A conceptual framework for integrating mathematics and science in the secondary classroom

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Abstract

This article presents a theoretical model for integrating mathematics and science in the secondary classroom. This model, Authentic Integration of Mathematics and Science (AIMS), promotes engagement with rich tasks which combine topics from mathematics and science to enable enhanced learning through structured inquiry, dialogue, and application of knowledge and skills from both subjects to relatable tasks. It is argued that this model will provide opportunities for students to retrieve previously learned material and explore key concepts from both disciplines in tandem, thereby strengthening retention and understanding. Application of this model should also support the enhancement of students' problem-solving skills and the facilitation of meaningful applications of mathematics to other disciplines in a sustainable manner. Attempts to integrate mathematics and science in the classroom are widely recommended but often encounter barriers such as deficiencies in teacher knowledge of their non-specialist subject, the inflexible nature of school timetables, and a dearth of instructional materials, amongst other issues. Lesson study is proposed as an effective means for operationalising the AIMS model and providing a framework which accounts for these barriers and allows for consistent implementation in tandem with single-subject instruction.

Keywords: Applications of mathematics; integrated pedagogy; rich tasks; science inquiry; transdisciplinary lessons.

Introduction

Demand for STEM (Science, Technology, Engineering, and Mathematics) skills required to meet economic challenges has increased thus placing greater emphasis on the improvement of STEM education in many nations (English 2016, Kelley & Knowles 2016; Marginson et al. 2013). This is due in no small part to the shift in emphasis from manufacturing to information and knowledge industries in world societies and economies due to rapid technological innovation (European Commission 2018). Creativity, problem solving, and critical thinking through the utilization of knowledge and skills from STEM are in high demand currently and will continue to increase in importance for future employment (OECD 2019; European Commission 2018). Analyses of the state of mathematics education around the world have led to recommendations that schools place greater emphasis on problem solving throughout the mathematics curriculum and meaningful application to other subjects, particularly science (e.g. Jerrim & Shure, 2016; DES 2017; NCTM 2018).

The Royal Society (2014, p.49) stated that science and mathematics education should adjust to accommodate the development of cross-disciplinary skills through “collaboration and open, dynamic dialogue within and across disciplines” as a complement to single-subject teaching. The STEM Task Force Report (2014, p.9) agrees that the STEM disciplines “cannot and should not be taught in isolation, just as they do not exist in isolation in the real world or the workforce”. Similarly, the Organisation for Economic Co-operation and Development [OECD] (2019) highlight the need for interdisciplinary knowledge to be developed among students in order to enable them to understand and solve complex problems. As such, integration of content from mathematics and other subjects through meaningful learning experiences has been highlighted as an area of important development for modern education systems.

Integration is a term which is interpreted variously. Here, it will be defined as the application of knowledge and skills from two or more disciplines to tasks which challenge students to explore phenomena of varying complexities (Honey, Pearson, & Schweingruber, 2014). This term has been further subdivided into various types which provide a continuum to better define the range of potential classroom approaches (see Table 1).

Table 1: Types of integration of subjects in the classroom (English 2016, p.2 adapted from Vasquez et al. 2013)

Type of Integration	**Characteristics**
Disciplinary	Concepts and skills are learned separately in each discipline.
Multidisciplinary	Concepts and skills are learned separately in each discipline but within a common theme.
Interdisciplinary	Closely linked concepts and skills are learned from two or more disciplines with the aim of deepening knowledge and skills.
Transdisciplinary	Knowledge and skills learned from two or more disciplines are applied to real-world problems and projects, thus helping to shape the learning experience.

Why Integrate?

The drive to promote STEM education has been rooted in the needs of societies and economies and, as a result, focus has typically been placed on addressing real life issues which invariably require the combination of knowledge and skills from different disciplines (Hazelkorn et al. 2015; Maass et al. 2019). This has led to an emphasis on not just improving student proficiency in the individual disciplines but also the promotion of means by which students can be challenged to combine learning in each subject (Maass et al. 2019). Initiatives such as the STEM@school project funded by the Flemish government in Belgium are becoming common place. This project aims to promote engagement in STEM projects at upper secondary level in order to increase student integrative learning experiences through real world problems (Knipprath et al. 2018). Similarly, the revised national core curriculum in Finland has resulted in the implementation of multidisciplinary modules within which students engage in learning which combines content from a range of subjects, often through a problem-based learning approach. The new core curriculum has made it compulsory for schools to run at least one of these modules once every school year (FNBE, 2016).

Allied to the proposed need to prepare learners to combine knowledge and skills from different disciplines, application of an integrative learning experience has the propensity to promote meaningful connections of concepts which typically leads to better and more sustained levels of learning (Weinstein et al. 2018; Honey et al. 2014; Becker & Park 2011). Science can often provide physical and/or visual representations of hither-to abstract mathematical concepts, thus not only forming connections between the subjects but providing students with enhanced representations and mental models (Ní Riordáin et al. 2015; Honey et al. 2014). Mathematics can also promote deeper understanding of scientific concepts by means of quantifying or numerically representing such phenomena (McBride & Silverman 1991).

Integrating mathematics and science in the classroom has previously produced encouraging outcomes with small to medium positive effect sizes for both mathematics (ES=0.27; SE=0.09) and science (ES=0.37; SE=0.12) reported by Hurley (2001) when analysing studies which compared an experimental group (integrated approach) to a control group (single subject approach). Recent applications of an integrated approach in Ireland have also been well received, gaining positive feedback from the teachers involved (Treacy & O'Donoghue, 2014). Much of the support for integration of mathematics and other subjects stems from the potential for more relevant and stimulating experiences which promotes problem solving, critical thinking, and retention (Stohlmann, Moore, & Roehrig 2012). While there are encouraging signs regarding this pedagogical approach, research in this area is sparse resulting in minimal evidence to support its implementation.

More recent studies appear to go beyond integrating mathematics and science by aiming to integrate most or all of the STEM subjects (e.g. Hobbs et al. 2018, Leung 2019, Gardner & Tillotson 2019), however there are some studies which maintain a focus on the integration of mathematics and science. Ní Riordáin et al. (2015) completed interventions in three secondary schools in Ireland within which they supported teachers to complete a three-week unit with their students on distance, speed, and time. Teachers in mathematics and science collaborated to form connections between the subjects within single-subject tuition and align their lessons so that learning within mathematics lessons could be utilised in science lessons and vice versa as the unit progressed. They found that teacher knowledge, collaboration, attitudes, and support played significant roles in the implementation of such lessons and units, while also identifying a range of challenges such as lack of curricular materials and time for planning.

A follow up to this study provided a professional support network of teachers and education researchers to aid these teachers to continue their development (Johnston et al. 2019). Teachers had more opportunities to share ideas, access curricular materials, and indicated they felt greater ownership over the design of lessons integrating mathematics and science. This may present a potential means for addressing some of the aforementioned barriers to integrating these subjects. Such barriers need to be considered in depth in order to develop a practical way forward for integrating mathematics and science.

Barriers to Integrating Mathematics and Science

Integrating mathematics and science, and potentially other subjects, has gained increased support and interest in the research community and within proposed educational reforms. However, any attempts to effectively apply this pedagogical approach must recognise that there are distinct barriers which have been repeatedly encountered. Such obstacles include the need to coordinate students and curricula, the extra time and effort required to implement this change, and deficiencies in instructional models and curricular materials (Beswick & Fraser 2019; Ní Riordáin, Johnston, & Walshe, 2015). While these factors do pose a challenge, a more pressing concern is that of teacher knowledge within both subjects as inadequate subject matter knowledge can often be a key reason for failure when attempting to integrate mathematics and science (Beswick & Fraser 2019; Ní Riordáin et al., 2015; Honey et al. 2014).

The level and complexity of knowledge held by a teacher affects what is done in classrooms and, as a consequence, also influences what students learn (Fennema & Franke, 1992). Students learn more from teachers who are skilled, experienced, and know what and how to teach (Rice, 2003). Integrating mathematics and science requires the teacher in question to have a certain level of both subject content knowledge and pedagogical content knowledge to educate students in both disciplines successfully (Frykholm & Glasson, 2005). Consequently, the knowledge required to effectively instruct students in an integrated setting is a vital element of the successful implementation of such lessons. Deficiencies in relation to such knowledge are regularly highlighted as barriers to effective integration of mathematics and science in the classroom, with teacher knowledge of their non-specialist subject often cited as an issue (e.g. Pardhan & Mohammad 2005; Ní Riordáin et al. 2015). In addition, it has been argued that it is vital to maintain the structure of disciplines such as mathematics and science so that subject-specific problems and challenges can be encountered to allow students to develop and improve specialised skills and knowledge (Honey et al., 2014). Similarly, the rigidity of the school timetable and an emphasis on single-subject final examinations have been cited as problems also (Stinson, Harkness, Meyer, & Stallworth, 2009).

Braskén et al. (2019) identified many of these issues when they explored a multidisciplinary module for Grade 9 students (age 15-16). This multidisciplinary module, implemented in a school in Finland as part of their new national curriculum, integrated mathematics and science along with other subjects from the humanities through a central theme of 'Energy' in 14 sessions (one hour each, two per week) over 7 weeks. While it was suggested that the module allowed for authentic learning opportunities, reservations were expressed regarding vague learning goals, difficulty combining a large range of subjects effectively, and lack of time and support for effective coordination and planning. These outcomes highlight the challenges, both practically and pedagogically, when engaging in integrative instruction.

Recognition of the barriers highlighted thus far has resulted in a range of challenges facing this field of education. The means by which educators can design quality learning experiences which integrate mathematics and science while also accounting for potential gaps in teachers' subject knowledge and pedagogical subject knowledge is a substantial challenge. The challenge grows larger when we add in the need to adapt to school timetables, develop curricular materials, and ensure key content specific to each subject is not neglected. Research so far in this field has made progress in relation to these challenges but no clear blueprint has been provided to sustainably incorporate integrative lessons into regular secondary education. If more education systems are to introduce some form of multidisciplinary modules in a similar manner to Finland, then this is pressing issue which needs to be addressed. This leads to the overall question underpinning this theoretical article: How can mathematics and science be effectively integrated in transdisciplinary lessons and implemented successfully and sustainably in secondary level education? A vital starting point is to consider how such lessons can be integrated into the current structures which generally focus on single-subject instruction.

Integrating mathematics and science as a complement to single-subject instruction

The aforementioned barriers to integrating mathematics and science in the secondary classroom need to be recognised and accounted for within any attempt to establish lessons of this nature in a sustainable manner. Moving away from lessons which focus on one discipline would require a significant transformation in the structure of a typical secondary education system. Allied to that, it is strongly recommended that educators within STEM subjects such as mathematics and science should avoid undermining student learning within these disciplines by attending to learning objectives and progressions specific to that subject (Honey et al., 2014; Tytler et al. 2019). Similarly, attention needs to be paid to the cognitive demands placed on students when integrating mathematics and science, as making connections between different disciplines within complex problems could overwhelm students and negatively impact on their learning and motivation (Honey et al., 2014; Kirschner, Sweller, & Clark 2006). As such, lessons which integrate mathematics and science need to be applied as a complement to single-subject instruction so that students establish a strong foundation of knowledge and understanding in the individual disciplines before facing the challenge of combining content from both areas.

I propose that this can be achieved by implementing lessons which integrate mathematics and science in 4-5 week intervals throughout the school year. Topics and/or concepts from both mathematics and science which students have studied previously should form the focus of these lessons. Such an approach has the value of allowing students to engage in distributed practice, while the inherent need to switch between important ideas and concepts from both mathematics and science within these lessons ensures the constant presence of interleaving. Distributed practice, which is the process of reviewing content on separate occasions across weeks or months, typically leads to better retention of learning (Rohrer 2015; Cepeda, Pashler, Vul, Wixted, & Rohrer, 2006). Interleaving involves the practice of different types of content within a lesson (e.g. concepts and processes from science and mathematics) and has been found to be quite beneficial to student learning and retention in mathematics and other disciplines (Rohrer, Dedrick, Hartwig, & Cheung, 2020; Pashler et al., 2007). A further benefit of this approach to integrating mathematics and science is that, even though prior single-subject instruction of each topic needs to have taken place, such instruction does not need to have occurred at the same time nor is it required to have been recent. This could best be defined as a 'disciplinary' approach, i.e. concepts and skills are learned separately in each discipline (see Table 1). Such an approach means that teachers of each discipline are not required to closely align the timing of instruction of any particular content in their curricula (i.e. a multidisciplinary approach) which could otherwise be a significant challenge and can sometimes hinder learning outcomes in either subject (English 2016).

Combining knowledge and skills from both subjects is a significant aspect of integration and one which requires careful planning. Knowledge is typically inflexible early in the learning process. Applying knowledge to very different contexts, i.e. 'far transfer', in the early stages of learning will typically result in failure (Willingham 2009). Teachers tend to focus more on providing challenges in subjects such as mathematics whereby students encounter problems with similar underlying structures and expected solution procedures, i.e. 'near transfer' (Renkl 2017). In these situations, the numerical values or objects (if the task being adjusted is a word problem) tend to be the characteristics varied. Students are just required to recognise which learned algorithm is required and apply it accurately to the problem. In contrast, 'far transfer' problems typically require an adapted approach to be applied in order to solve the problem or complete the task. 'Far transfer' is typically required for successful engagement with tasks which require the application of knowledge and skills from one or both disciplines in cross-curricular contexts.

Establishing strong foundational knowledge is important in order to apply understanding in a flexible manner (Perkins and Salomon 1989). 'Worked examples' which provide well-structured and clear overviews of the applications of relevant algorithms or problem-solving approaches have been demonstrated to aid such development and form an important precursor for effective engagement with 'far transfer' tasks (Cooper and Sweller 1987, Renkl 2017).

Similarly, exploring and comparing the underlying deep structure of problems typically leads to enhanced transfer (Minervino et al. 2017). As such, establishing foundational learning in a topic with effective single-subject teaching strategies is necessary prior to challenging students to apply their understanding in cross-curricular contexts. Using a context that is already familiar, e.g. a scientific concept studied previously, should aid the transfer process also as it reduces cognitive load due to the familiarity of the context. Establishing such an approach should best support learners to engage in 'far transfer' of mathematical and scientific knowledge into different contexts, particularly transdisciplinary contexts.

Authentic Integration of Mathematics and Science

Designing lessons which integrate mathematics and science can adopt a range of approaches as suggested by the various types that have been previously observed (e.g. see Table 1). The Authentic Integration of Mathematics and Science (AIMS) model (see Fig. 1) offers a blueprint for creating effective transdisciplinary lessons based on a set of key principles. A transdisciplinary approach can be defined as instances where "knowledge and skills learned from two or more disciplines are applied to real-world problems and projects, thus helping to shape the learning experience" (English 2016, p.2). Central to these transdisciplinary lessons will be rich tasks which integrate concepts from both mathematics and science simultaneously. Establishing lessons of this nature would place greater emphasis on problem solving and the application of meaningful, cross-curricular activities in the mathematics classroom which has been regularly recommended (e.g. The Royal Society 2014; Jerrim & Shure, 2016; DES 2017; NCTM 2018).

Construction of this model has its foundations in the Authentic Instruction model refined during the 1990s by Newmann and associates (see Newmann et al. 2007). The Authentic Instruction model was selected due to its suitability for supporting complex intellectual tasks which can often require knowledge and skills from multiple disciplines as well as the proven value of its application across a range of subjects. Research carried out between 1990 and 2003 indicated that student outcomes in a range of subjects (grades 3-12) were superior when they experienced higher levels of Authentic Instruction compared to students who experienced lower levels of Authentic Instruction (Newmann et al. 2007). The Authentic Instruction model is based on three criteria: construction of knowledge, disciplined inquiry, and value beyond school. These criteria were adapted to suit the challenges and requirements of integrating mathematics and science while also applying guidance from research in related fields. These three characteristics, along with research in the field, inspired and informed the development of the three key characteristics of the AIMS model observed on the outer layer of the visual representation (see Fig. 1).

The key characteristics of the AIMS model include opportunities for students to consolidate and synthesise the knowledge and skills developed in previous single-subject lessons; a focussed, structured approach to inquiry within the rich task accompanied by opportunities for dialogue; and applications of learners' knowledge to relatable scenarios. Each of these characteristics will be present in the formulation of a rich task which will be the main element within such a lesson.

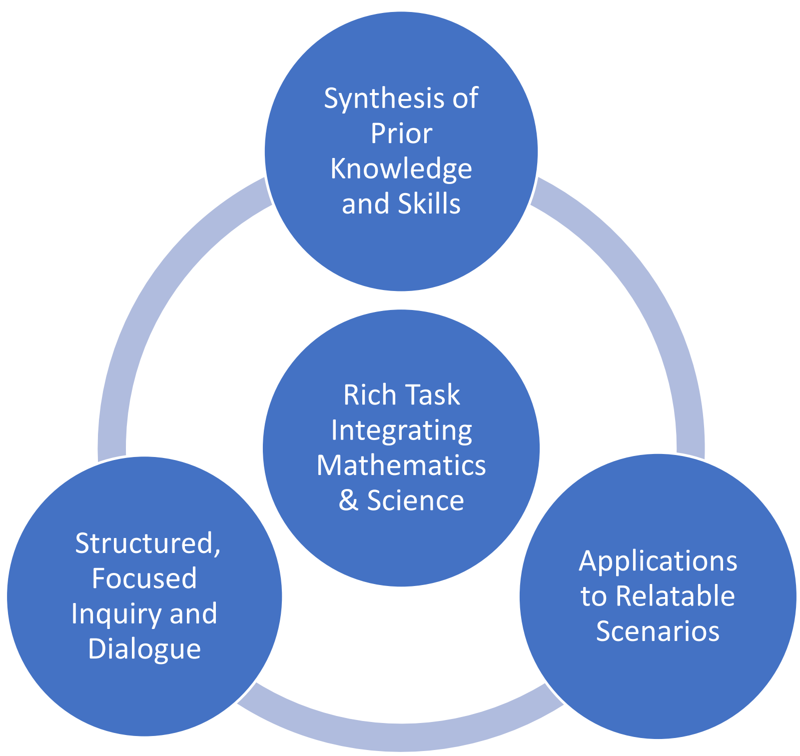

Figure 1: Authentic Integration of Mathematics and Science (AIMS) model.

Rich Tasks

Rich tasks form a central element of an AIMS lesson so that students may engage in inquiry, problem solving, and analysis of relatable representations of what may otherwise be abstract concepts in mathematics and/or science. Rich tasks are typically open in nature, promote problem solving, often link to the real world, and require combinations of knowledge, skills, and procedures in order to achieve success. Simple applications of learned procedures or algorithms (i.e. 'near transfer') would not be sufficient to engage effectively in such tasks. Rich tasks have been demonstrated to support the development of mathematical fluency while also retaining the potential to better engage students when compared to traditional practice problems (Foster 2018). The characteristics of rich tasks also align well with inquiry-based approaches which are commonly applied in science contexts and have been proven to aid student interest, enjoyment, and other positive dispositions towards science (Cairns & Areepattamannil 2019).

Establishing the means to connect and integrate abstract representations of a concept with concrete representations of the same concept have been found to enhance understanding and learning (Richland, Zur, & Holyoak 2007). Rich tasks offer the scope to achieve this. An example of a task of this nature would be the calculation of target heart rate during exercise. This task allows for exploration of the workings of the cardiovascular system while also providing opportunities for application of knowledge of algebra, percentages, ratio and proportion. Tasks of this nature should be carefully structured to offer the required guidance and scaffolding for students with explicit identification of concepts from mathematics and science. Such an approach is needed due to the observation that connecting ideas productively across different disciplines can be quite challenging (Honey et al., 2014).

Synthesis of Prior Knowledge and Skills

Suitable prior knowledge is vital in order for learners to master new ideas and make connections between existing knowledge (Day & Goldstone, 2012; Renkl 2017). Once a foundation of knowledge and understanding in a particular topic or concept has been established, it is imperative to provide opportunities to make connections between concepts to strengthen understanding and recall. Achieving this characteristic of the AIMS model requires that content from both disciplines is explored in sufficient depth. Mathematics should not just be used as a tool for enabling the completion of a task in science and science should not be viewed as a means to provide a useful context for mathematical concepts.

Opportunities to combine previous learning from both mathematics and science also allows for a logical progression through a student's learning pathway as recommended in the revised version of Bloom's Taxonomy (Anderson et al., 2001). Students can potentially progress from challenges of being able to remember, understand, and apply within a single domain towards challenges involving analysis, evaluation, and creation across domains of knowledge. Complementing single subject instruction – which establishes foundational learning – with transdisciplinary lessons creates the platform for such experiences. However, teachers should be aware that combining and applying previously learned knowledge and skills in contexts other than which they were learned typically requires careful planning and support (Honey et al. 2014).

This characteristic of the AIMS model also ensures students experience the benefits of interleaving and distributed practice. The process of switching between knowledge and skills in mathematics and science in order to effectively engage in a rich task should lead to enhanced retention of learning. Rohrer et al. (2020) demonstrated that practice of this nature, i.e. interleaving, had a strong effect on student retention in mathematics at lower secondary level when learning was assessed one month after instruction (d = 0.83). Similarly, the engagement with previously learned material after a period of time (distributed practice) is an inherent element of this AIMS characteristic as students are returning to content with which they have engaged previously. Distributed practice has also been demonstrated to significantly enhance student retention of learning (Rohrer 2015; Cepeda, Pashler, Vul, Wixted, & Rohrer, 2006).

Structured, Focused Inquiry and Dialogue

Opportunities to apply knowledge and skills to new contexts through inquiry is a vital part of the learning process. Mourshed et al. (2017) in their analysis of student performance in PISA 2015 science indicated that students who experienced a blend of teacher-directed instruction (most of their lessons) and inquiry-based instruction (some lessons) experienced the best achievement outcomes. Teig et al. (2018) came to similar conclusions when exploring data related to 8th grade

Norwegian science students in TIMSS 2015. Providing inquiry opportunities through lessons of this nature can complement prior single subject instruction in which foundational knowledge and skills have been developed. However, this should be monitored carefully to ensure that student inquiries maintain a clear focus and result in high-quality learning (Newmann et al. 2007; Dennis & O'Hair 2010).

Applying previously learned concepts and knowledge to new contexts (i.e. transfer) is quite challenging and is typically most successful when accompanied by careful support and structure from the teacher (Kirschner et al., 2006; Day & Goldstone, 2012). Learners often require explicit cues to recognise opportunities for application of prior learning to new contexts. As such, teachers need to carefully structure rich tasks which integrate mathematics and science so that learners are given sufficient support and guidance to utilise their prior knowledge effectively as they progress. Such guidance can often be achieved through effective dialogue and discussion.

Dialogue goes beyond closed questions which follow a sequence of 'teacher question – pupil response – teacher feedback' by promoting higher-order thinking through purposeful questioning and discussion (Alexander 2017). Regular dialogue between teachers and learners is vital in order to provide cues and scaffolding as well as advancing and checking for student understanding. Similarly, dialogue between learners, where suitable, allows for the development of a shared understanding. This can be particularly beneficial when students express and argue differences of opinion in small group settings (Howe et al. 2019). Implementing dialogic teaching has been proven through large-scale randomised control trials to have a significant impact on learning in mathematics, science, and English (Alexander 2018). Providing opportunities for students to explain and describe ideas in detail in one-to-one, small group, or whole class settings enables them to organise and integrate understanding thereby enhancing their learning (Weinstein et al. 2018). An enhanced focus on dialogue would also provide opportunities to improve transfer by exploring the deep structure of a problem to better understand the underlying concepts (Minervino et al. 2017). Providing opportunities to engage in such dialogue is a natural element of rich tasks and should enable students to synthesise and advance their learning in mathematics and science.

Applications to Relatable Scenarios

Concepts within mathematics and science can often be abstract in nature, thus difficult for students to fully understand. Creating tasks which allow for these concepts to be applied to relatable scenarios can provide enhanced representations of these concepts which can lead to improved understanding. Typically, learning is more effective when students can map a new idea onto one with which they are already familiar, with this being particularly true in relation to mathematics (Richland et al., 2007). Such a concept is central to the instruction theory for mathematics known as Realistic Mathematics Education (RME). RME has underpinned the teaching of mathematics in the Netherlands since the 1970s and has been successfully applied in the UK and US (van den Heuvel-Panhuizen et al. 2020). Posing problems to which students may relate is considered vital in RME so that they can recognise the importance and meaning of the mathematics they are learning (van den Heuvel-Panhuizen et al. 2020). Incorporating this characteristic into the rich task will aid the learning process during these transdisciplinary lessons and may also improve motivation to engage with the content.

Operationalizing AIMS through Lesson Study

Determining the design features of transdisciplinary lessons is only one part of the challenge. Identifying how these lessons can be sustainably incorporated into regular schooling is typically a greater challenge due to the barriers mentioned previously. This section will address these barriers and present the means by which the AIMS model can be operationalized to overcome these potential issues.

A significant barrier to integrating mathematics and science which has been identified regularly is the deficiencies in content knowledge and pedagogic content knowledge that the teacher may have in their non-specialist or 'other' subject (Ní Riordáin et al. 2015; Honey et al. 2014). Allied to that, a lack of curricular resources and time to plan transdisciplinary lessons negatively impact the implementation of these lessons (Beswick & Fraser 2019; Stinson et al. 2009; Ní Riordáin et al. 2015). Adopting a team-teaching approach to planning, delivering, and reviewing these transdisciplinary lessons – a somewhat adapted form of Lesson Study – may aid in overcoming these significant barriers.

Lesson study has been branded as "practice-based professional development" (Huang et al. 2016, p.425). It typically adopts the pattern whereby educators engage in cycles of collaborative planning, observation, and reflection upon student learning within lessons (Lewis 2016). This approach, which originated in Asia and has gained in popularity around the world in the past decade, has been applied to enhance mathematics teachers' expertise and create high quality lessons in a range of settings (Huang et al. 2016). It has also been argued that Lesson Study is an effective means by which the gap between theory and practice may be bridged due to the positioning of teachers as key stakeholders in the adaptation and application of research recommendations (Kieran et al. 2013). As the challenge of integrating mathematics and science is a growing area of educational research, positioning teachers at the centre of this process through Lesson Study could provide greater insights into overcoming the practical challenges as well as enhancing the theoretical framework in this field.

Adopting a Lesson Study approach would enable teachers, one mathematics and one science, to select a lesson appropriate to the stage of learning and the needs of their students, tailor it to suit, deliver it as a pair, and then reflect upon the learning which took place. This would ensure that deficiencies in content knowledge and pedagogic content knowledge that either teacher may have would be addressed and improved through collaboration. It would also provide a structure within which teachers could adequately assign and utilise the time needed to plan transdisciplinary lessons that are suitable to their unique contexts. Implementing regular Lesson Study cycles also aligns with a key recommendation of Honey et al. (2014, p.8) as they argued that those engaged in integrating STEM subjects "should explicitly ground their efforts in an iterative model of educational improvement".

Collaborative approaches in such circumstances have been successful in the past as Tytler et al. (2019) reported on two projects implemented to support secondary school teachers in Australia to collaborate to integrate STEM subjects in their lessons. They found that teachers became more confident in the value of interdisciplinary project-based learning as they progressed in the planning and implementation of such lessons. Interaction with other teachers proved to be vital as it enhanced professional learning, knowledge within the 'other subject', and was a valuable source of ideas for lessons. As mentioned previously, Johnston et al. (2019)

reported similar findings in relation to the establishment of a professional support network for teachers integrating mathematics and science. The challenge of crossing the 'boundary' between familiar and unfamiliar domains (i.e. teachers developing expertise in the teaching of their 'other subject') provides opportunities for professional learning also (Hobbs et al. 2018). This process of professional learning within STEM subjects is characterised by identifying learning needs; accessing resources to support this learning; reflecting upon practice; and transforming practice and identity (Hobbs et al. 2018). Recognition of these stages of professional learning through 'boundary crossing' would appear to align effectively with the Lesson Study framework.

Given that curricular materials are vital in aiding this process, I have constructed ten lesson guides (available on request) which each provide a broad outline of a lesson based on the AIMS model which teachers can adapt to their setting and student needs. These lesson guides combine typical topics from Year 9 UK mathematics and science curricula. One such lesson guide is based on the 'Karvonen Method' for calculating target heart rate (THR) during exercise which challenges students to combine knowledge and understanding in science (Cardiovascular System) and mathematics (algebra, number, ratio & proportion) (see Fig. 2). Within the rich task, students are challenged to work out their THR for exercising at various levels of intensity using the formula: $THR = ((HR_{max} - HR_{rest}) \times \%\ intensity) + HR_{rest}$. This rich task should support discussion regarding the reasons why heart rate increases when exercising intensely thus offering opportunities to develop further understanding of the need for blood to be pumped through our bodies. It should also provide opportunities for meaningful applications of mathematics. The structure of the rich task can be adapted to suitable levels of challenge depending on the student group and provides a range of potential avenues for dialogue and exploration. For example, extensions such as graphing functions and calculating intensity level of exercise for a given heart rate can be incorporated into the lesson to further enhance understanding of key concepts.

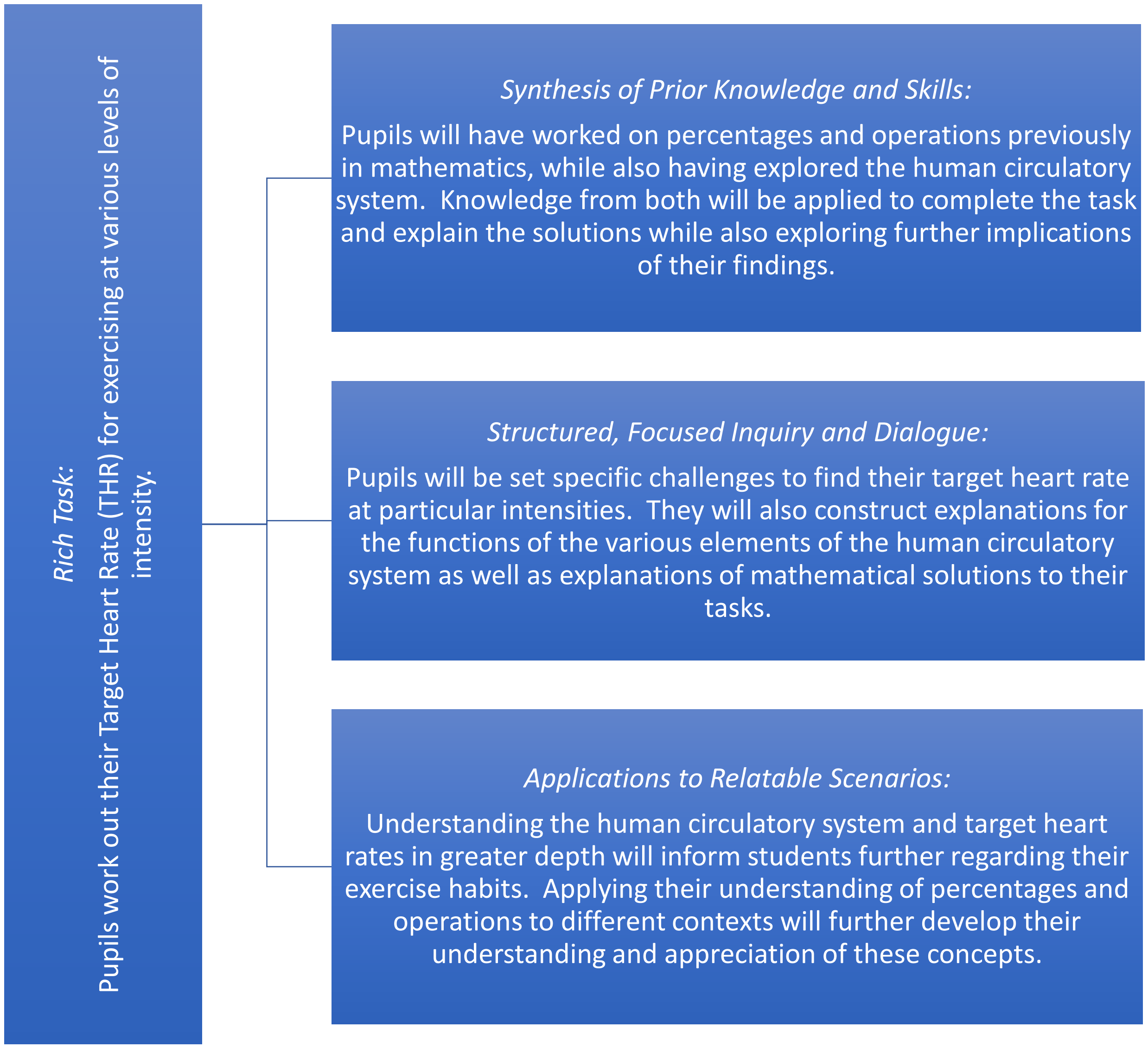

Figure 2: AIMS model applied to the design of a lesson focussing on the application of the 'Karvonen Method' to calculate target heart rate during exercise.

Engaging with lesson ideas and outlines for integrating mathematics and science has been observed to positively impact interdisciplinary pedagogical content knowledge thus better preparing teachers for engaging with transdisciplinary lessons (An 2017). Providing materials of this nature and a research-informed model for designing transdisciplinary lessons along with professional development and support should effectively equip educators to overcome a range of barriers to integration explored previously. Delivering a lesson of this nature approximately every 4-5 weeks would appear to be best to allow teachers to identify opportunities within a typically inflexible school timetable to engage in team teaching without much disruption. A monthly cycle would also ensure students regularly engage with transdisciplinary lessons in a sustainable manner to complement typical single-subject learning.

Conclusion

Adopting the AIMS model when integrating mathematics and science provides opportunities to apply and combine prior learning to new contexts in a meaningful manner. Team teaching of these lessons by mathematics and science teachers every 4-5 weeks should be viable, even when considering the restrictions of a typical timetable. Utilising a Lesson Study approach involving both mathematics and science teachers will also provide a research-informed structure to enhance cyclical planning, delivery, and reflection upon transdisciplinary lessons. Such an approach ensures that any gaps in teacher knowledge within either subject can be overcome in the planning and delivery of these lessons. Similarly, it provides opportunities for teachers to engage in professional learning through ‘boundary crossing’ which should enhance their overall capabilities. Positioning these lessons at monthly intervals allows for plenty of time to plan and review while also ensuring that there is a suitable balance between single-subject instruction and lessons which are integrative in nature.

The AIMS model offers opportunities for teachers to assess the learning that has taken place in previous single-subject lessons. The challenge of retrieving previously learned material both tests and strengthens students’ retention of that material (Pashler et al., 2007), while the challenge of applying this material in a new context provides the teacher with an insight into the depth of student understanding. Similarly, teachers and students often mistakenly rely upon their performance during acquisition of knowledge and skills as an indicator of the associated long-term learning (Soderstrom & Bjork, 2015). A wealth of empirical evidence indicates that significant changes in performance regularly fail to translate into corresponding variations in learning and, conversely, that substantial learning can occur without the presence of any performance gains (Soderstrom & Bjork, 2015). It is recommended that learners should be provided with regular opportunities to revisit material previously studied to strengthen retention and understanding as alluded to previously. Integrating mathematics and science in the manner outlined here would provide such opportunities on a regular basis.

The challenge of integrating mathematics and science (or any other subjects) is fraught with a plethora of barriers. These include deficiencies in teacher knowledge of their non-specialist subject, the rigid nature of the school timetable, lack of instructional models and materials, and the need to maintain single-subject instruction in a suitable balance with any attempts at integrative learning experiences. Applying the research-informed AIMS model to the design and delivery of transdisciplinary lessons which integrate mathematics and science within a Lesson Study structure should counteract most or potentially all of these barriers. Providing a range of instructional resources, professional support, and positioning these lessons at 4-5 week intervals provides a platform for teachers to engage in the Lesson Study process effectively while also ensuring transdisciplinary lessons regularly complement single-subject instruction. This may provide the blueprint for establishing a steady diet of lessons which integrate mathematics and science in secondary schools rather than depending on erratic engagements which may be beneficial but rarely sustained.

Competing interests

The author declares that he has no competing interests.

Data Availability

Data sharing not applicable to this article as no datasets were generated or analysed during the current study.

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