

MOVING CLOSER TO SOLVING THE ‘MATHEMATICS PROBLEM’ WHERE IT ARISES: THE INFLUENCE OF THE ‘TYPICAL’ IRISH PRE-TERTIARY MATHEMATICS EXPERIENCE ON UNDER-PREPAREDNESS AMONG NUMERATE ENTRANTS

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The contribution of mathematics proficiency to the numerate (e.g. science, engineering) disciplines is well-documented. Internationally concern exists that graduates of these disciplines are persistently demonstrating mathematical deficiencies i.e. the ‘Mathematics problem’. A connection has been made between this problem and the worldwide reportage of numerate entrants to numerate courses demonstrating insufficient and incomplete mathematical knowledge. While the characteristics of ‘at risk’ students are more or less equivalent internationally, the same consensus does not exist in relation to the causes of this phenomenon. There is growing consensus however that ‘under-preparedness’ is caused by the apparent mismatch between the nature of the pre-tertiary and subsequent tertiary level mathematics experience. This study focuses the ‘Mathematics problem’ from an Irish perspective, exploring the nature of the ‘typical’ pre-tertiary mathematics experience, which in turn provides insight into the nature of the transition required of Irish students entering the numerate disciplines. Brousseau’s ‘didactical contract’ is used as an intellectual tool to uncover the features of the mathematics experience in two case classrooms in Irish upper secondary schools (Senior Cycle). While the authors are both professional mathematics educators and therefore acutely aware of prevalent classroom practice, the restrictive nature of contract and its implications for students’ future mathematics education left all concerned astounded.

1. The ‘Mathematics Problem’- a global issue!

Mathematics has always been noted for its potential to prepare students for life, work and further education. Undoubtedly we all need mathematics at some stage of our lives. In more recent years, however, mathematics education has been deemed a priority among education systems worldwide for its potential to foster advanced thinking skills strategies and qualities (e.g. perseverance, problem solving) essential within the numerate disciplines (e.g. science, engineering) [1]. In light of the economic implications, it is understandable that the existence of substandard mathematics ability among numerate graduates i.e. ‘The Mathematics Problem’ is a source of international concern (e.g. U.K., U.S.) [2, 3]. Consensus exists that this phenomenon is directly related to the fact that numerate entrants, in many cases, are deemed ‘at-risk’ or ‘under-prepared’ i.e. demonstrate mathematical skills deemed inadequate for courses [2, 5]. Ireland is no exception to the rule. While there is a dearth in the volume of research in this area, a number of Third Level institutions (e.g. Cork Institute of Technology, University of Cork, and University of Limerick (U.L.)) have reported substandard mathematics facility among entrants from the mid-nineteen eighties onwards [2, 4, 5, 6, 7, 8].

Internationally (e.g. U.S., Australia, U.K., Ireland) much of the mathematics that ‘under-prepared’ students find difficult astonishes lecturers. Frequently these students demonstrate substandard numeric/algebraic facility as well as limited problem solving skills. Gaps in mathematical knowledge are also prevalent (e.g. trigonometry and

complex numbers) [2, 5, 6, 8, 9]. It has been proposed that many ‘at risk’ students do not possess the mathematical skills required to cope with everyday life not to mention studying Mathematics in college [2, 6].

What causes further unease is the fact the terminal school examination results (U.K., Ireland) fail to provide any indication of preparedness [2, 10]. In the Irish context, for example, 31% of U.L. students who achieved Ordinary Level ‘A1’ and ‘A2’ grades at Leaving Certificate, were diagnosed as being ‘at-risk’ [2]. Universities also report that grade depreciation is a reality [2, 7].

Third level institutions have little alternative but to ‘pick up the pieces’ i.e. such students must reach the same standards of excellence despite their ‘substandard’ preparation in Mathematics [3]. In order to maintain exit standards upon course completion, a growing number of institutions internationally have reacted positively by placing support strategies in place [11]. Approaches vary from adaptation/lengthening of courses to testing and intervention initiatives [10, 12].

The authors’ work within the U.L. initiative (testing and intervention) provided invaluable and unexpected insight into the nature of the problem [13]. Prior to this, the authors, like so many others, would have associated failure in Mathematics at Tertiary Level exclusively with a lack of commitment on the part of students. This insight into the causes/sources of mathematical under-preparedness challenged the authors’ preconceptions and subsequently prompted the initiation of an investigation into the nature of the problem within the Irish context [14].

2. The Uniqueness of the Irish ‘Mathematics Problem’

While a thorough review of the relevant literature highlighted that internal factors within Tertiary Level institutions can potentially trigger or exacerbate poor mathematical performance (e.g. large class groups, lecture format), the vast volume of evidence suggests that the inability of many students to make a successful transition to tertiary mathematics is the primary cause [2, 15]. The fact that Irish Third level entrants are diagnosed as ‘under-prepared’ despite the reality that First Year Mathematics Courses at Tertiary Level generally overlap and extend the school Leaving Certificate Syllabus, supports this belief [2, 4].

When investigating the transition issue in further depth, the uniqueness of the ‘Irish’ Mathematics problem soon became apparent. While the U.K. universities have pinpointed the increasingly heterogeneous nature of the student population i.e. acceptance of vocational entrants and mature students etc. as the primary cause of mathematical under-preparedness, Irish entrants do not exhibit the same degree of diversity. Although the entrance routes to Irish Third Level institutions are now more varied than ever before (e.g. Post Leaving Certificate (P.L.C.)/FETAC awards; Mature Students), the Irish system is still deemed homogeneous in nature i.e. the vast majority of students enter through the State School Leaving Certificate/C.A.O. system [2, 16]. Acknowledging such differing contexts, the authors sought to answer the following question: Why are Irish students exhibiting the same problems on entering university Mathematics courses as their U.K. counterparts?

Given the fact that Irish students seem to have two distinct advantages over their U.K. counterparts; namely the fact that they are obliged to study mathematics for the entire duration of their secondary school experience (five or six years) and achieve at least the minimum qualification; the sameness of the descriptions of the problem in the U.K. and Ireland was cause for even more alarm.

On reflection the authors concluded that the nature of Irish entrants' pre-tertiary mathematics education i.e. in secondary school directly influences his/her ability to make a smooth transition to Tertiary Level mathematics-intensive courses. Consequently, the authors sought to explore the nature of the 'typical' pre-tertiary mathematics experience (Upper Secondary school mathematics) of Irish pupils and in turn investigate the potential contribution of this experience on students' ability to make a successful transition to Tertiary level mathematics. This fact-finding process did not seek a 'culprit' for the underachievement. The purpose of the study was to gain valuable insight into the closed world of the 'typical' mathematics classroom with a view to suggesting alternative, more effective practices where relevant [14].

3 The Study

In order to identify the unknown factors within pre-tertiary mathematics experiences that determine the extent to which one can successfully make the transition to Tertiary Mathematics-intensive courses, it was essential to collect data that provided an in-depth and realistic portrayal of the behaviours and attitudes of all interested parties (i.e. both teachers and students). Therefore a case study approach was adopted. The study focused on two fifth year class groups (i.e. pupils studying the first of a two year Senior Cycle Mathematics syllabus) selected from a co-educational and a single sex (All girls) School. The selection process was based on the voluntary co-operation of the class teachers and principals of each school. Within this report, the former class group is referred to as the 'Mixed group' and the latter the 'Single Sex group'. All concerned were assured of total confidentiality regarding the location and identity (i.e. the use of pseudo-names). T1 and T2 were the codes given to the 'Mixed' and 'Single Sex' teachers respectively.

As the authors wished to explore possible variations in either teacher/student attitudes or behaviours that could possibly be associated with ability, a variety of levels was requested. While the Single Sex group was studying the Higher level course, the mixed group was an Ordinary level class. The teachers were asked to treat the lesson preparation of lessons observed as for any other mathematics lesson they gave.

While it is not proposed that this study is representative of all Senior Cycle mathematics classrooms, due to the narrow focus achieved by a case study approach, the authors believed that it represented a valuable starting point. This study potentially offered all interested parties 'food for thought' and guidelines for future progress and development [14].

3.1 Theoretical framework

Brousseau's concept of 'didactical contract' was identified as an appropriate intellectual tool to illustrate fully the findings of this study. Brousseau's research suggests that a child's acquisition in a mathematical learning situation is not simply regulated by her level of intelligence, but is equally affected by many other relationships i.e. an unspoken didactical contract exists between the participants of every mathematics classroom determining the roles, behaviours and attitudes of all 'actors' (teacher and students) within the classroom situation [17]. Lim's (2000) interpretation and use of Brousseau's 'didactical contract' were especially relevant to this study. Lim concluded that seven common elements were apparent almost exclusively in the four classrooms observed in that study. The authors could strongly identify with the conditions (i.e. traditional, predictable approach) described in Lim's

‘contract’ and felt that they were indicative of the Irish exam-orientated Senior Cycle mathematics classroom. Such compatibility facilitated the adoption of the ‘didactical contract’ as an appropriate conceptual framework to guide both data collection and analysis of classroom practices and behaviours [18].

3.2 Methodology

3.2.1. Data Collection. Qualitative non-participant observation was deemed the optimum strategy for data collection because Brousseau [17, p. 226] reported ‘the hypothetical process of finding a contract is the contract’ i.e. it was essential to undertake an in-depth study of the routine happenings (i.e. behaviours, interactions, and attitudes) of the mathematics classroom. This method allowed the author (MH) to capture all participants in their natural settings in everyday conditions [14]. In order to facilitate participants in overcoming initial caution, it was decided that observation should take place over an extended period (10 weeks). The author was positioned at the rear of the classroom throughout the classroom observations, making no attempt to participate or interfere in activities or classroom discourse. It was hoped, therefore that data collected would reflect participants ‘everyday’ practices i.e. increase validity [19]. Maximum objectivity and validity of findings was also ensured through the use of a structured data collection strategy i.e. observer checklist consisting of factors considered critical in gaining a holistic picture of the classroom environment [14, 19].

A multi-method approach was adopted, to facilitate triangulation and increase the reliability and validity of the findings. Complementary methods included reflection and interviewing. The author kept a reflective journal throughout the observation period. The reflection process, guided by a purpose-developed ‘reflection guide’, took place after every session and facilitated the author with an opportunity to look beyond the description, thus discovering overall patterns and conclusions. Semi-formal interviews were subsequently utilized (after the final observation). This method proved to be an effective means of information backup, providing confirmation and clarification of behaviours observed. Questions also focused on participants’ thoughts, beliefs and attitudes; factors which were almost impossible to detect through observation alone [19].

3.2.2. Data Analysis. The data collection process led to the accumulation of a plethora of random information which needed to be analysed. As it was not possible or necessary to examine every action or utterance, the focus was narrowed to factors deemed influential to the nature of the ‘didactical contract’ present in each classroom [19]. Constant comparative analysis ensured that all findings were both grounded and relevant. Initially the establishment of broad codes e.g. ‘Exam reference’, ‘Routine’ pulled the wealth of data into an elementary structure. Through a succession of examinations, the author found that many of the codes were subsets of others and therefore could be merged. Such overlap highlights the richness of the data, as substantial relationships existed between units. The final themes were: ‘Exam-oriented mathematics’, ‘Daily mathematics class’ and ‘Quality of interaction’.

4. Findings

The experiences of two relatively ‘typical’ Senior Cycle Mathematics classes highlighted various practices, which may contribute to the inadequate transition made

by a sizeable number of pupils to Tertiary Level mathematics-intensive courses in Ireland [14].

4.1 Exam-oriented mathematics (Exam Focus)

Within the Senior Cycle classes observed, exam-focus, whether mentioned or implied by action, was the central concern. Both teachers and pupils alike demonstrated the implicit conviction that the Leaving Certificate Examination is the principal reason for studying Mathematics at Senior Cycle. Consequently the Leaving Certificate exam was regularly used as a motivational ploy to gain and maintain interest and concentration levels among pupils. Comments like *'This is a full question in the Leaving Certificate and it's very easy to do well in if you practice'* (Observation (O) 1, T1) were especially popular and a central component of each lesson observed. Further evidence lies in the fact that the only voluntary pupil interaction within the majority of the lessons observed were queries relating to exam questions.

Pacing of lessons observed was also influenced by the obligation to cover topics required for the Leaving Certificate exam. While a challenging pace was evident in both settings, it was more pronounced in the Honours class. In turn the evident obsession with topic progression affected pupil behaviour. One student linked her passivity with pacing: *'I feel I can't ask the questions I want or need to even though she says 'Well any problems with the homework' ... I just feel she's always giving the impression that she's under pressure to get the course done. It's always as if we have to move on...I don't want to hold the class back'* (Interview (I), T2: Anna).

4.2 Daily Mathematics Class

The exam-oriented environments observed also determined many aspects of the day-to-day mathematics lesson e.g. the predominant methodology. The statement *'...the same more or less everyday...I'm afraid-quite boring'* (I: T1) depicts the fact that set routine is a central characteristic of both settings. The predominant resources were the blackboard and the primary text. The focus was on the mastery of algorithmic procedures as isolated skills, with only rare connections made to other relevant subjects or every-day links.

Methodologies were traditional in nature and teacher-centred, ranging from exposition to consolidation and practice. Throughout the investigation period, many examples of quick-fix approaches and drill came to the fore. Both teachers adopted a 'reductionist orientation', where teacher was equated with 'effective teller' [20]. Pupils were constantly provided with 'ready-made' mechanisms. One example became evident in a lesson focusing on quadratic inequalities. The teacher stated *'In order to remember the n-shaped graph-remember n stands for negative'* (x^2 coefficient) (O3-T1). Consequently mathematics for these pupils entailed manipulating numbers and letters and filling in the right formula. Students expected a 'learn-off' approach: *'I like the way she goes through the steps and breaks the examples down...'* (I, T2: Olivia).

'Problem-solving' in these settings was limited to practised text-based story problems with one 'right' answer. The limited nature of the students' problem solving skills became apparent from the typical pupil's response when problems strayed slightly from the textbook format: *'It's grand doing all the section questions, but when they start mixing topics, it's impossible to know where to start'* (I, T2: Marian). Confirmation of pupils' 'situated learning' came from one of the teachers, who stated *'If they couldn't do an exercise that is slightly different, they wouldn't try- the majority would leave it blank...'* (I: T1) [21].

Another source of concern was the fact that none of the pupils interviewed had ever completed practical or investigation work in their post primary mathematics education to date.

4.3 Quality of Interactions

Dialogue, for the most part, was teacher-initiated. As questions generally lacked direction e.g. ‘*Anyone not get it?*’ (O1-T1), the opportunity to assess the level or even presence of understanding was lost. The statement ‘*Sometimes you feel like you’re taking it down like a robot. You’re really not involved in it...*’ (I, T2: Olivia) reflects pupils’ beliefs that their role was primarily a passive one of listener and copier. Students were largely unwilling to publicly share their thoughts for fear of making an error and facing public embarrassment. Two-way and even pupil-initiated interaction was only plausible during the ‘practice and consolidation’ stage of the lesson.

In their effort to promote positive attitudes, both teachers repeatedly cajoled the groups, offering endless positive reinforcement regardless of relevance or accuracy. In her bid to encourage the pupils, the Ordinary Level teacher demonstrated very flexible expectations. As a result, an inability to recall even the most basic elements of previous topics was deemed acceptable: ‘*...They expect to be spoon-fed at all times, not attempting homework if it looks too hard...*’ (I: T2) [14].

5. The nature of the ‘didactical contract’ in the classrooms investigated

The collection and analysis stages facilitated the formulation of a ‘didactical contract’, which was representative of the actions and attitudes of all participants in both educational settings. This unspoken contract reflected a negotiated agreement between the participants observed, consisting of both consensual and involuntary demands on all concerned. While much of the contract conditions represented both classrooms, some variations between the two groupings became apparent. The author utilised the word ‘should’ throughout the contract, signifying that these deductions are objective interpretations of the data analysis process. The common elements of this ‘didactical contract’ are as follows:

- The Leaving Certificate terminal exam should be the central aim of the Mathematics class. This exam should be the core component of each lesson, present as a sole motivation to learn a new topic. The teacher should present work referring to its inclusion/importance in the Leaving Certificate and provide details of the gain/ loss of marks at every opportunity. All class tests should be based on previous exam questions. No time should be wasted in class, as the adequate chapters for exam preparation require completion in the shortest possible time.
- The teacher should introduce the lesson by correcting the homework swiftly and orally if possible and move onto a new topic. Blackboard work should be curtailed, unless pupils have major problems and ask for help. During the homework correction, the teacher should interact mainly with the group, asking for the answer to the exercises. The introduction of a new topic should consist of the illustration of a number of worked examples on the blackboard. After pupils have copied these into their copies, they should practise similar exercises from the course text entitled, *Text and Tests*. During individualised practice work, the teacher should circulate, offering personal feedback and help to all needy pupils. The pupils should use this opportunity to ask questions regarding the homework etc. Before the bell, similar type exercises should be set for homework from the primary text. Homework should be attempted in order for pupils to gauge how much they actually know.

- The teacher should not depart from the set lesson routine unless the class are preparing for a test, in which case the entire class should be used to review formulas, techniques and standard problems. The teacher should, if possible, get something new done everyday, as pace is vital. The standard lesson should never include practical work or investigation or make logical links to other subjects or everyday life unless it is directly relevant to the exam e.g. Statistics.
- The teacher should not ask pupils complicated individualised questions, as the pupils should not be publicly scrutinised. Such questioning in class should be pitched at the group, to allow more outspoken and well-able pupils to volunteer. The only directed questions that should be permitted are those on new topics or on straightforward workings.
- The teacher should ‘break things down’ and simplify for the pupils. The pupils in turn should listen and try to learn off. The teacher should provide pupils with tricks to remember mathematics methods and a step-by-step breakdown of problem-solving techniques i.e. the use of trigger phrases. Pupils should not be expected to persevere with difficult questions or to be able to solve questions requiring a combination of procedures.
- Pupils should not be expected to remember previous topics and the teacher should be patient and ready to re-explain any section/method on request. The pupils should listen and learn, participation in class activities is not vital. Once the minimum standard is achieved i.e. homework attempted, on task in class and able to answer directed questions, the teacher shouldn’t request any more. The teacher should always be positive and encouraging even if pupils are not working to their ability.
- Any individual teacher or pupil should not interfere with the contract described above, even if they are unhappy with many elements of it. Pupils should not interrupt the lesson unnecessarily, thus holding back the group, even if they are confused. Pupils should work on passively in class even if they require extra blackboard reference, examples or time on a particular topic. The teacher should not ask directed questions that may demand thought and reflection, or set questions which have more complex wording or layout to that of the main text. The class form should not be disrupted, as the present momentum is deemed vital in order to keep on target in the pursuit of exam success. All loose ends and confusion should be dealt with during seatwork or after class [14, p. 159-161].

6. Reliability of the Findings

The authors have already acknowledged that this study does not provide a comprehensive picture of the pre-tertiary situation and undoubtedly there is ample scope for further national and international research into this phenomenon. Despite this fact the study does provide invaluable insight into the nature of the ‘Mathematics problem’ in Ireland and is consistent with the concern and discontent evident among the relevant research in the field.

Agreement exists within the relevant literature that the Irish Second Level Education System is extremely ‘exam-oriented’ [5, 14, 22]. The powerful backwash effect of the Mathematics Leaving Certificate examination on what and how it is taught is an ongoing cause for concern [22]. The fact that the terminal examination is the sole means of assessment causes many teachers to utilize ‘course dilution’ i.e. omit sections of the syllabus for the purposes of examinations [4, 5]. This approach directly contributes to the existence of gaps among numerate entrants’ knowledge [8].

National and international studies also concur that exam-focused teaching within the Irish context is ‘traditional’ in nature, prioritising recall and routine procedures, while

pupils remain largely passive, relying on rote memory and special-purpose algorithms as alternatives to understanding [2, 4, 8, 23]. The textbook has been found to more influential in planning than the curriculum [22]. The lack of reference to the potential role of the subject to pupils' own lives or other related subject areas has also been deemed unsatisfactory [8, 23]

Data also exists on the effects of the implemented curriculum (i.e. the attained curriculum). From a national perspective, the Chief Examiner's Report (2000) was especially critical of the serious decline in ability, effort and understanding among Leaving Certificate students at all levels. A source of particular concern was many students' apparent inability to demonstrate relational understanding [24]. In more recent years the N.C.C.A. (2006) report that "...many students leave school with only a superficial understanding of the subject and little or no conceptual knowledge" [8, p. 36].

The mathematical performance of Irish Second Level pupils in the various international studies of achievement e.g. Programme for International Student Assessment (PISA) has been consistently 'satisfactory'. While Irish pupils perform better than the international average on tasks involving basic mathematical operations, their more limited ability to use higher-level mathematical thinking e.g. problem solving (required by the numerate disciplines) is a particular cause for concern [25, 26].

7. Food for Thought

While there is no doubt that Mathematics teachers, alongside their peers, are endeavouring to do what's 'best' for their pupils, this study illustrates that the present obsession with 'exam-oriented' practices serves to narrow such pupils' future potential. This and other studies strongly suggest that the prevailing inflexible and unresponsive learning environment promotes mediocre learning and poor study habits; is serving the students and economy poorly [27]. The reality is, however, that there is no incentive to change because students can achieve high standards in the predominantly abstract, context-free Leaving Certificate examinations which focuses predominantly on memorisation, the practice of technique and ability to spot cues [8, 14, 23].

While Mathematics-intensive courses at Tertiary Level need independent learners possessing conceptual and transferable skills required to solve unfamiliar problems, the development of these essential skills is not fostered within the classrooms studied. Unfortunately the pressure on schools to deliver good examination results seriously weakens the greater aim of providing a high quality mathematics education [28].

8. In Pursuit of Change: Aspirations

If Second Level mathematics education is to adequately prepare pupils to participate in mathematics courses at Tertiary Level, they must leave this level of their education in a 'high energy state' i.e. demonstrate a predisposition to ...confront any problem given to them/invented by themselves with their previous learning in Mathematics, in an active and accessible state and with the assumption that they have the ability to progress [29, p. 105].

If our pupils are to demonstrate this ideal mathematical state, considerable changes in the quality of Second level mathematics provision are inevitable.

Firstly, all interested parties (e.g. practitioners, Department of Education and Science) must become fully informed of the importance of mathematics and the current phenomenon, and realise that pupils' achievement in the Leaving Certificate mathematics examination is only one aspect of their mathematics education. Widespread awareness of the prerequisite mathematical skills and abilities required by pupils hoping to enter Tertiary Level mathematics-intensive courses would prove invaluable [8, 14].

Second Level mathematics practitioners must alter the predominant classroom practices. A move away from over-reliance on traditional approaches is a must. It is necessary that the belief that mathematics is useful and vital for all people is portrayed through the use of integration and real-world connections. There should be an emphasis on 'connection-making' i.e. moving from the known to the unknown, thus promoting understanding. Pupils need to experience open-ended relevant problems, which require perseverance and risk-taking. In such an environment mistakes and confusion are considered as opportunities for learning, rather than a source of embarrassment or discomfort that should be avoided if possible [14].

The authors are cognisant, however, that many of the contributing factors associated with the predominant examination-focused practices lie outside teachers' control. A focused alteration in national policy is prerequisite to real change. Consequently, much hope can be gained from the government's recent move to seek a 'root and branch' review of Second Level mathematics education with special reference to Senior Cycle mathematics. The fact that many of the recommendations of this study have been subsequently reflected in the relevant national discussion and consultation papers is also heartening [5, 8].

The National Council for Curriculum and Assessment publications also reflect the authors' belief that the alteration of the mode of assessment, to include project work for example, would foster the development of problem-solving and higher-order thinking skills [5, 8, 14]. There are also proposals to change the curriculum to include 'realistic mathematics' applications and facilitate the development of a range of skills. Support for teachers at all levels has also been deemed a precondition to genuine change. If the authors' recommendations become a reality, the envisaged 'root and branch' reform will move one step closer to tackling the 'mathematics problem' where it arises [8].

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