SPOON-FEEDING TO TONGUE-BITING: AN EVOLVING INSTRUCTIONAL FRAMEWORK FOR PRIMARY SCHOOL MATHEMATICS

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In this paper, I examine the evolution of an instructional framework for primary mathematics during a sustained, on-site professional development project with one case study school. The project attempts to bridge the gap between the perpetually-reported issues with pedagogical practices in Irish mathematics lessons and those espoused as best practice in international literature; and, to a lesser extent those highlighted in the Primary School Mathematics Curriculum. This instructional framework is used in an attempt to support teachers in addressing these pedagogical shortcomings, in addition to enriching the quality of mathematics lessons through a heightened emphasis on mathematical thinking. The paper draws on one aspect of my doctoral thesis and so this partial analysis is limited to lesson observations and teacher interviews. Findings were that the instructional framework changed considerably during the project. Teachers reported finding this instructional framework to be very useful in their classroom teaching, particularly as a planning and a reflection tool. Findings suggest that the transition from the traditional role of the teacher to a more facilitative role where teachers and pupils have an equal voice was particularly challenging for teachers.

INTRODUCTION

Since the introduction of the 1999 Primary School Curriculum in Ireland three reports have been published by the Inspectorate regarding teaching and learning in mathematics. Similar concerns in mathematics lessons are highlighted across all three reports:

- an over-reliance on whole-class teaching in a majority of classrooms including teacher-dominated discussion;
- classroom environments where pupils are not provided with adequate opportunities to work collaboratively;
- an over-reliance on textbooks as the chief teaching aid;
- insufficient provision and use of resources, in particular, concrete materials; and
- insufficient differentiation to meet the needs of children with varying learning abilities and needs.

Furthermore, the *Review of English, Mathematics and Visual Arts* (DES, 2005) reported that in more than two-thirds of mathematics classrooms, teacher talk dominated where pupils worked individually and silently for excessive periods. Unsurprisingly, the recommendations from these reports include that the over-reliance on textbooks as the primary teaching aid should be discontinued; pupils should be encouraged to use a range of reasoning and problem-solving strategies; teachers' awareness of the potential of co-operative or collaborative learning should be heightened; talk and discussion should feature more prominently in mathematics lessons; and that pupils should ‘have access to the objects, equipment and materials necessary for them to discover, learn and consolidate their learning’ (DES, 2010, p.17). These findings regarding the over-reliance on textbooks in Irish primary classrooms are mirrored in national and international assessments (e.g. TIMSS, 1995; NAMA, 1999; NAMA, 2004; NAMA, 2009; NAMIS, 2010). Furthermore, the Inspectors’ findings regarding the insufficient use of concrete materials are also corroborated in national assessments (e.g. NAMA, 2004; NAMA, 2009). Finally, the findings regarding the insufficient opportunities for collaborative learning are mirrored by the NCCA (2005) findings that whole-class teaching was the organisational setting most used, followed closely by individual work; whilst there was only limited use of pair or group work.

However, it would be naïve to assume that pair or group work should be championed at the expense of whole-class discussion or indeed that whole-class teaching is undesirable. Dooley’s (2010, p. 229) research in Irish mathematics classrooms highlights the potential of whole-class discussion suggesting that “extended whole-class conversation can be a vehicle for the construction of mathematical insight.” The need for various organisation settings is further illuminated by her finding that the effect of group work increased pupils’ contribution in whole-class discussion in that ‘pupils often consolidated or generated ideas at this stage…” (p. 236). However, the quality of whole-class discussion is important and Dooley (p. 253) contends that “the construction of insight by pupils in whole-class settings is a complex interaction of task, classroom discourse style and pupil engagement” where the discourse embraces a conjectural atmosphere. Similarly, analysis of a Fourth class mathematics lesson in an Irish primary school leads NicMhuirí (2011) to assert that the discourse was not truly mathematical and that opportunities for high level mathematical thinking were limited. Both studies illuminate the pivotal role played by discourse styles, in particular, teacher follow-up moves during mathematical conversations.

So, general agreement exists that the pedagogical approaches employed in Irish primary mathematics lessons are misaligned with the constructivist principles which are advocated in the Primary School Mathematics Curriculum (PSMC) and so need to be reformed and enhanced. Achieving this in individual classrooms is challenging; attempting to do this at a whole-school level is even more ambitious. Thus, teachers require guidance and support in attempting to address these perpetually-reported pedagogical shortcomings. In this paper, I analyse one particular aspect of my doctoral thesis – the evolution of an instructional framework for teaching and learning mathematics. This instructional framework is used in an attempt to support teachers in addressing the pedagogical shortcomings outlined above, in addition to enriching the quality of mathematics lessons through a heightened emphasis on mathematical thinking.

OUTLINE OF RESEARCH PROJECT

The research project aimed to explore the experiences and perspectives of primary school pupils and teachers during the implementation of a reform approach to mathematics. The case study school is a mixed gender, vertical school in a small rural town. At the beginning of the study, the school was in existence for three years resulting from an amalgamation of an all-girls' and an all-boys' school. There were 205 pupils enrolled in the school during the project: 103 girls and 102 boys. The participants in the study included all of the teaching staff of the case study school which comprised of thirteen teachers: an administrative principal, eight class teachers and four support teachers.

METHODOLOGY

The methodology involved collaborative on-site professional development (PD) whereby the teachers were firstly up-skilled in the use of an instructional framework and secondly, the teachers collaboratively devised mathematics lessons which they subsequently taught. After analysis of their standardised test results, the school chose the strand of Measures as the focus for the project. The research project took place over a year. The PD aspect of the research took place over two school terms or seven months and was rolled out in two phases. Phase One focused on an instructional framework devised by Hiebert, Carpenter, Fennema, Fuson, Wearne, Murray, Olivier & Human (1997) and was implemented through the topic of Length. Phase Two focused on a revised version of this instructional framework and was implemented through the topic of Weight. The PD was mainly collaborative, for example, sharing of expertise and the generation of collaborative lesson plans; however, for the first few weeks of each phase, the PD was front-loaded with input from the researcher in order to up-skill teachers in:

1. an evidence-based instructional framework for teaching and learning mathematics with understanding;
2. recommended approaches in the school-selected strand of Measures, in particular, the strand units of Length/Weight; and
3. pupil conceptions/misconceptions in the school-selected strand units of Length/Weight.

Both the PD and the data collection took place simultaneously. The following data collection instruments were used: observation, review of documents, individual interviews, focus groups and field notes. Although all teachers were invited to participate in the PD, early in the site visit, teachers were asked to express an interest in being 'tracked' throughout the study. All teachers volunteered to be 'tracked'; however, only four teachers were chosen. Data from these teachers and classrooms form the bulk of the research. Two mathematics lessons were observed in each of the 'tracker' classes – one in each phase. Although multiple data sources were collected, the focus of this paper – the evolution of the instructional framework – relies chiefly on lesson observation and teacher interviews.

ANALYSIS

Rationale for using the instructional framework

Although a significant amount of research has been conducted regarding the teaching and learning of mathematics in primary schools, very little tangible guidance for teachers appears to be available in one place. In order to garner information about good practice, it is often necessary for teachers to draw from many varied, often far flung sources. Using this instructional framework was an attempt to highlight good practice for teachers in a meaningful, user-friendly format. Furthermore, the Instructional Framework for Teaching and Learning Mathematics with Understanding (Hiebert et al, 1997) was chosen as a tool for guiding teachers "toward designing classrooms that encourage understanding" (p. xv) because its genesis lies in multiple research projects. Four mathematics teaching and learning projects contributed to the development of this framework: Cognitively Guided Instruction at the University of Wisconsin-Madison; Conceptually Based Instruction at the University of Delaware; Problem Centred Learning at the University of Stellenbosch; and Supporting Ten-Structured Thinking at Northwestern University. The framework "within which teachers can reflect on their own practice, and think again about what it means to teach for understanding" (p.xix) arose out of five years of collaboration between the researchers in all four projects. Although the projects were quite different a "rather striking consensus about the features of classrooms that are essential for supporting students' understanding" (p.xix) grew. These core features form the basis of the instructional framework.

Evolution of the instructional framework

Phase One

Table 1 below illustrates Hiebert et al's (1997) instructional framework. This is a summary of the book in which each dimension has a corresponding chapter. The teachers analysed this framework in detail, for example, groups of teachers each studied, discussed and analysed different chapters of the book before summarising the main points and sharing with other groups.

Table 1: Summary of dimensions and core features of classrooms that promote understanding (Hiebert et al, 1997)

Dimensions	Core Features
Nature of Classroom Tasks	Make mathematics problematic Connect with where students are Leave something behind of mathematical value
Role of the Teacher	Select tasks with goals in mind Share essential information Establish classroom culture
Social Culture of the Classroom	Ideas and methods are valued Students choose and share their methods Mistakes are learning sites for everyone Correctness resides in mathematical argument
Mathematical Tools as Learning Supports	Meaning for tools must be constructed by each user Used with purpose – to solve problems Used for recording, communicating and thinking
Equity and Accessibility	Tasks are accessible to all students Every student is heard Every student contributes

Similarly, Table 2 outlines the same instructional framework but with one addendum – an additional information column. This additional information was extrapolated from various chapters of the book and was included as a more detailed reference guide for teachers. Both versions were used with teachers during the PD in Phase One; however, teachers reported using Table 2 more often, both as a reference guide and when planning mathematics lessons.

Table 2: Dimensions and core features and additional information of classrooms that promote understanding

Dimensions	Core Features	Additional Information
Nature of Classroom Tasks	Make mathematics problematic Connect with where students are Leave something behind of mathematical value	Tasks should encourage reflection and communication Tasks should allow students to use tools Tasks should leave behind important residue - there are 2 types of residue: a) insights into the structure of mathematics (mathematical relationships) and b) strategies or methods for solving problems
Role of the Teacher	Select tasks with goals in mind Share essential information Establish classroom culture	Explanations and demonstrations by the students become more important than those by the teacher Teachers need to select sequences of tasks not just individual tasks Teachers should remove themselves from a position of authority (deciding whether answers are correct) in order to promote the autonomy of students' intellectual activity
Social Culture of the Classroom	Ideas and methods are valued Students choose and share their methods Mistakes are learning sites for everyone Correctness resides in mathematical argument	Students work together to solve problems and interact intensively about solution methods Collaboration depends on communication and social interaction (individual work can be followed by small group or whole class discussion of methods and ideas) Students must learn to live with a certain amount of uncertainty
Mathematical Tools as Learning Supports	Meaning for tools must be constructed by each user Used with purpose – to solve problems Used for recording, communicating and thinking	Tools include oral language, physical materials and symbols Students develop meaning for tools by actively using them in a variety of situations, to solve a variety of problems Using tools can free our thinking for more creative activities
Equity and Accessibility	Tasks are accessible to all students Every student is heard Every student contributes	All ideas and methods are potential learning sites A variety of ideas are essential for fuelling rich discussions Each person learns to respect and value each other's thinking

During the initial stages of PD, concerns regarding this type of approach were raised by several teachers. Examples of these concerns included a) the very different role of the teacher to that of the traditional role, in particular, *when* teachers should 'tell'; b) the ability of less-able pupils to be involved in problematic tasks and any subsequent self-esteem issues; and c) the suitability of this approach for certain types of mathematics, for example, the possibility of this approach being more suitable for traditionally hands-on content such as Length and less suitable to more "abstract content such as Number". Finally, questions were also posed regarding the possible reactions of parents to this type of approach, in particular, parents of more-able pupils considering the collective responsibility for understanding which is espoused in this approach. Teachers expressed concerns regarding negative reactions from parents if their children have to regularly help others, clarify ideas for others and explain concepts to others. However, despite these concerns, teachers experimented with the implementation of this approach.

Following several weeks of implementation, the tracker teachers were very complimentary about the instructional framework, in particular, its use as a teaching aid. The reported benefits of using the instructional framework included a) using it as a reflection and planning tool for teaching; b) ensuring a focus on problem solving; c) encouraging pupil explanations; and d) giving pupils control of their own learning. Interestingly, all of these teachers reported the same challenge in using the framework – the role of the teacher. In particular, teachers reported a difficulty in changing from a more traditional teacher role to one where teachers and pupils have an equal voice:

> Well the biggest challenge would be the teacher stepping back and trying to give that control…rather than the teacher voice it is the pupil voice that needs to be heard. That was the biggest challenge.

> Teacher has removed him/herself from a position of authority – that was hard…

> I suppose the biggest challenge was that change where you weren't the teacher anymore. That is a big thing for a teacher. That was the biggest thing.

Another teacher mirrored this latter view that the teacher role was being relinquished by "handing over the role of the teacher to the children." Although teachers found this role-change challenging, they also highlighted some benefits to it, for example, teachers viewed it as giving more agency to pupils by "holding back and letting them (the pupils) come around to solutions' and 'the teacher was purely observational really while they (the pupils) were working rather than the teacher spoon-feeding which we tended to do. …That is our nature to spoon-feed them everything." Another teacher illuminated the dichotomy that exists between the teacher telling and the pupils telling:

> So the teacher trying to talk less and giving control over to the children was great, it took that little bit of biting your tongue but once they got into it like they were great…and rather than you telling them, they were telling you. It was great.

Based on this teacher feedback, the suggestion that the role of the teacher should be further clarified in any revised instructional framework is unsurprising. In particular, teachers requested clarification regarding the amount of guidance to give pupils, for example, "It is just to know have you said too much or too little". In other words, by stepping *back* are teachers stepping *out* or stepping *aside*? Teachers referred to this change in many ways from "stepping back" to "handing over" to "letting them off" to "biting your tongue" to "observation" to "facilitation". Derived from this, teachers suggested the need for additional guidance by including "the language of questioning and affirming comments" in any revision of the framework. The uncertainty regarding questioning and guidance was a very real one for teachers and is highlighted by one of the tracker teachers:

> Should you ask any question or by asking a certain question are you giving them (the pupils) too much…are you like helping them to discover? Do you want to ask the bare essentials and let them completely come up with everything or do you want to steer the question?...so you want questions that are going to get them completely thinking kind of openly or do you want a kind of steering question? It is important to know what type of question to ask because you weren't sure like, should I even have said that, or is that still me being in control…

This teacher role uncertainty was also evident in the observed lessons in Phase One. For the majority of these, teachers had completely 'stepped back' and were providing little or no guidance to pupils. Teachers appeared to have taken on an observational role rather than a proactive, facilitative role. This is despite the fact that some teachers had reported taking on a facilitative role. So facilitation, its meanings and practical application in mathematics lessons needed to feature in the Phase Two PD and also in any revised instructional framework.

Observations of the mathematics lessons also revealed that although ample collaborative opportunities were evident through pair and group work, there was very little cross-fertilisation between the groups, in that learning seemed to remain within each small group and was not shared with other groups or with the whole class. Arising from this, sharing and building on mathematical thinking was not obvious at a whole-class level because whole-class mathematical discussion did not take place. This is regrettable considering Dooley's (2010) finding that group work can increase pupils' contributions to whole-class discussion. Moreover, the problems did not appear to be particularly rich or challenging. In conclusion, based on the teacher interviews and lesson observations, the following chief additions were necessary in an effort to refine and enhance the instructional framework for Phase Two:

- an emphasis on the teacher's role as an active, skilled facilitator;
- inclusion of teacher talk which encourages reflection and communication, in particular, the language needed for facilitation and the questioning needed for discovery learning;
- an emphasis on whole-class discussion in developing mathematical thinking including reasoning, and revision of conjectures and solutions;
- an emphasis on students using language to refine, revise, clarify, build on and communicate mathematical thinking;
- inclusion of revoicing to deepen mathematical understanding and to enrich mathematical thinking; and

- an emphasis on students choosing to record verbally, concretely, pictorially/graphically, symbolically or in written form.

Two dimensions of Hiebert et al's instructional framework appeared to be misplaced – Social Culture of the Classroom and Equity and Accessibility appeared to be pre-requisites for a certain type of classroom. Therefore, in the revised instructional framework, these dimensions were combined and moved to a foreword focusing on pre-requisites for a classroom environment. Three dimensions remained so combined with the new dimension regarding teacher talk, the revised instructional framework had four dimensions all beginning with T – Tasks, Teachers, Tools, Talk; hence, the revised instructional framework evolved into the 4Ts Instructional Framework for Maths (see Table 3).

Table 3: The 4Ts instructional framework for maths

4Ts	Core Features	Additional Information
Tasks	Make mathematics problematic Connect with where students are Select tasks with goals in mind Leave something behind of mathematical value	Tasks should encourage reflection and communication Tasks should allow students to use tools Teachers need to select sequences of tasks not just individual tasks Tasks should leave behind important residue - there are 2 types of residue: a) insights into the structure of mathematics (mathematical relationships) and b) strategies or methods for solving problems
Teacher	Take on an active, skilled facilitator role Share essential information Establish classroom culture Encourage revision of conjectures	Teachers facilitate discussion and value silence (Pratt, 2002) in the course of mathematical discussions Explanations and demonstrations by the students become more important than those by the teacher Teachers remove themselves from a position of authority (deciding whether answers are correct) in order to promote the autonomy of students' mathematical thinking Revising conjectures (Lampert, 2001) and solutions is encouraged Reasoning is used to judge whether a conjecture/solution is mathematically sound
Tools	Meaning for tools must be constructed by each user Used with purpose – to solve problems Used for recording, communicating and thinking	Tools include oral language, physical materials, pictures/diagrams (Askew, 2012) and symbols Students develop meaning for tools by actively using them in a variety of situations, to solve a variety of problems Using tools can free our thinking for more creative activities Students choose to record verbally, concretely, pictorially/graphically, symbolically or in written form
Talk	Teacher talk encourages reflection and communication Students use language to refine, revise, clarify and communicate mathematical thinking Talk is used to encourage and communicate reasoning	Teacher questioning is open-ended and probing Teacher responses are neutral (Pratt, 2002) and encourage further discussion Students share, clarify and refine their mathematical ideas through facilitated discussion Revoicing is used to deepen mathematical understanding and to share mathematical thinking Students build on the mathematical ideas of others Talk is used to encourage and communicate reasoning – both in verbal and in written form (either in a journal or a notebook)

Phase Two

The 4Ts Instructional Framework for Maths was used to teach the strand unit Weight. Due to space restrictions, the Pre-requisites foreword, in addition to the suggestions for teacher language and questioning, are not included in this paper; however, these aspects featured as part of the revised instructional framework.

The observed differences in maths lessons between Phases One and Two were noteworthy. In particular, the sharing and promotion of mathematical thinking appeared to be more prevalent in the lessons in Phase Two. This finding is consistent with the teacher feedback from the Phase Two interviews. It is important to note that these observations are reflective of the observed lessons only. They cannot be used to generalise about other maths lessons, either in these tracker classes during these phases or indeed in other classes in the school. A number of similarities and differences are outlined in Table 4. The similarities are denoted by italics.

Table 4: Observed similarities and differences in maths lessons between phases one and two

	Phase One Lessons	**Phase Two Lessons**
Hands-on tasks	*Use of hands-on tasks in collaborative group settings*	*Use of hands-on tasks in collaborative group settings*
Concrete materials	*Use of concrete materials*	*Use of concrete materials*
Discovery learning	*Use of discovery learning*	*Use of discovery learning*
Collaboration	*Use of extensive pair and group work*	*Use of extensive pair and group work*
Groupings	Use of mostly ability groupings	Use of mostly mixed-ability groupings
Whole-class discussion	Little if any whole-class discussion was evident	Whole-class discussion was used extensively to share ideas and concepts following small-group work
Problem solving	Some problem solving evident but mainly related to a practical task	Problem solving was evident in all maths lessons and ranged from the practical through to abstract
Types of problems	Many of the problems were simple, routine, or lower-order in nature	Most of the problems were rich and higher-order in nature
Sharing solution methods	Little evidence of how pupils solved a problem or task	Evidence of pupils sharing and explaining how they arrived at solution methods, for example, using representative materials, drawing a table, trial and error, discussion, etc.
Role of teacher	Teachers appeared unsure of their role and often stepped back completely from the lesson	Teachers appeared more sure of their role and took on a facilitative role in asking probing, open-ended questions, promoting the sharing

		of ideas, suggesting revoicing, etc.
Mathematical authority	Mathematical authority appeared to reside with pairs and groups of pupils rather than with the teacher	Mathematical authority appeared to reside with pupils following whole-class discussion rather than with the teacher
Communicating mathematical ideas	Little evidence of pupils communicating their mathematical ideas	Evidence of pupils communicating their mathematical ideas to the whole class
Refining mathematical ideas	Little evidence of pupils refining their mathematical ideas	Evidence of pupils sharing and then refining their mathematical ideas
Learning logs	Learning logs were often descriptive in nature – often describing the task rather than reflecting on the learning	Learning logs appeared to be more reflective of the learning that took place. This may have been aided by teacher-suggested prompts, for example, What I learned…; What helped me to learn…; etc.

In summation, rich problem solving in addition to communicating and expressing mathematical thinking appeared to be notably more prevalent in the lessons in Phase Two compared to those in Phase One. Furthermore, teachers appeared to be more confident and relaxed with the approach and importantly, with their role as facilitator within this approach. It is important to note that the mathematics lessons in Phase One satisfy most of the recommendations from the Inspectorate reports. These include the judicious use of textbooks; pupils using a range of problem solving strategies; collaborative learning; focussed use of materials and resources; and talk and discussion. However, as outlined in Table 4, although this is a marked improvement from traditional mathematics practice, a dearth in communicating and expressing still exists in these lessons. It is only in the Phase Two lessons that these absent elements became interwoven with the other pedagogical elements resulting in classrooms where pupils were thinking deeply and communicating frequently about mathematics.

DISCUSSION

The evolution of the instructional framework during the project was chiefly influenced by the experiences of teachers coupled with lesson observation and professional reading. The corresponding changes in the mathematics lessons were striking; however, a word of caution is necessary. This instructional framework was not developed in a vacuum; neither was it implemented by teachers in a vacuum. It was one segment of a sustained, on-site professional development programme that included mathematical content knowledge, mathematical pedagogical knowledge, reformed pedagogical approaches, pupil feedback from mathematics lessons, professional dialogue, collaborative planning, professional reading, video footage and podcasts. The changes to classroom practice cannot therefore be solely attributed to the 4Ts Instructional Framework for Maths. The process of this collective endeavour is important.

CONCLUDING REMARKS

Teachers' experiences of using the 4Ts Instructional Framework for Maths were extremely positive in this case study school. Classroom practice appears to have changed considerably in the tracker classes for the strand units of Length and Weight. Communicating ideas, reasoning, refining mathematical thinking, rich problem solving and democratic collaboration appear to be to the forefront of these classrooms. However, challenges have also been apparent throughout this evolutionary process including challenges to teacher beliefs, in addition to time constraints. The teacher's role as an active, skilled facilitator appeared to pose the utmost challenge for teachers, particularly when facilitating mathematical discussion. However, the importance of interpretive flexibility and professional discretion cannot be over emphasised. Successful mathematics classrooms do not mean conforming to a highly prescribed method of teaching (Stigler & Hiebert, 1999; Hiebert et al, 1997). Instead, it means "taking ownership of a system of instruction, and then fleshing out its core features in a way that makes sense for a particular teacher in a particular setting" (Hiebert et al, p. 14). In this way, the 4Ts Instructional Framework for Maths can act as a temporary bridge or scaffold between recommended pedagogical features and pupil learning where planning, implementation and reflection feature in an iterative feedback loop. Equally, it has the potential to guide teachers in progressing from (in the words of one of the teachers) "spoon-feeding to biting your tongue" … and beyond!

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