share miscellaneous information or elaborate further on issues they felt strongly about (Draper, 2006). It was deemed necessary

to break the survey into 6 subsections addressing the feelings and experiences of the various members of the cohort in relation to the initiative i.e. A: All Students; B: Non-participants in Project; C: Participants in the Diagnostic test; D: Participants in Diagnostic Test but not support sessions; E: Participants in Support Sessions; F: Overall. On becoming accustomed to the possible questions used in similar educational settings e.g. Huntley (2005); Murchan et al (2005); a draft survey was developed. Methods utilised to enhance the reliability and validity of the survey included consultation with a ‘jury of experts’ regarding the ‘content validity’ of the survey and the subsequent piloting to a small group of initiative participants from a different cohort (n=17) (Opie, 2004). During the administration of the survey to the cohort (Monday/Tuesday 24<sup>th</sup>/25<sup>th</sup> of April (Week 11)), standard conditions were established. The sample size was 282 students, 147 of whom had participated in some stage of the initiative. The numbers who completed Sections A-F of the survey were 282, 135, 120, 39, 106 and 282 respectively.

It was also decided to administer a mathematics post-test at the end of the initiative as an evaluation tool, focusing on participants who completed both the pre- and post- tests. The use of the same instrument for the post-test was deemed inappropriate due to the threat of ‘testing’ to internal validity (Mertens, 2005). Therefore a new instrument was developed for the post-test. During pre-test creation, the author developed a large number of items from which to construct equivalent forms. The ‘equivalent form reliability’ was tested by administering both pre and post-tests to the same prospective teachers who were non-participants in the initiative (n= 8) at the same time in a bid to check the correlation of scores i.e. the Guttman Split Half method. The coefficient of equivalence was .959 suggesting that the score received by an individual was about the same on both forms (McMillan and Schumacher, 2001). The post-test in the preliminary study was administered in Week 12 (Thursday 4<sup>th</sup> May: 10 a.m.-1p.m), following notification and information the previous week. Seventeen student-teachers took the post-test, 8 of who had completed the pre-test. In terms of analysis, pre-test/post-test comparison was done through the use of a paired sample t-test measure (Mc Millan and Schumacher, 2001).

### **Findings and Conclusions**

The multi-method approach proved effective in adding depth and breadth to the findings given the complementary nature of the various approaches. The author became more fully informed regarding the participants’ beliefs and experiences, thus facilitating reflection regarding the successes of the existing service and the potential for modification. Sharipo’s (1987: 290) four criteria for effectiveness were central to the evaluation of the initiative.

The first component of the evaluation framework addresses the ‘Treatment effectiveness’ i.e. the evidence of change or improvement within participants of the intervention (Sharipo, 1987). The comparison of pre- and post-test performance found a statistically significant improvement (p= 0.01) from the small group of intervention participants involved (n= 8). As the preliminary phase of the action research did not have a control group, there is no way of knowing how these same student-teachers would have fared if they had not undertaken the support. The reality of a very small sample is also an issue. Unfortunately, the effectiveness of this aspect of the evaluation was hampered by the problem of ‘experimental mortality’

(Mertens, 2005). The author believed that this was due to the late timing of the post-test within a prolonged initiative. Also the timing of the post-test may not have been convenient for all prospective teachers. The author, aware that such results should be interpreted cautiously sought to present such findings in light of other findings e.g. post-survey (Crompton, 1999).

In relation to the survey outcomes, positive feedback comes from the fact that 86.3% of the intervention participants (n= 82) reported feeling '*...more prepared to teach mathematics at senior primary level*' (a/sa) (item 46). In terms of confidence, the finding that 78.2% (n= 79) reported gaining '*...confidence about teaching mathematics at senior level since participating in the sessions*' (item 42) is also an encouraging indicator. Similar feedback was provided directly to the author (MH): "*From the students' perspective, it is very beneficial... they are very content with the sessions and are already feeling more confident...*" (Reflective Journal (R.J.), Week (W) 9: Wed). However while these findings were to be welcomed, there is no objective data suggesting that such changes in attitude/confidence had occurred. The author intended to introduce an appropriate pre-post survey within the main study.

The second element of the evaluation framework refers to the 'integrity of the intervention' i.e. the extent to which the planned or intended initiative is implemented. A number of issues came to the fore under this category, which required attention. While the author was content that in general the initiative was implemented as intended (e.g. communication of information, the administration of pre-test) a number of unsatisfactory aspects became apparent. Firstly, with regard to the accessibility of the initiative to all student-teachers, the survey found that 18.4% (n=51) of respondents (participants/non-participants) disagreed (d/sd) that "*The times...allocated for testing/follow up sessions facilitated me to attend if I chose to*" (item 8). In fact almost half of the prospective teachers who did not partake in any aspect of the initiative (47.3%-n= 62) reported "*I did not partake due to other timetable commitments*" (a/sa) (item 11). However the qualitative statements suggest that in many cases rather than being physically unable to attend one of the three sessions in place, prospective teachers' workload and prioritisation of commitments was the main source of this difficulty: "*I had too much work to do to be going to extra classes*". It also became apparent that many students had difficulty with the day in question e.g. '*Don't have all sessions on the same day as some days we only have one hour off during the day anyway*'. During the intervention stage, the author (MH) became aware of this issue: "*...Because a number of students have 6 hours on Wednesday, their only free session is 1p.m. Therefore in order for them to attend this session, they have no lunch break. ...undoubtedly, the 'final' timetable does not represent the demands made on this cohort*" (R.J., W9: Wed). The author envisaged the formal allocation of timeslots on the cohorts' timetable labelled 'Professional Mathematics- Elective' would go some way to avoiding the timetabling of tutorials at the respective times. Also a request was made for the sessions to be spread over more than one day to facilitate students to avoid their 'busy' day.

The timing of the initiative within the semester was also perceived by the author as unsatisfactory. While 86.5% (n= 103) of those who completed the test reported being '*...content with the time which lapsed between test completion and feedback*' (a/sa) (item 19),

those who reported discontent or uncertainty reflected the author's beliefs on the issue. It was perceived that the time lapse between test administration and feedback of three weeks was unsatisfactory and that it limited many prospective teachers opportunities to attend support sessions as course work and exams began to exert pressure from week 8 onwards, peaking at week 10: *"I became aware today that ...students have projects/coursework due over the next few weeks"* (R.J. W10, Wed). The usage statistics also provide further support. While 92 prospective teachers attended the initial session (week 5), attendance decreased substantially as the weeks went on with the attendance for the three weeks from week 10 onwards were 45, 36 and 25 respectively. *"Overall I feel that a lack of continuity (Week 6- no session) and late start in the semester as well as timetabling issues had a negative impact on attendance"* (R.J., W13: Wed). In terms of timing, the various findings suggest that it is necessary that subsequent initiatives must be focused early in the semester thus avoiding a clash with times where students have very heavy workloads i.e. end of semester. This can only be achieved through prompt feedback and timely commencement of intervention. Unlike the preliminary stage, in light of this ambition, the author (MH) felt it necessary to correct all the tests herself with just a single week of a turnover.

When questioned about the type of feedback received, almost one third (31%) of responding pre-test participants declined to concur that they were *'...satisfied with the nature of feedback...'* (20 (d/sd), 16 (n)) (item 20). The subsequent items provided additional information regarding the issue. Only 54.6% (65 students (a/sa)) of respondents believed that *'The detail received in the test feedback gave me clear insight of my mathematical strengths/weaknesses'* (item 21) (22.7% d/sd (n= 27); 22.7% n (n=27)). The author was acutely aware of this shortcoming: *"Only one or two people on receiving their preliminary feedback have requested individual meetings. Unfortunately this means that the majority have a very vague sense of their strengths and weaknesses"* (R.J. W5, Mon). A number of students communicated sentiments such as: *'I would have liked to know the questions I got wrong in the test'*. While one student reported *'The fact that I didn't get adequate feedback was my own fault'*, the author was determined to address this issue in the main study. In terms of feedback, the author envisaged providing all pre-test participants with optimum opportunity to view their work and gain insight into their errors and strengths/weaknesses. The absence of a cut-off point was also an issue, reflected in the fact that one student teacher reported that *'deciding whether/not to attend the follow-up sessions because of the results of the test'* as his/her **'greatest challenge'** (item 48). In fact 85.9% (n=103) of those who had completed the pre-test concurred (a/sa) that *'Students should be given a cut-off point e.g. 20 correct, below which attendance at follow-up sessions is 'strongly advised''* (item 22). The introduction of this 'cut-off point' would serve to encourage low achievers to avail of the support available, in light of the fact that 31% (n=9) of those who achieved 0-20 (out of 41) in the pre-test attended no support sessions. However decisions were required regarding the position of the cut-off point.

Another issue which influenced the integrity of the intervention was the fact that it was possible that the approach taken by the respective presenters within the intervention stage differed, providing participants with varying experiences depending on the sessions they attend. The author intended that in the main study, she would assume sole responsibility for

intervention preparation and presentation to ensure that the focus and emphasis proposed were consistently implemented across all sessions.

‘Social Validity’ of an intervention refers to the participants’ beliefs regarding the success of the initiative in achieving its goals (Sharipo, 1987). There were high levels of satisfaction among respondents with the overall organisation and value of the initiative. While 93.5% (n= 121) agreed that *‘The project was organised and arranged well’* (item 44), 95% (n=132) acknowledged that *‘The project offered a valuable service to students’* (item 45)). However dissatisfaction regarding the constraints e.g. time, workload, other commitments was also communicated. While a number of intervention participants contested the suggestion that they possessed *‘...adequate mathematics subject matter knowledge to teach mathematics effectively to senior classes’* (item 5) (11%- 14 students), the percentages reporting feeling better prepared and more confident as outlined above speak for themselves. Also 86.4% (n= 89) of the intervention participants reported that their *‘... mathematical needs were met by the follow-up sessions’* (a/sa) (item 38). The qualitative comments (‘greatest success’ of the initiative/‘overall comments’) further support quantitative findings e.g. *‘greater understanding of topics’*; *‘consolidating knowledge’*.

The final criterion ‘acceptability of the intervention’ focuses on the degree to which the cohort ‘likes’ the means of intervention i.e. the procedures. The popularity of the initiative is reflected in the fact that a total of 210 prospective teachers within the cohort participated in some aspect of initiative. The usage statistics reported that 121 prospective teachers within the cohort partook in the intervention stage. The perceived benefits of the service are illustrated by the fact that many of the students chose to return to attend more than one session. While the mean attendance was 2.98 sessions, 64.5% of the intervention group attended two or more sessions. In fact while 33.1% of participants (n= 40) attended 4-5 sessions, one tenth (9.1% (n= 11)) of this group either attended six or seven sessions. Although the initiative timeslots proved problematic for a sub-group of the cohort, there was strong overall support for the timing of the initiative within the course i.e. 75.9% (n= 205) reporting it was **‘appropriately placed’** (item 9). While dissatisfaction with the communication of pre-test performance has been previously discussed, the feedback in relation to the **‘content’** of the pre-test (item 18) and *‘time allocated’* to its administration (item 17) reflected high levels of satisfaction among participants (88.2% (n= 105) a/sa and 91.5% (n=109) a/sa respectively). In terms of the characteristics of intervention the feedback was extremely affirmative in relation to amount (item 28: 89.1% (n= 90) **‘about right’**) and nature of content (item 35: 93.3% (n= 97) a/sa). However over a quarter (26.4%) did not believe *‘the time allocated to the course (weeks/class duration) was adequate’* (item 41). The author was also dissatisfied with this characteristic and intended that the intervention programme would be extended in the main study. In terms of the approach taken within sessions, 82.1% (n= 83) reported finding **‘...the sessions interesting’** (a/sa) (item 36). Almost two thirds of respondents believed that they *‘...had an opportunity to be active learners’* (item 36) (65.4% (n= 67) a/sa) and that *‘During the sessions there was a discursive atmosphere’* (item 37) (61.6% (n=61) a/sa). In light of the fact that 28.7% (n= 29) of the intervention only attended one or two sessions and were not in a strong position to comment on the approach taken in the support programme, the author was extremely satisfied with this response. Overall the evidence from the various sources suggests

that with some exceptions, the nature of the initiative was well received. The fact that the author would present all of the support sessions meant that there was a guarantee this in the main study.

While the evaluation process suggested that the intervention did cause positive change, it also allowed the author to learn a number of valuable lessons regarding constraint and shortcomings of the initiative within the initial cycle and indicated potential improvements. The author was ‘...anxious that the project will build on the lessons learned from the first cycle’ (R.J. Summer).

## REFERENCES

- Ball, D.L., H.C. Hill and H. Bass, 2005. ‘Knowing Mathematics for Teaching- Who Knows Mathematics Well Enough to Teach Third Grade, and How can we decide?’ *American Educator*, Fall: 15-46.
- Cohen, L., L. Manion and K. Morrison, 2000. *Research Methods in Education*, 5<sup>th</sup> Edition. London and New York: Routledge Falmer Academic Publishers.
- Conference Board of the Mathematical Sciences (CBMS), 2001 a. *The Mathematical Education of Teachers*. Providence RI and Washington DC: American Mathematical Society and Mathematical Association of America, [Accessed on-line], [http://www.cbmsweb.org/MET\\_Document/index.htm](http://www.cbmsweb.org/MET_Document/index.htm) , Last accessed on 17/09/2008.
- Corcoran, D., 2005 a. ‘An Exploration of the Mathematical Literacy of Irish Students Preparing to be Primary School Teachers Knowledge of Irish Primary Pre-Service Teachers.’ In: *Proceedings of the First National Conference on Research in Mathematics Education (MEI 1) 16th, 17th September, 2005: Theme: 'Opening Doors'*, S. Close, D. Corcoran and T. Dooley, eds. Dublin: St. Patrick’s College of Education, 220-233.
- Corcoran, D., 2005 b. ‘Mathematics Subject Knowledge of Irish Pre-Service Primary Teachers’, Paper presented at European of Educational Research, UCD, Dublin, 7- 10 September, 2005, [Accessed on-line], <http://www.leeds.ac.uk/educol/documents/144080.doc>, Last accessed on 12/02/2008.
- Crompton, P., 1999 ‘Evaluation: A Practical Guide to Methods’, for LDTI: the Learning Technology Dissemination Initiative, [Accessed on-line], <http://www.icbl.hw.ac.uk/lti/implementing-it/eval.htm>, Last accessed on 12/05/2007.
- Delaney, S., 2008 a. ‘Unacknowledged Expertise-Irish teacher’s mathematical knowledge for teaching. *InTouch*, October: 41-45.
- Delaney, S., 2008 b. ‘Knowledge for Practice: The mathematical demands of primary teaching’. *InTouch*, November: 40-42.
- Department of Education and Science (DES), 2002. *Preparing Teachers for the 21st Century: Report of the Working Group on Primary Preservice Teacher Education*. Dublin: Stationary Office.

- Draper, S., 2006. 'Observing, Measuring, or evaluating courseware: A conceptual introduction', [Accessed on-line], <http://www.icbl.hw.ac.uk/lti/implementing-it/measure.htm>, Last accessed 27/02/2006.
- Elliot, J., 1991. *Action Research for Educational Change*. Milton Keynes: Open University Press.
- Expert Group on Future Skills Needs (E.G.S.F.N.), 2008. *Statement on Raising National Mathematical Achievement*, Dublin: EGFSN, [Accessed on-line], <http://www.forfas.ie/media/Raising%20National%20Mathematical%20Achievement%20-%20Final%20WEB.pdf>, Last accessed on 02/06/2009.
- Hill, H.C., B. Rowan and D.L. Ball, 2005. 'Effects of Teachers' Mathematical Knowledge on Student Achievement'. *American Educational Research Journal*, 42 (2): 371-406.
- Hourigan, M. and O' Donoghue, J., 2007 a. 'Mathematical Under-Preparedness: The Influence of the Pre-tertiary mathematics experiences on students' ability to make a successful transition to tertiary level mathematics courses in Ireland'. *International Journal of Mathematical Education in Science and Technology*, 38(4): 461-477.
- Hourigan, M. and O' Donoghue, J., 2007 b. 'The Challenges Facing Pre-service Education: Addressing the Issue of Mathematics Subject Matter Knowledge among Prospective Primary Teachers.' In: *Proceedings of Second National Conference on Research in Mathematics Education (MEI2) Theme: 'Walking the Talk'-Using Mathematics Education Research*, Dublin, 14-15 September, S. Close, D. Corcoran and T. Dooley, eds. Dublin: St Patrick's College, 323-338.
- Huntley, R., 2005. 'An Evaluation of Primary Trainees' view of the Subject Knowledge Audit Process.' In: *Proceedings of the British Society for Research into Learning Mathematics* 23(2) University of Oxford, 7 June, J. Williams, ed, 63-68, [Accessed on-line], <http://www.bsrlm.org.uk/IPs/ip23-2/BSRLM-IP-23-2-Full.pdf>, Last accessed on 08/04/2009.
- Leavy, A. and N. O' Loughlin, 2006. 'Preservice Teachers Understanding of the Mean: Moving Beyond the Arithmetic Average'. *Journal of Mathematics Teacher Education*, 9: 53-90.
- McMillan, J.H. and S. Schumacher, 2001. *Research in Education: A Conceptual Introduction*, 5<sup>th</sup> Edition. New York: Longman.
- Mertens, D.M., 2005. *Research and Evaluation in Education and Psychology: Integrating Diversity with Quantitative, Qualitative, and Mixed Methods*, California: Sage Publications.
- Mullan, Y. and J. Travers 2005. 'Educational Disadvantage and Mathematics: The Impact of the Numberworld intervention programme on the number knowledge of children in a Junior Infant class.' In: *Proceedings of the First National Conference on Research in Mathematics Education (MEI 1) 16th, 17th September, 2005: Theme: 'Opening Doors'*, S. Close, T. Dooley and D. Corcoran, eds. Dublin: St. Patrick's College of Education, 64-79.

- Murchan, D., A. Loxley, K. Johnston, M. Quinn and H. Fitzgerald, 2005. *Evaluation of the Primary Curriculum Support Programme (P.C.S.P.): Final Report*. Dublin: Education Department, University of Dublin, Trinity College.
- Murphy, M., 2002. *An Investigation into Mathematical Under-preparedness among Third Level Entrants: The Possible Contribution of the Second Level Mathematics Experience*, Unpublished M.Sc. Thesis, University of Limerick, Ireland.
- National Council for Curriculum and Assessment (NCCA), 2006. *Review of Mathematics in Post-Primary Education: Report on the Consultation*. Dublin: Stationary Office.
- Oldham, E., 2005. 'Solving Problems with Student-Teachers: Reactions and Reflections 1995-2005.' In: *Proceedings of the First National Conference on Research in Mathematics Education (MEI 1) 16th, 17th September, 2005: Theme: 'Opening Doors'*, S. Close, T. Dooley and D. Corcoran, eds. Dublin: St. Patrick's College of Education, 250-264.
- Opie, C. (ed.), 2004. *Doing Educational Research: A Guide to First Time Researchers*. London: Sage Publishers.
- Shapiro, E.S., 1987. 'Intervention Research Methodology in School Psychology'. *School Psychology Review*, 16 (3): 290-305.
- Shulman, L., 1986. 'Those who understand: knowledge growth in teaching'. *Educational Researcher*, 15(2): 4-14.
- Starkings, S., 2005. 'Numeracy and Teacher Training'. *MSOR Connections*, 5(2): 1-3.