Getting the balance right

Much of the number work carried out in primary classrooms contains elements of algebraic reasoning. In fact, equals sign work in junior and senior infants underpins much of the algebraic reasoning used in later years. In the 1st class algebra strand, the Primary School Mathematics Curriculum (PSMC) refers to the need for children to 'understand the use of a frame to show the presence of an unknown number (e.g. 3 + 5 = ❑)'. In order to address this objective and set appropriate foundations for later work in equations and inequalities, primary school children must develop a relational understanding of equality.

Assessing childrens' understandings of equality

While the equals sign is introduced to children as young as five or six; the concept of equality is not simple. To investigate your pupils' understandings of equality, present them with the following problem and record their responses:

8 + 4 = ❑ + 5

Operational view of equality

The following (incorrect) responses indicate an operational view of equality:

8 + 4 = 12 + 5
8 + 4 = 17 + 5

When children give responses of 12 or 17, they believe that the equals sign means 'the answer is'. Consequently it seems logical to such children that when you see an equals sign you should perform the calculation that precedes the equals sign and that the number to the right of the equals sign is the answer to that calculation. They see the equals sign as a unidirectional operator situated between the input (to the left) and the output (to the right). This 'operator' view is the result of an overemphasis on problems of the form a + b = ❑.

Relational view of equality

The following (correct) response indicates a relational view of equality:

8 + 4 = 7 + 5

Children with a relational view of equality believe that the equals sign means 'is the same value as'. Such children understand that the amounts either side of the equals sign are relationally the same. They then search for a value that when placed in the frame (❑) will result in both sides of the equals sign balancing each other.

Developing relational understandings

We describe a sequence of teaching approaches (visual, narrative, kinaesthetic, symbolic) that we found supportive in developing a relational understanding of equality with fourth class children. Some strategies, however, are suitable for use as early as senior infants.

1 Introduction: asking what does the equals sign mean?

Image 1: the equals sign

Show an image of the equals sign and ask 'Can you explain what the equal sign means?' Children's answers provide insights into whether they hold a relational or operational view of equality. One child, who held an operational view, responded; "It's the thing going to the answer that the two numbers equal". In contrast, a child with a relational view responded; "It actually means 'is the same as' because one plus one is the same as two".

2 The same but different!

We then emphasised that expressions may have the same value but look different. This reinforces the idea that equality may involve the same value on both sides of the equals sign even though both sides may look quite different. We selected the context of money as it is a context that children could relate easily to and it is helpful in reinforcing the idea that a variety of different combinations may still have equal value. Examples we used included:

€1 = 50c + 50c   25c x 4 = €1
50c + 50c = 25c x 4

3 Making a human equation

Human equations may also be used to further explore the relational concept of equality. The only materials needed for this activity are symbols of the equals sign and operations (addition, subtraction, multiplication and division). Select a number of children to stand in groups of certain sizes at the top of the classroom. The remaining children have to position the equals sign and one of the operations signs so that the (human) quantities balance around the equals sign. In the example (image 2), twelve children are in a line. Six are in one group, four in another, and the final group consists of two children. The class selected the calculation 6= 4+2. This activity challenges the notion that when you see the equals sign you write down the answer. Present a number of problems that increase in complexity and utilise a variety of operations.

Image 2: human equation

4 Exploring equality through children's literature

The story *Equal Shmequel* (by Virginia Kroll) is another novel way to introduce the see-saw analogy to develop relational understanding of equality among children. The story centres on a mouse and her woodland animal friends who decide to play a friendly game of tug of war. They struggle to work out how to make both teams equal so that the tug of war is fair. They solve the problem by distributing animals either side of a see-saw until both sides balance. Balancing different sized animals reinforces the notion that things can be the same value but at the same time look different. If possible, project images from the book as you read the story aloud.

Image 3: making predictions

As the *Equal Shmequel* story is being read, children can communicate their own predictions regarding who would win using their hands to demonstrate a kinaesthetic notion of balance. This reinforces the idea that the value of quantities either side of the equals sign has to balance each other.

5 Using pan balances

Many of the previous approaches focus on the balance notion of equality i.e. the left and right hand sides of number sentences must balance. Children can use pan balances to 'solve' number sentences by determining the value which balances

The equals sign

both sides. This activity can be used effectively, using simpler number sentences, with children in the infant classes. The children in the textbox are working on the problem 8 = ❑ + 3. Subsequently a *drawstring bag* can be introduced. It provides a visual representation of the 'unknown' and illustrates that the 'unknown' must make both sides of the equals sign balance. Start with a simple problems such as 8 = ❑ + 3. Place eight marbles on one side of the pan balance. Place three marbles on the other side of the balance. The sides will not be balanced. Slowly add a drawstring bag (with 5 marbles hidden inside) and watch the balance level out. Children should be given opportunities to share and justify their predictions.

6 Representing and solving word problems

All of the previous approaches provide pupils with appropriate understanding required to interpret, translate and solve word problems using equations. Encourage children to write problem scenarios as equations and represent unknown quantities with frames (❑). This provides opportunities to translate between problem scenarios and their algebraic representation. Consider using a balance to support pupils' explorations. We found that while some groups used pan balances from the outset, several only used the pan balances to check their solutions. Children should be encouraged to come up with several ways that any one scenario could be represented algebraically. We observed such flexibility among the children we worked with. An example of a starter problem scenario to try (see website for more challenging examples): *there were 30 children on a bouncing castle. Some of these children went inside to have a drink. There were 16 children left on the bouncing castle. How many children went inside to have a drink?* 30 - ❑ = 16 (or 16 + ❑ = 30)

There were 42 pupils in Ms Mahon's fourth class. A few pupils joined the class in January. Then there were 50 in the class. How many pupils joined the class? 42 + ❑ = 50 (or ❑ + 42 = 50).

Cian and his friends took part in a local charity cycle between Magnolia Primary School and Sunflower Primary School. The schools are 10 kilometres apart. It took them 2 hours to complete the cycle. What was their average speed? 10 = ❑ x 2 (or ❑ = 10 ÷ 2)

Teacher:	Did you ever see a problem like this? (*Teacher writes on the board: 8 = ❑ + 3.*) Eight equals something unknown plus three.
Children:	Yes (*nod in agreement*).
Teacher:	Okay I am going to show you here on my scales how to think about the 'something unknown'. (*Teacher places marbles in the left side of the scale one-by-one.*)We put 1, 2, 3, 4, 5, 6, 7, 8 in here. (*Teacher places marbles in the right side of the scale one-by-one.*) And I am going to put 1, 2, 3 on this side. Is the scale balanced?
Children:	No.
Teacher:	Now I am going to put my unknown on this side. (*Teacher places the drawstring bag in the right side of the scale.*)
Teacher:	What has happened when I add in my unknown?
Children:	It balances.
Teacher:	Yes. So we don't know what is in the bag. How will we work it out?

7 True or false statements

True/false number sentences were used as an assessment strategy. Each child had a card with 'true' printed on one side and 'false' on the other side. Sample equations presented:

12 + 28 = 5 x 8

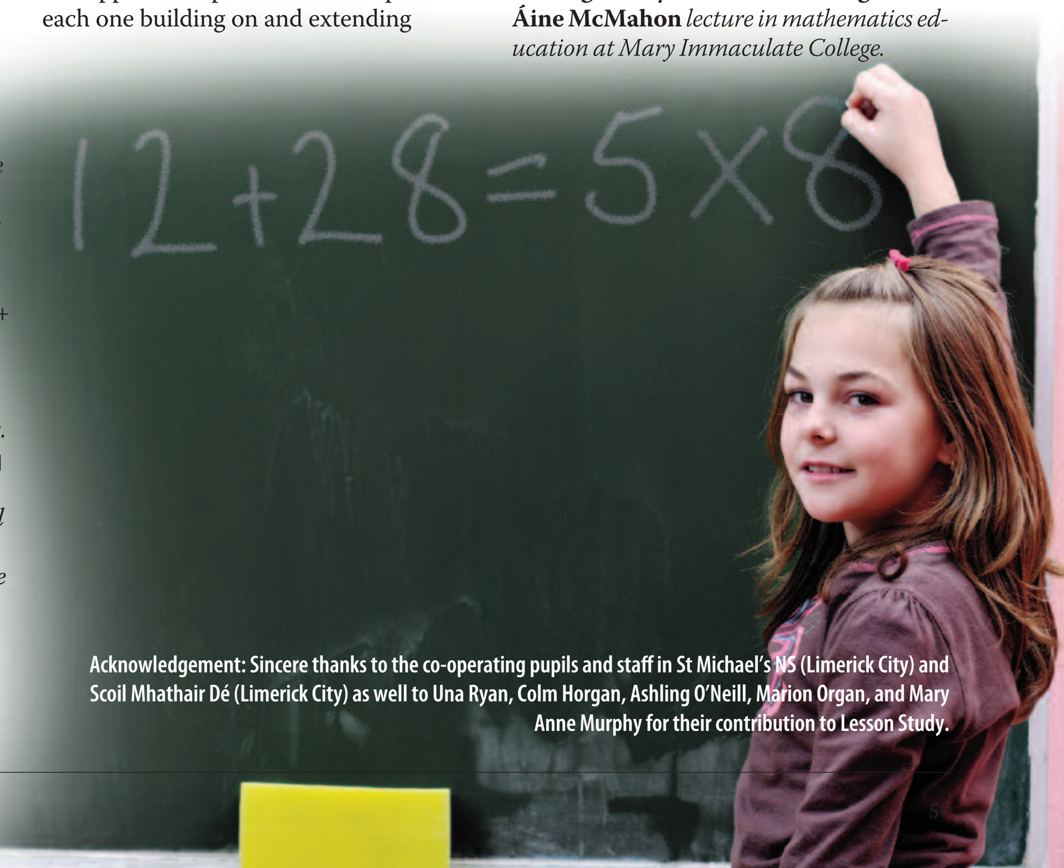
11 + 16 = 17 + 9

After providing children with adequate time to make a decision regarding the accuracy of the statements, they indicated their conclusion using the true/false card. Performance on the equations indicated whether children had developed a relational understanding of equality.

In conclusion

The approaches presented are sequential, each one building on and extending understandings of equality. While initial activities encourage children to informally explain how both sides of a number sentence can be balanced (e.g. coins, human equations), later activities provide opportunities to create number sentences using frames to represent the unknown. We found that the variety of contexts used drew on children's imaginations and were motivating. This made the process of learning an enjoyable one. At the end of the sequence of instruction children were at ease in representing a single scenario in several different algebraic forms thus demystifying the activity of writing equations.

Aisling Leavy, **Mairéad Hourigan** *and* **Áine McMahon** *lecture in mathematics education at Mary Immaculate College.*



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